Fördelen med det andra beviset är att det lätt kan generaliseras till $\sum_{k=1}^{n} k^2$

Sats $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

B: $(k+1)^3 = k^3+3k^2+3k+1 \Rightarrow (k+1)^3 - k^3 = 3k^2+3k+1$

$$\Rightarrow (n+1)^3 - 1^3 = (n+1)^3 + n^3 + (n-1)^3 + \dots + 3^3 + 2^3 - \left(n^3+(n-1)^3+\dots+2^3+1^3\right)$$

$$= \sum_{k=1}^{n}(k+1)^3 - \sum_{k=1}^{n} k^3 = \sum_{k=1}^{n}\left((k+1)^3 - k^3\right) = \sum_{k=1}^{n} 3k^2+3k+1 =$$

$$= 3\sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 = \underline{3\sum_{k=1}^{n} k^2 + 3\,\frac{n(n+1)}{2} + n}$$

$$\therefore \sum_{k=1}^{n} k^2 = \frac{1}{3}\left((n+1)^3 - 1 - \frac{3\,n(n+1)}{2} - n\right) =$$

$$= \frac{1}{3}\left(n^3+3n^2+3n+1-1-n - \frac{3\,n(n+1)}{2}\right) =$$

$$= \frac{1}{3}\left(n^3+\frac{3}{2}n^2+\frac{1}{2}n\right) = \frac{n\left(n^2+\frac{3}{2}n+\frac{1}{2}\right)}{3} = \frac{n(2n^2+3n+1)}{6} =$$

$$= \frac{n(n+1)(2n+1)}{6} \quad \blacksquare$$

Sats (Geometrisk summa)

$$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1} = \frac{1-x^{n+1}}{1-x}$$

B: $S = \sum_{k=0}^{n} x^k = 1+x+x^2+\dots+x^{n-1}+x^n$; $xS = x+x^2+\dots+x^n+x^{n+1}$

$$\therefore S - xS = (1-x)S = 1+x+\dots+x^{n-1}+x^n - \left(x+x^2+\dots+x^n+x^{n+1}\right) = 1-x^{n+1} \quad \blacksquare$$