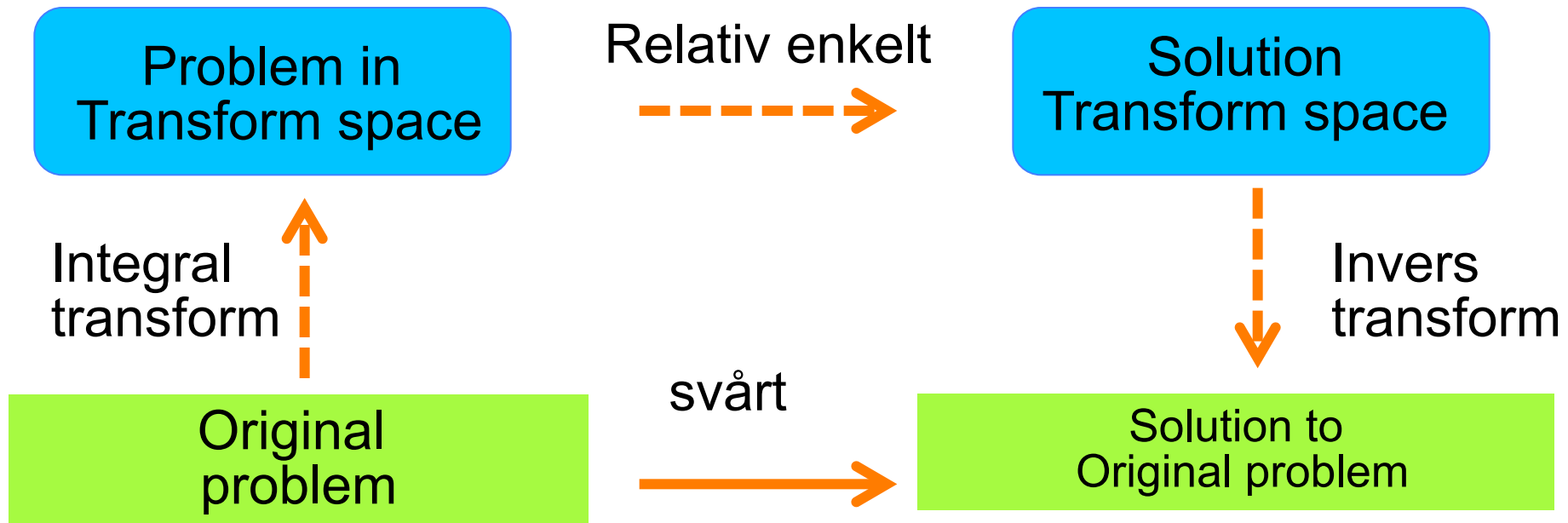


Idag

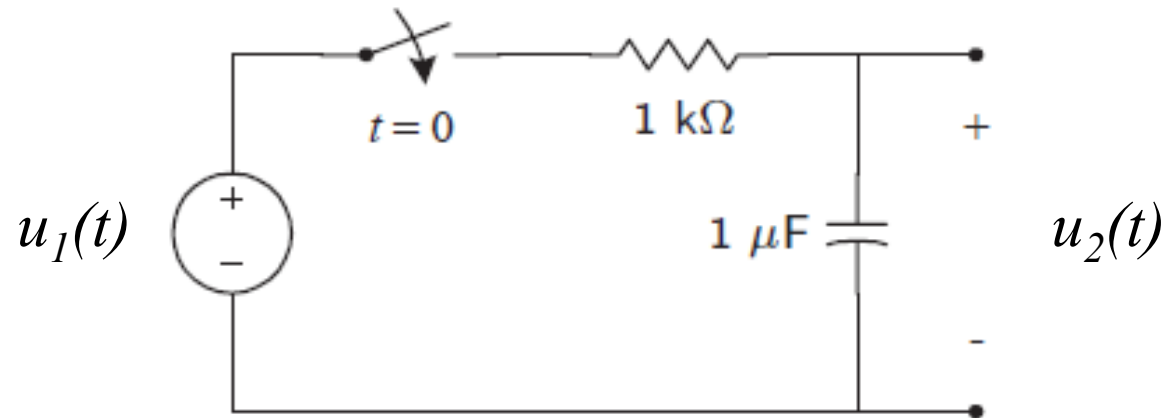
1. **System och dess överföringsfunktion**
2. **S-planet & sammanhang med frekvensfunktionen**
3. **Impulssvar**
4. **Stegsvar**

Varför är (Fourier/Laplace) transformer bra att ha

(1) Förenklar livet



Krets

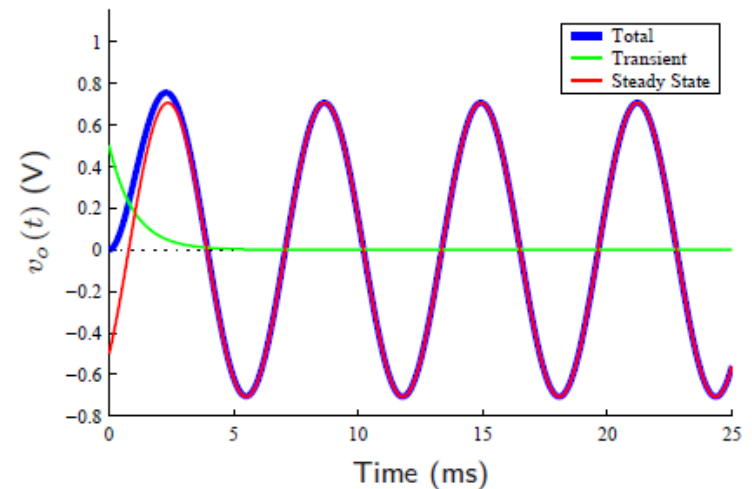


$$v_o(t) = \frac{1}{2}e^{-t/0.001} + \frac{1}{\sqrt{2}}\sin(1000t - 45^\circ)$$

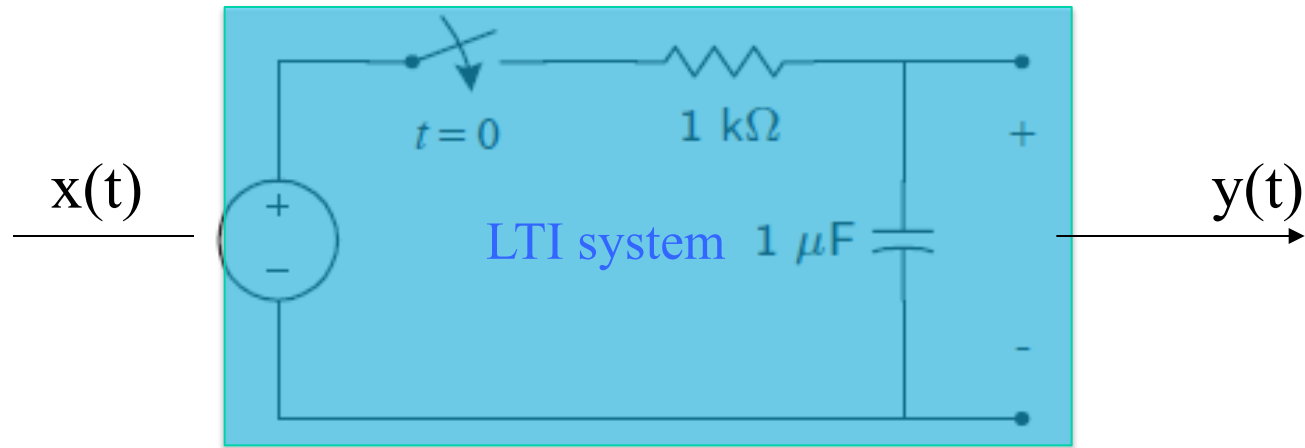
$$= v_{\text{tr}}(t) + v_{\text{ss}}(t)$$

$$v_{\text{tr}}(t) = \frac{1}{2}e^{-t/0.001}$$

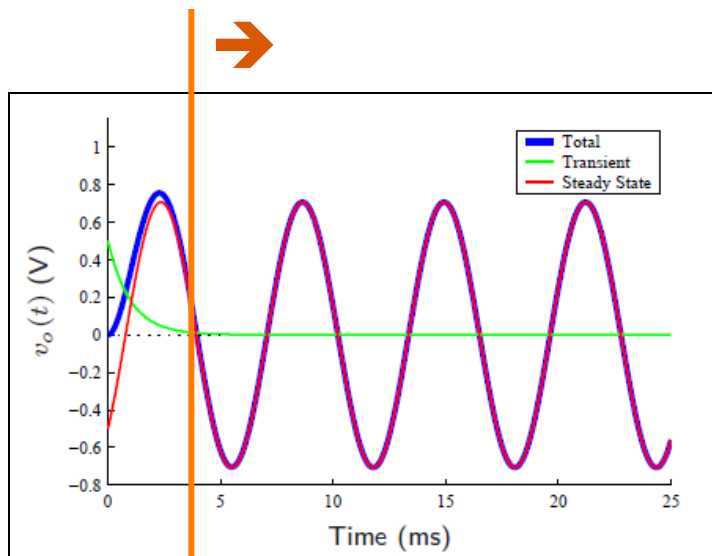
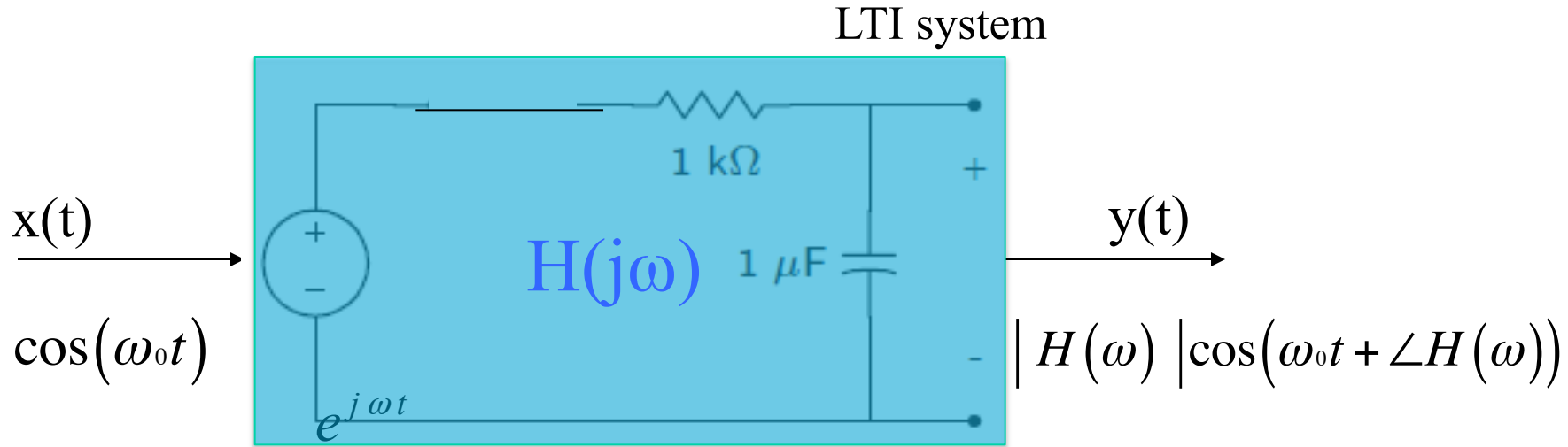
$$v_{\text{ss}}(t) = \frac{1}{\sqrt{2}}\sin(1000t - 45^\circ)$$



Krets → System

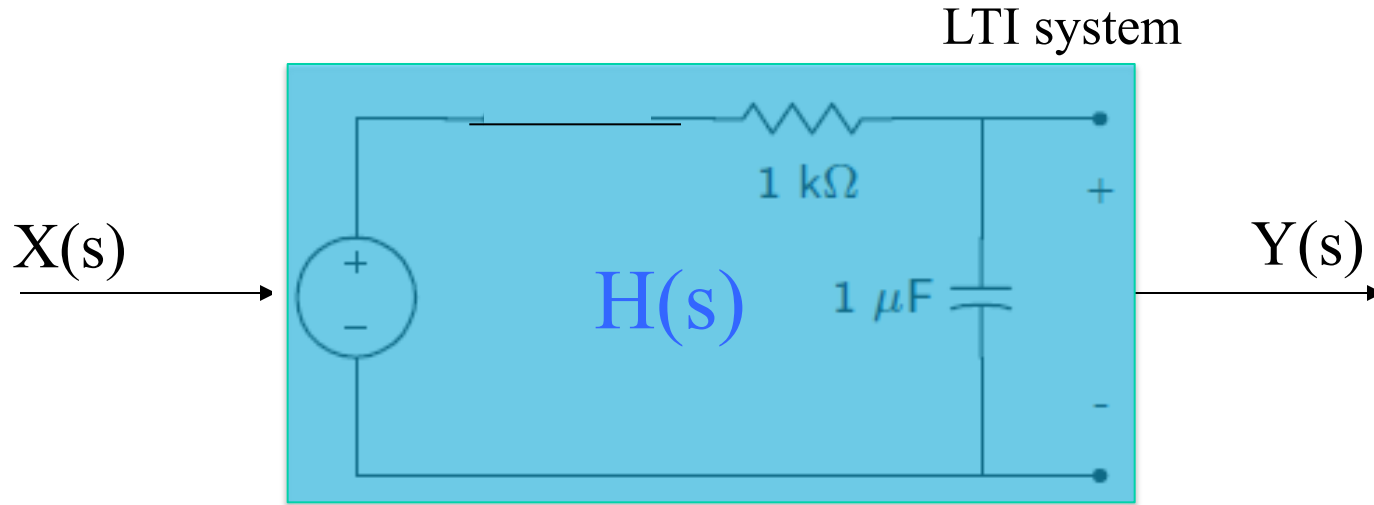


System representering: Frekvenssvår/Stationärtsvår



$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

System representering: Överförningsfunktion

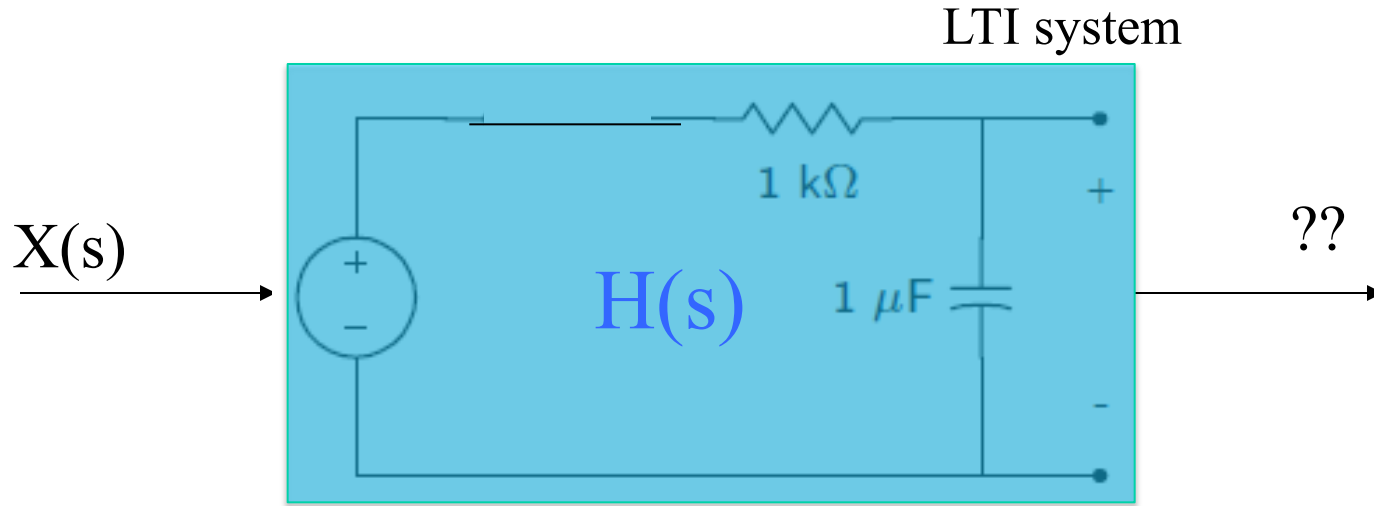


$$H(s) = \frac{Y(s)}{X(s)}$$

!Obs!

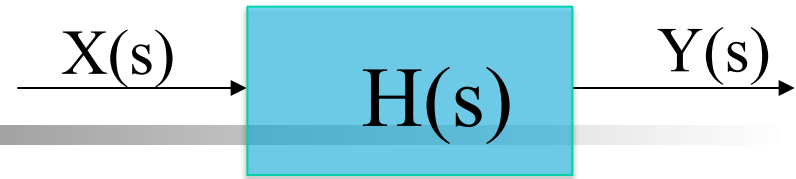
$$y(0) = y'(0) = y''(0) = y^{(3)}(0) = \dots = y^{(n-1)}(0) = 0$$
$$x(0) = x'(0) = x''(0) = x^{(3)}(0) = \dots = x^{(n-1)}(0) = 0$$

System representering: Överförningsfunktion



$$Y(s) = H(s)X(s)$$

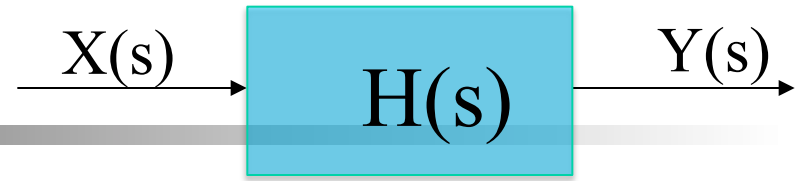
Exempel



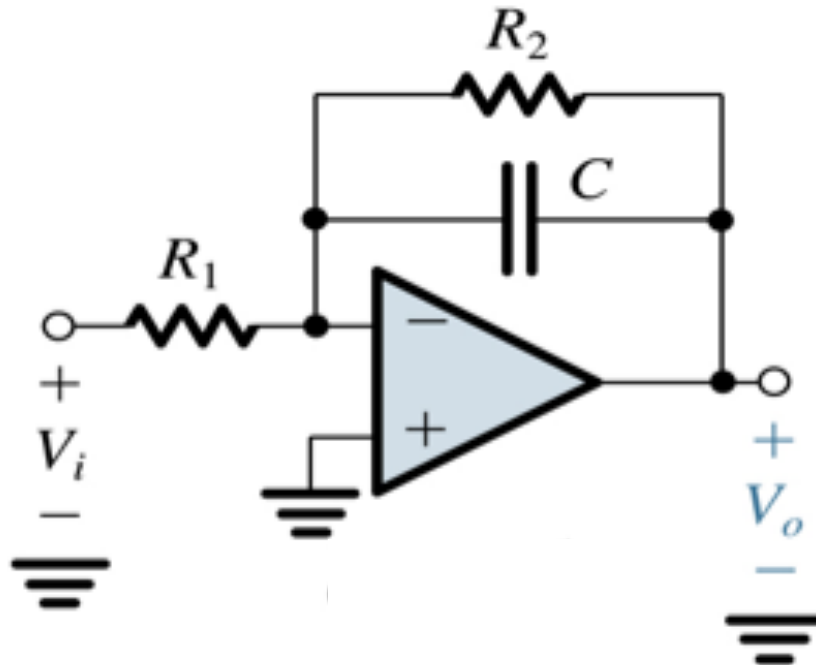
Bestäm $H(s)$ för system som beskriv av differentialekvationen:

$$y''(t) + 4y'(t) + 10y(t) = x'(t) + 2x(t)$$

Exempel (2)



Bestäm $H(s)$ för följande krets (filter):



Överförningsfunktion: Hur den ser ut

Om systemet beskrivs av en differentialekvation av typen

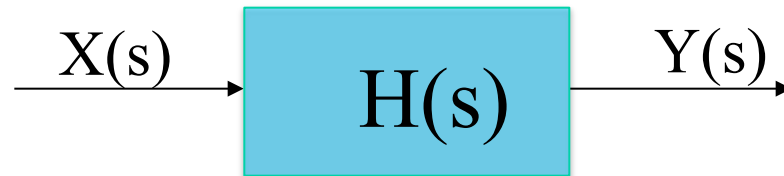
$$a_n x^{(n)} + a_{n-1} x^{(n-1)} + \dots + a_0 x = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \dots + b_0 u$$

Så kommer överföringsfunktionen att vara en rationell funktion i s

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}$$

Den är ett viktigt verktyg för att studera systemets egenskaper.

Undersökning av S-plan



$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}$$

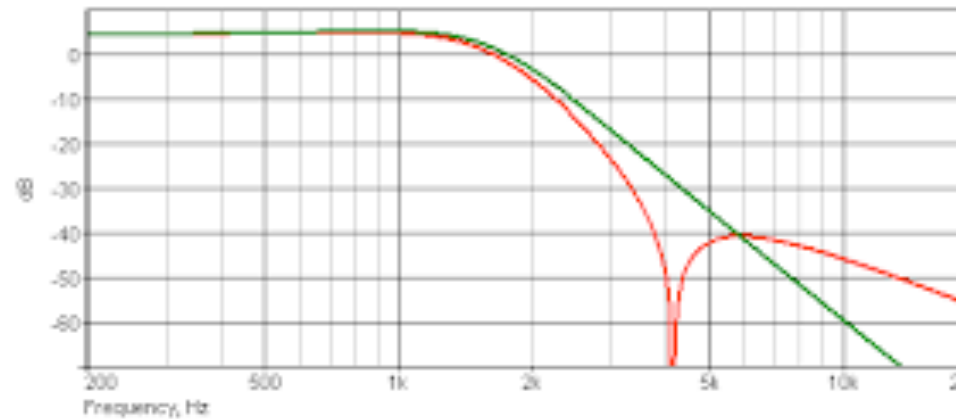
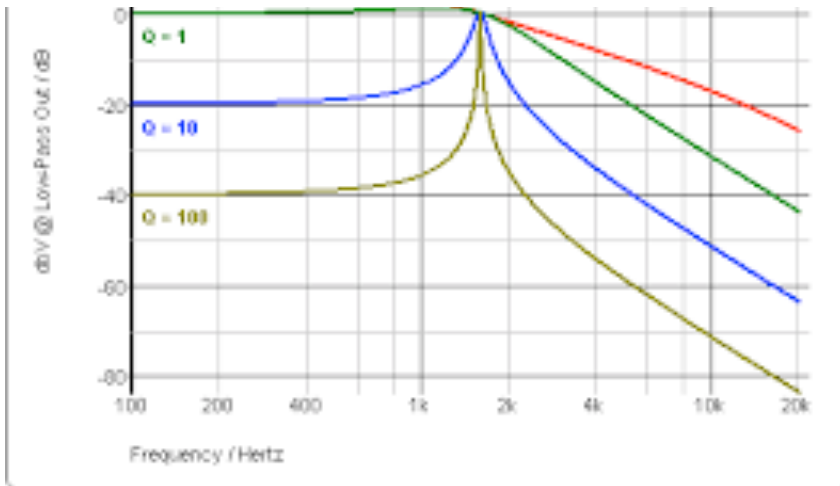
nollställe ↴

$$H(s) = \frac{K(s - z_1)(s - z_2) \dots (s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_n)}$$

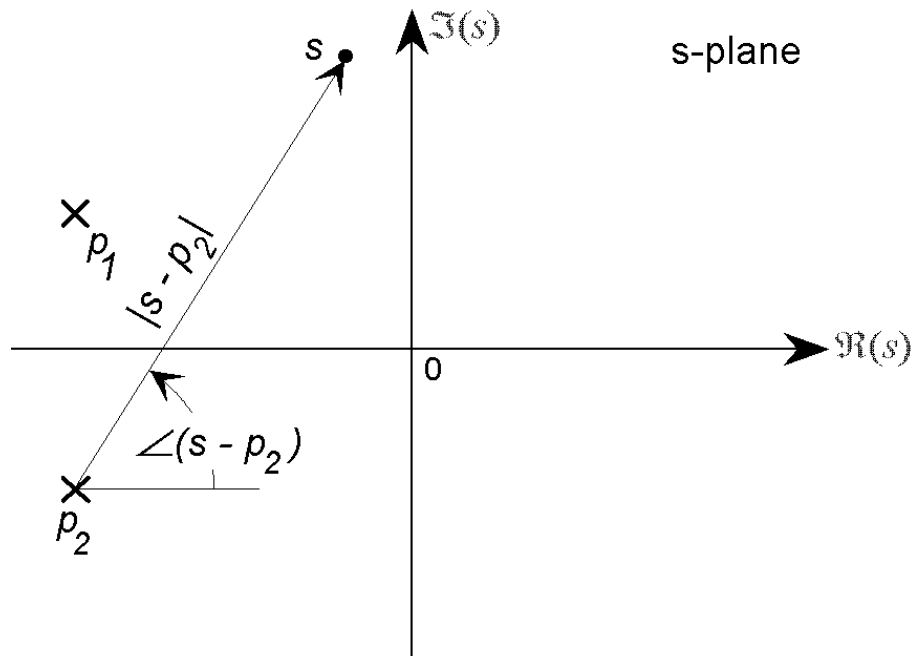
pol ↴

Frekvensfunktion & Bode

$H(j\omega)$ av en 1:a ordning system



Beloppsyttta & fasytta

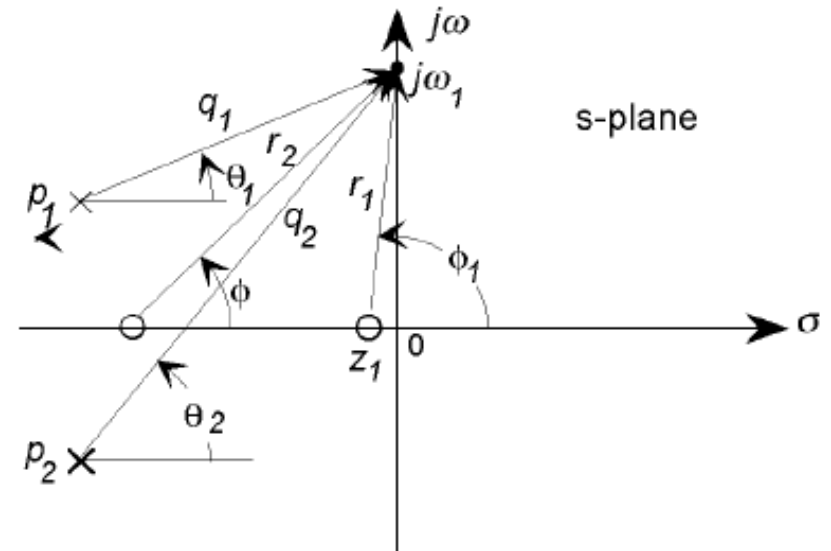
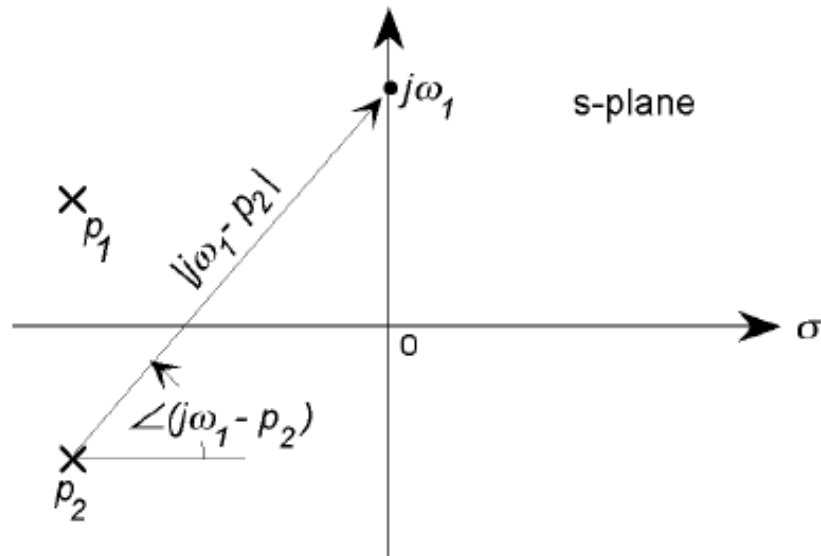


$$|s - p_i| = \sqrt{(\sigma - \sigma_i)^2 + (\omega - \omega_i)^2},$$
$$\angle(s - p_i) = \tan^{-1} \left(\frac{\omega - \omega_i}{\sigma - \sigma_i} \right)$$

$$|H(s)| = K \frac{\prod_{i=1}^m |(s - z_i)|}{\prod_{i=1}^n |(s - p_i)|}$$

$$\angle H(s) = \sum_{i=1}^m \angle(s - z_i) - \sum_{i=1}^n \angle(s - p_i)$$

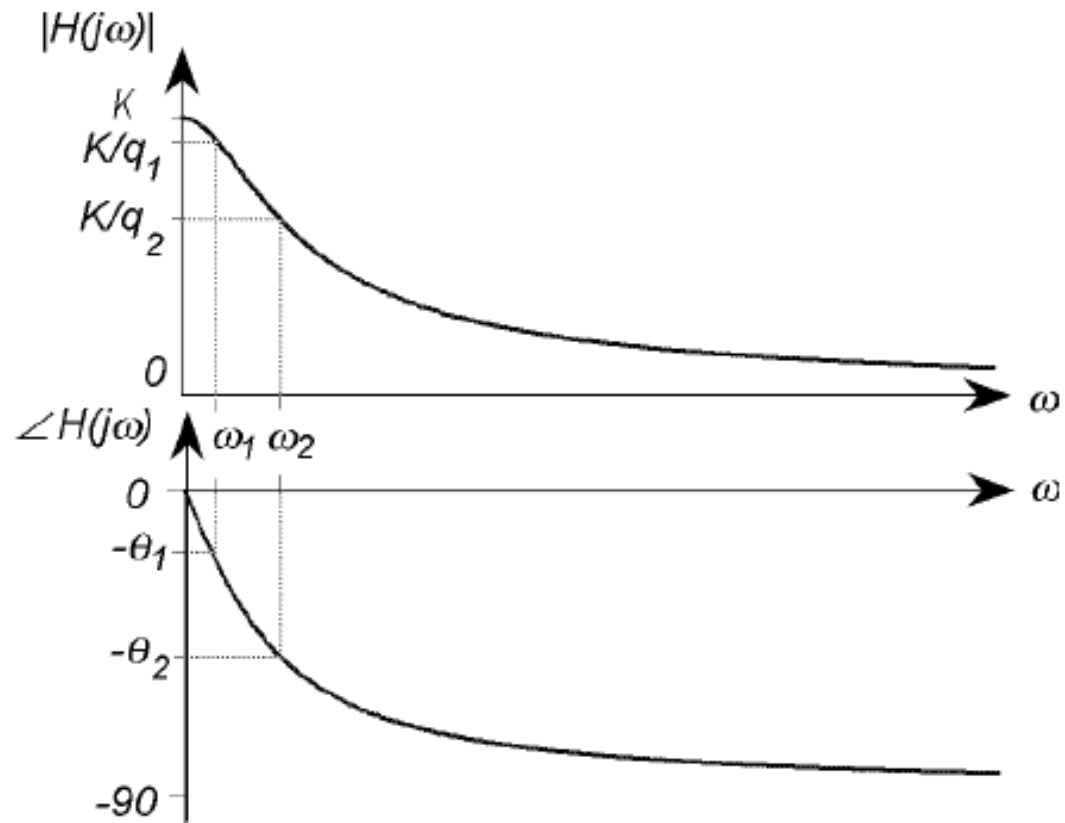
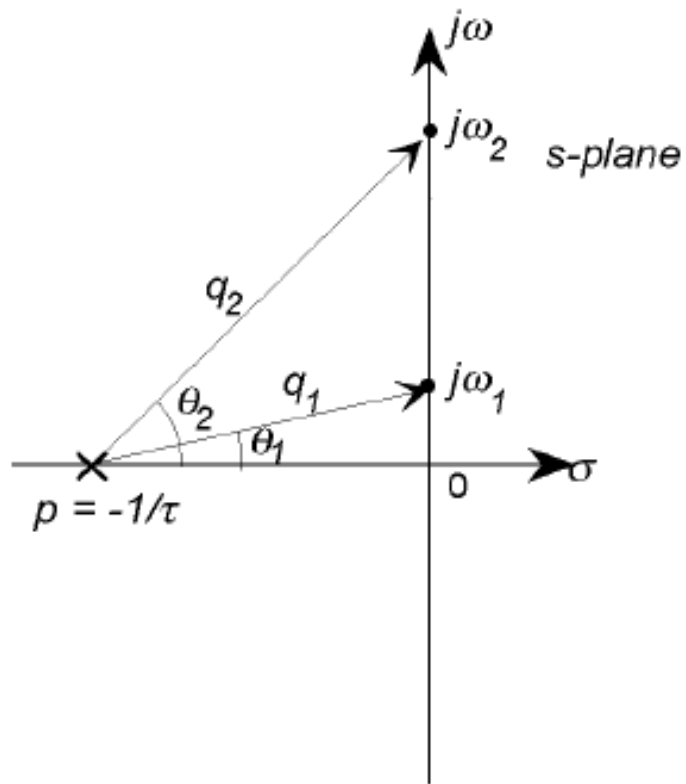
S-plan & Frekvensfunktion



$$|H(j\omega)| = K \frac{r_1 \dots r_m}{q_1 \dots q_n}$$

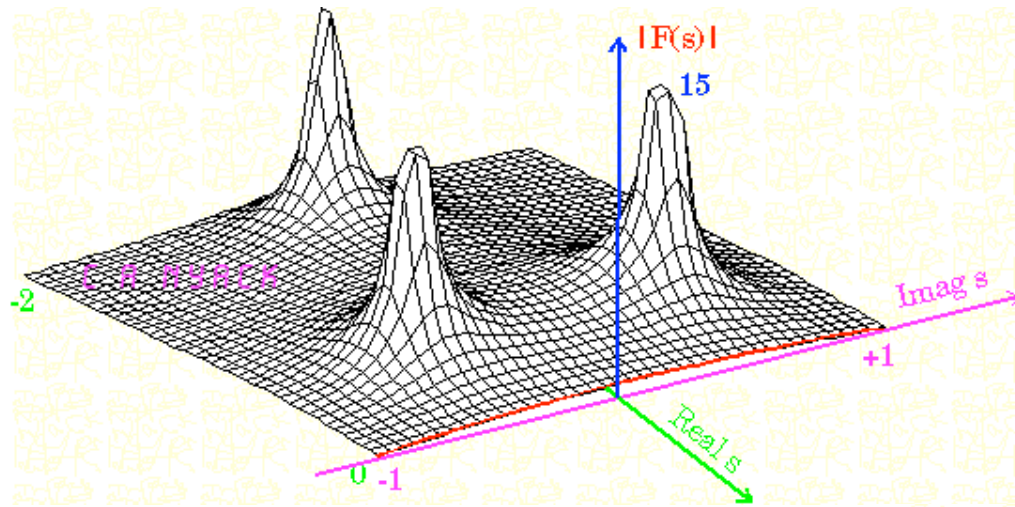
$$\angle H(j\omega) = (\phi_1 + \dots + \phi_m) - (\theta_1 + \dots + \theta_n)$$

S-plan & Frekvensfunktion

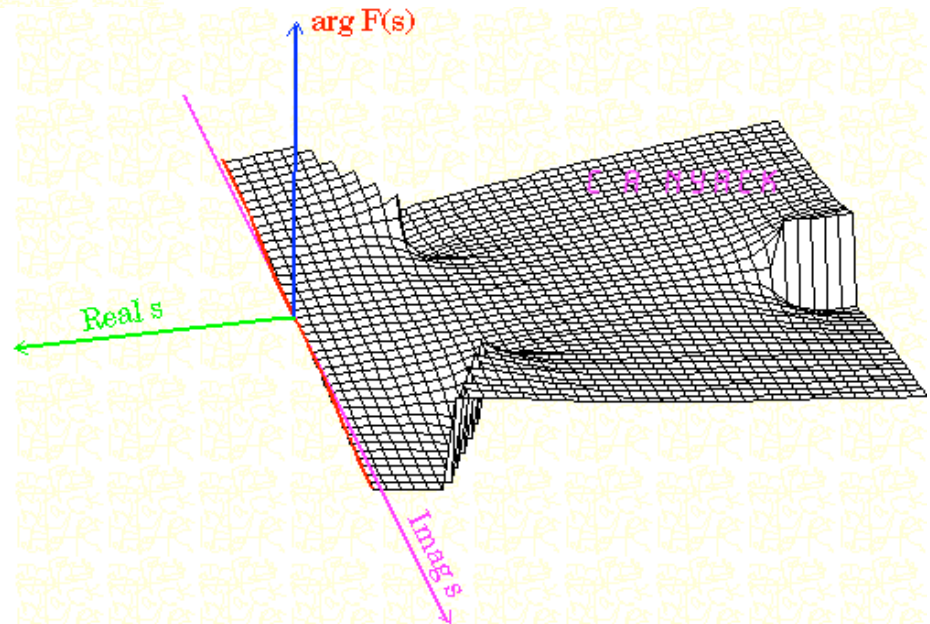


Poler & nolor i s-plan

Beloppsyttan



Fasyttan



1 reel och 2 komplexa poler

<http://www.jhu.edu/%7Esignals/>

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Signals
Systems
Control

demonstrations

**Joy of Convolution**

A Java applet that performs graphical convolution of continuous-time signals on the screen. Select from provided signals, or draw a signal with the mouse. Includes an audio introduction with suggested exercises and a multiple-choice quiz. (Prepared by Steven Crutchfield, Fall 1996.)

**Joy of Convolution (Discrete Time)**

A Java applet that performs graphical convolution of discrete-time signals on the screen. Select from provided signals, or draw a signal with the mouse. Includes an audio introduction with suggested exercises and a multiple-choice quiz. (Original applet by Steven Crutchfield, Summer 1997, is available [here](#). Update by [Michael Ross](#), Fall, 2001.)

**Interactive Lecture Module: Continuous-Time LTI Systems and Convolution**

A combination of Java Script, RealAudio, technical presentation on the screen, and Java applets that is used at Johns Hopkins to complement classroom lectures on the discrete-time case. (Applets by Steven Crutchfield, interface by Mark Nesky, Spring 1998.)

**Fourier Series Approximation**

A Java applet that displays Fourier series approximations and corresponding magnitude and phase spectra of a periodic continuous-time signal. You can select from provided signals, or draw a signal with the mouse. (Original Applet by Steven Crutchfield, Fall 1996, update by [Hsi Chen Lee](#) Summer, 1999.)

**Listen to Fourier Series**

Sound is used to introduce basic notions of Fourier series, including harmonic content and filtering. If you have a RealAudio player, click [here](#) for a

Internet

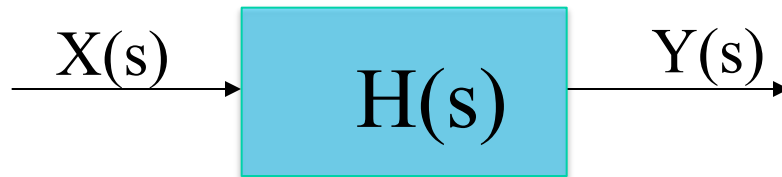
Analys



Design

Varför är (Fourier/Laplace) transformer bra att ha

(2) Tillåter systemdesign för en 'fågel'perspektiv



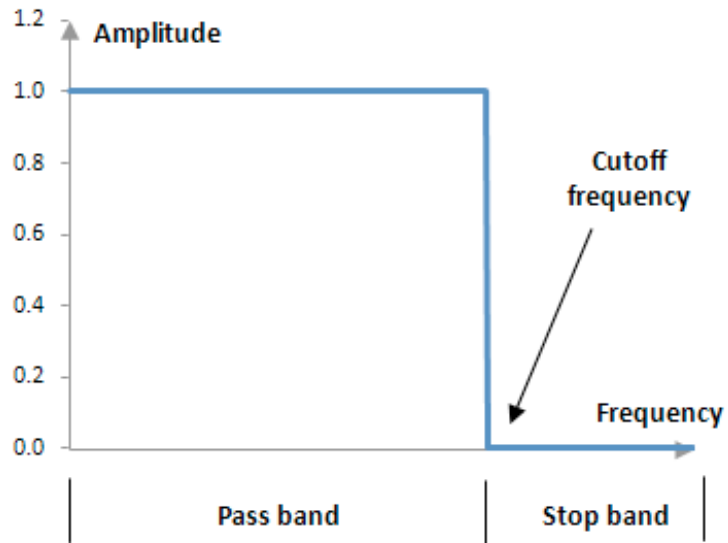
Jag vill ha en krets som filtrerar bort alla frekvenser över 100KHz

(a) Jag tillåter ingen översväng

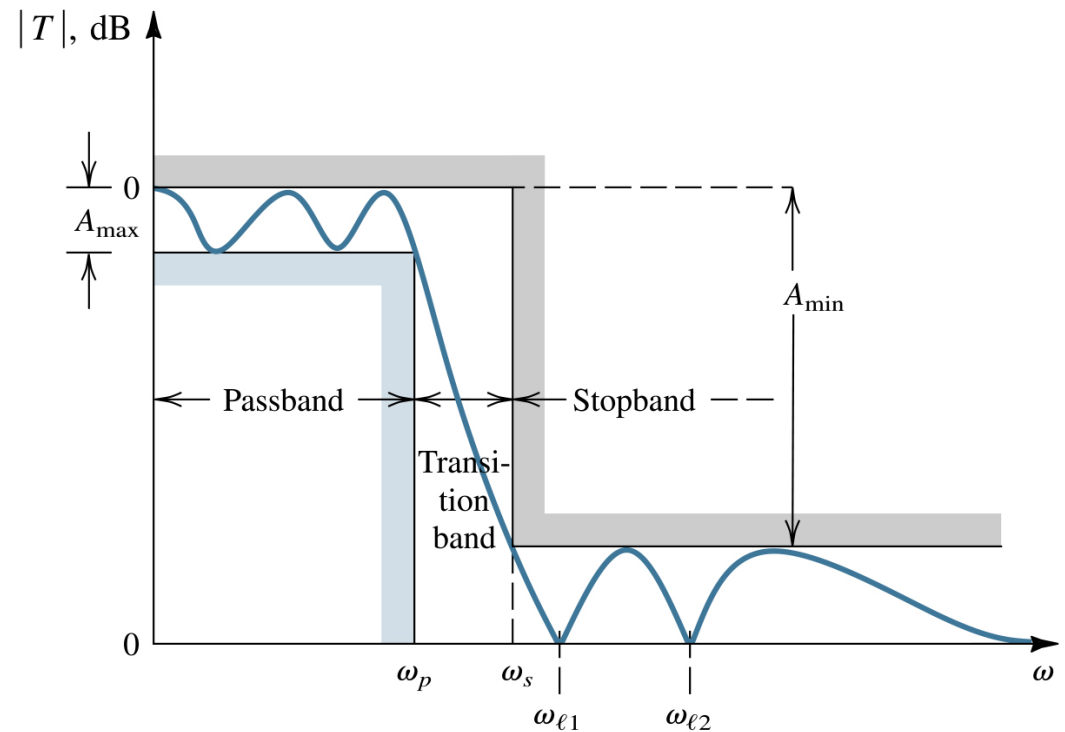
(b) Liten översväng är ok, men redan $f=101\text{kHz}$ måste filtreras bort

Filter

Ideal filter

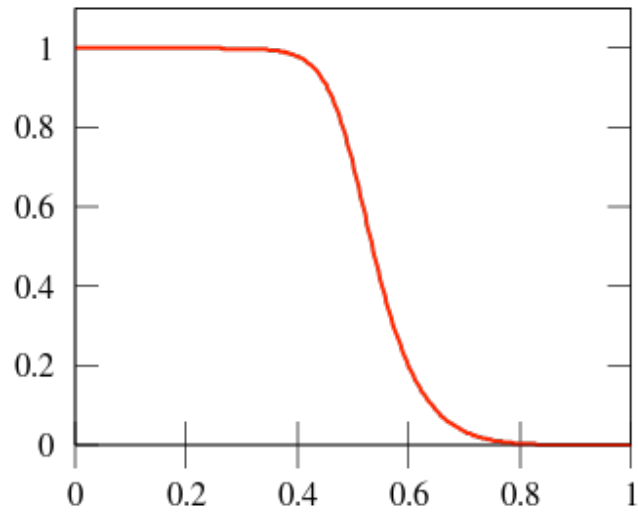


Real filter

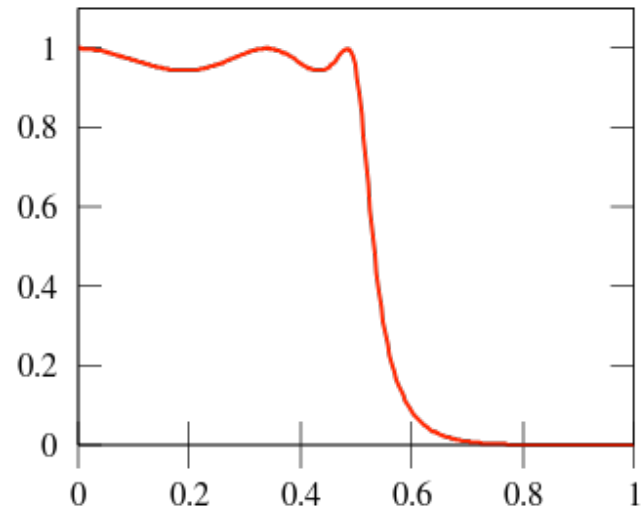


Filter

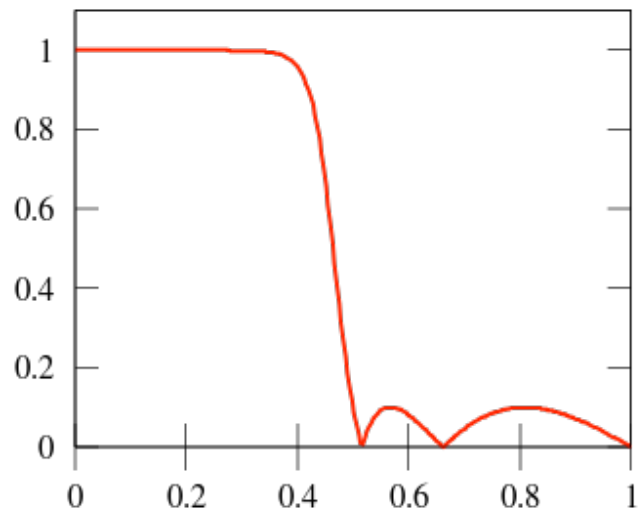
Butterworth



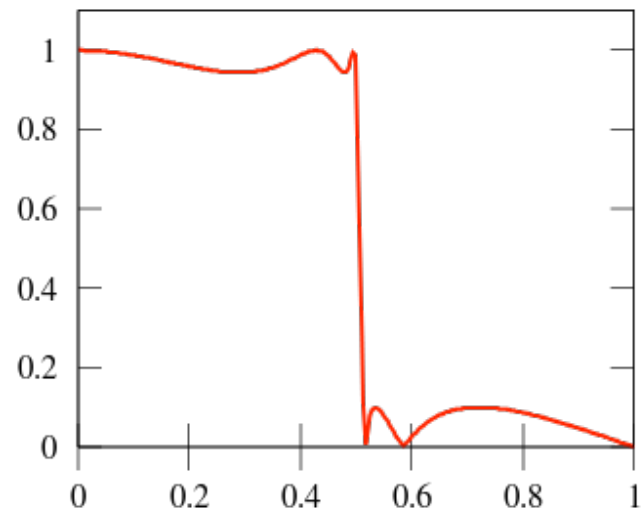
Chebyshev type 1



Chebyshev type 2

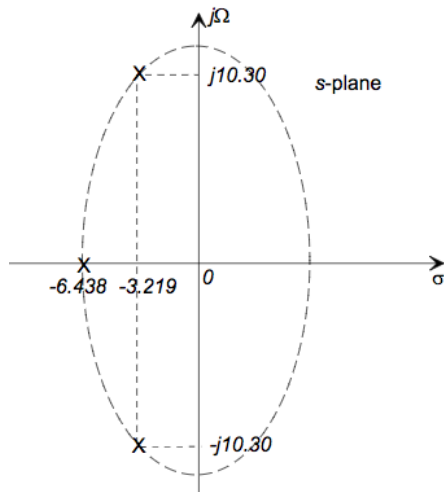
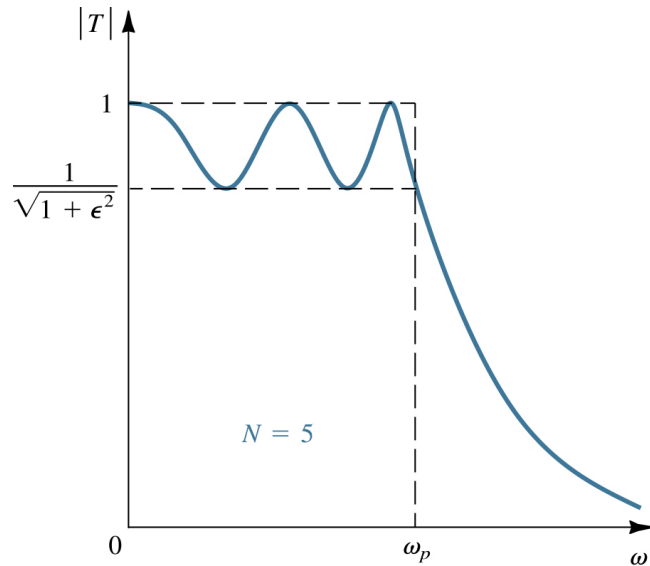


Elliptic

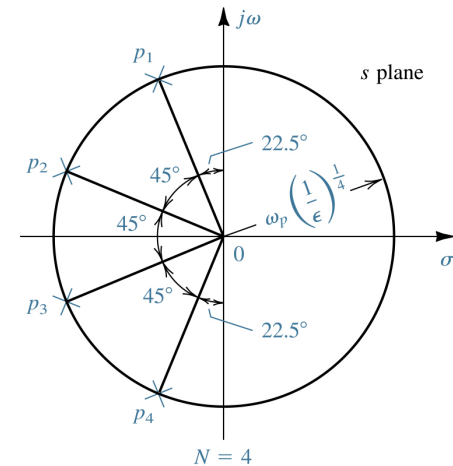
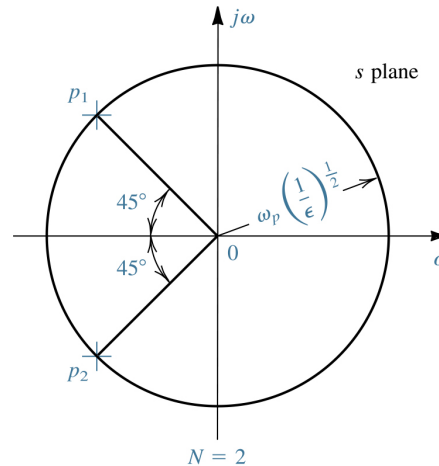
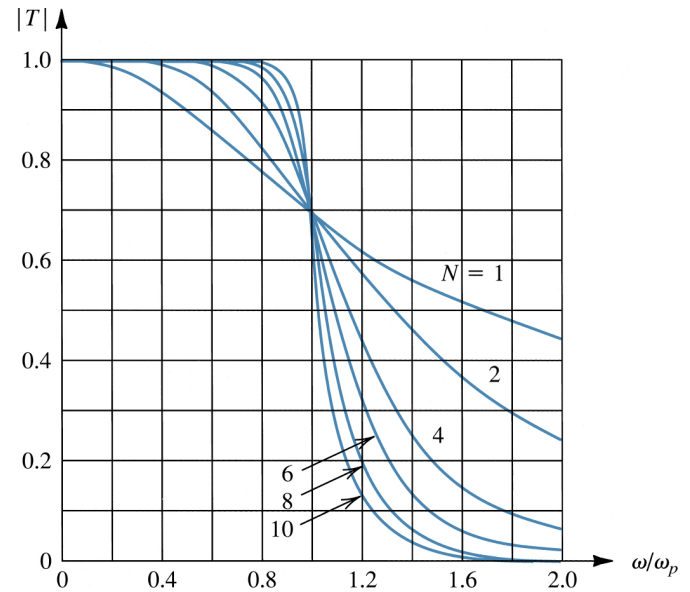


Filter design: polplacering

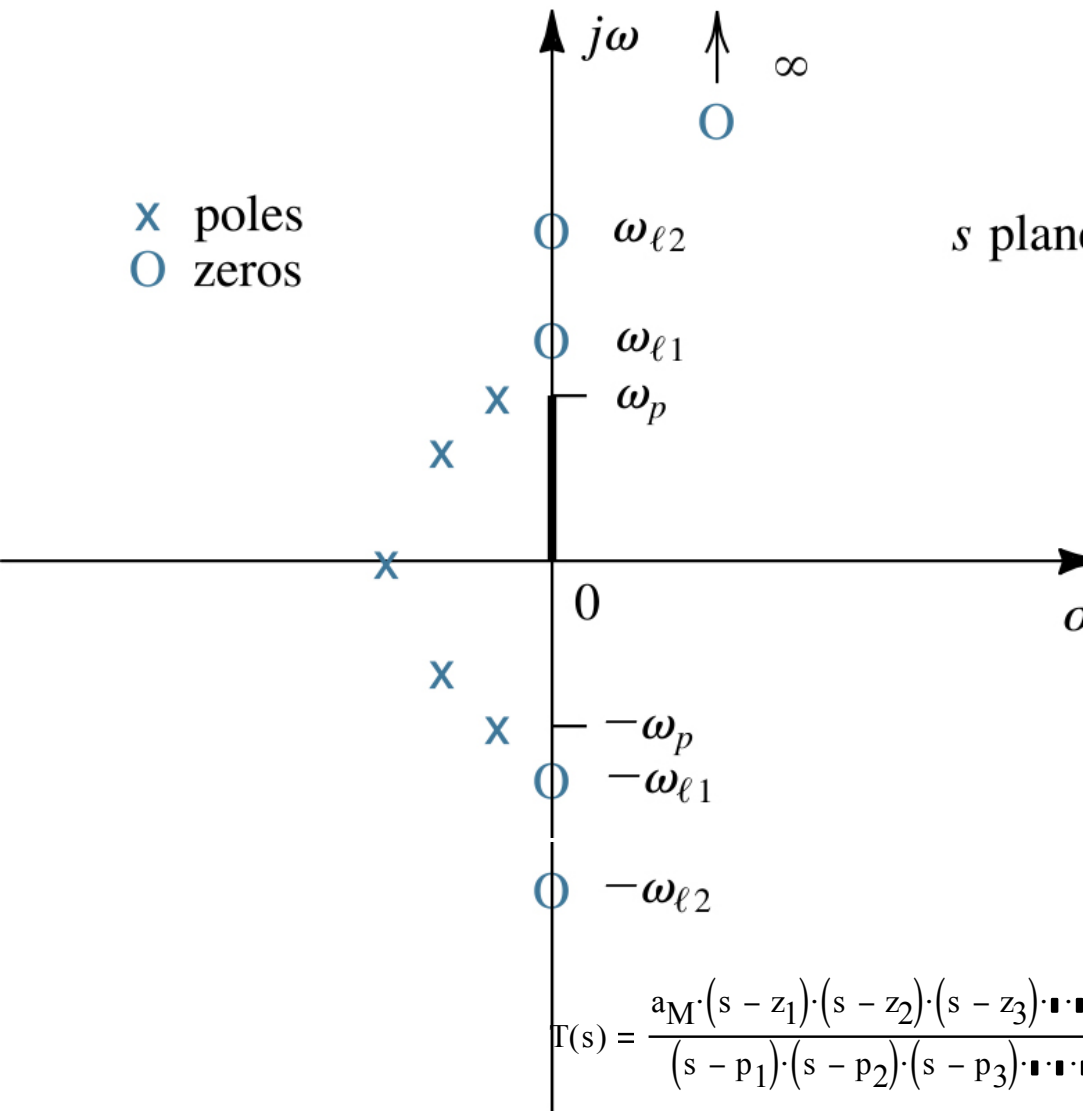
Chebyshev



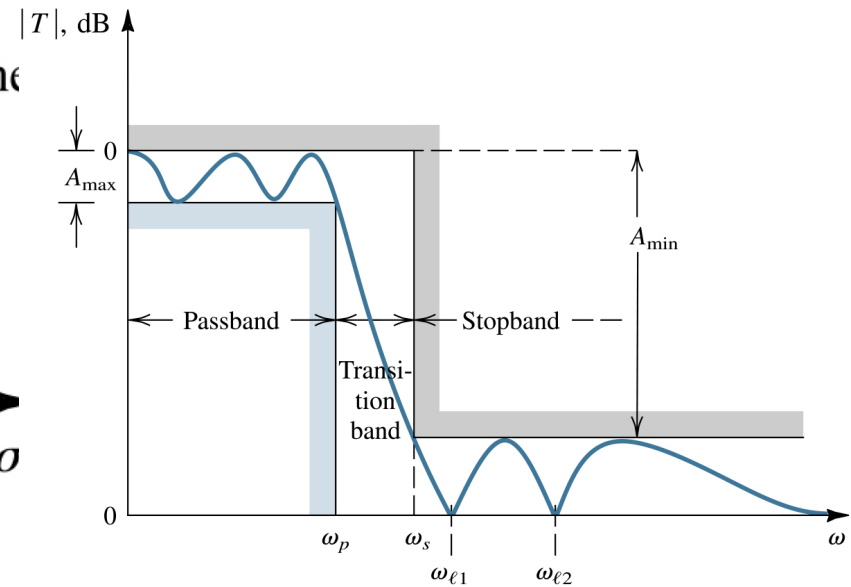
Butterworth



Filter



s plane



transfer function zeros or transmission zeros

transfer function poles or the natural poles

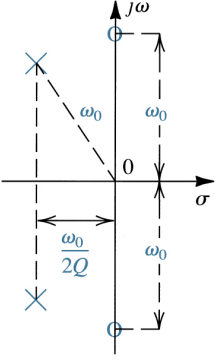
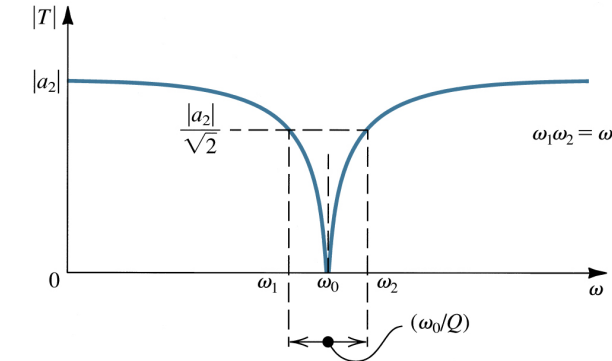
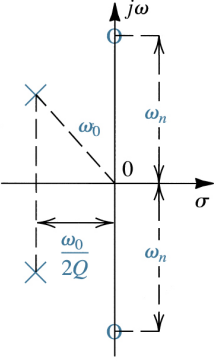
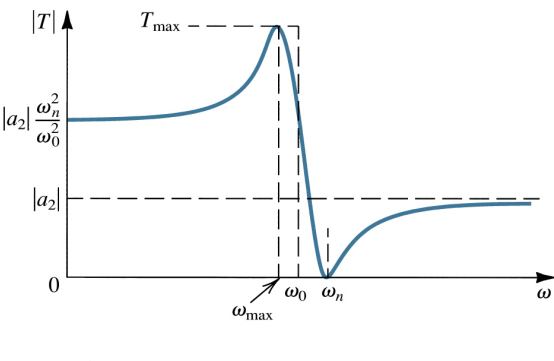
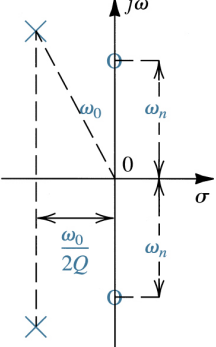
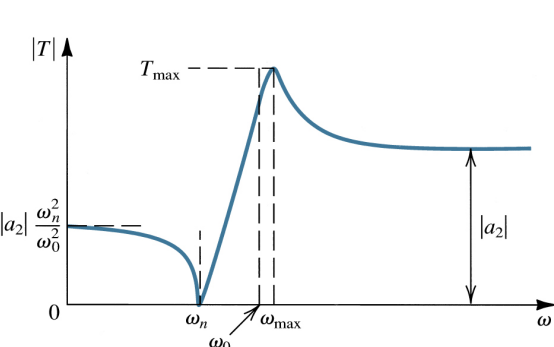
Filter design: eksempel på 1:a ordning filtrar

Filter Type and $T(s)$	s -Plane Singularities	Bode Plot for $ T $	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$			$CR = \frac{1}{\omega_0}$ dc gain = 1	$CR_2 = \frac{1}{\omega_0}$ dc gain = $-\frac{R_2}{R_1}$
(b) High-Pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$			$CR = \frac{1}{\omega_0}$ High-frequency gain = 1	$CR_1 = \frac{1}{\omega_0}$ High-frequency gain = $-\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			$(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_0}{a_1}$ dc gain = $\frac{R_2}{R_1 + R_2}$ HF gain = $\frac{C_1}{C_1 + C_2}$	$C_2 R_2 = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ dc gain = $-\frac{R_2}{R_1}$ HF gain = $-\frac{C_1}{C_2}$

Filterdesign: eksempel på 2:a ordning filter

Filter Type and $T(s)$	s -Plane Singularities	$ T $
(a) Low-Pass (LP) $T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ dc gain = $\frac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain = a_2		
(c) Bandpass (BP) $T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain = $\frac{a_1 Q}{\omega_0}$		

Filterdesign: eksempel på 2:a ordning filter forts...

(d) Notch $T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ dc gain = high-frequency gain = a_2		
(e) Low-Pass Notch (LPN) $T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$ high-frequency gain = a_2		 $\omega_{\max} = \omega_0 \sqrt{\frac{\left(\frac{\omega_n^2}{\omega_0^2}\right)\left(1 - \frac{1}{2Q^2}\right) - 1}{\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1}}$
(f) High-Pass Notch (HPN) $T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$ high-frequency gain = a_2		 $T_{\max} = \frac{ a_2 }{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$

LTI system Representering

Varje LTI system kan unikt karakterizeras med sitt

Impulssvar $h(t)$

svar av systemet till en impulsfunktionen

Frekvenssvar $H(j\omega)$:

svar av systemet till en komplex exponentiella $e^{j\omega}$ för alla frekvenser f .

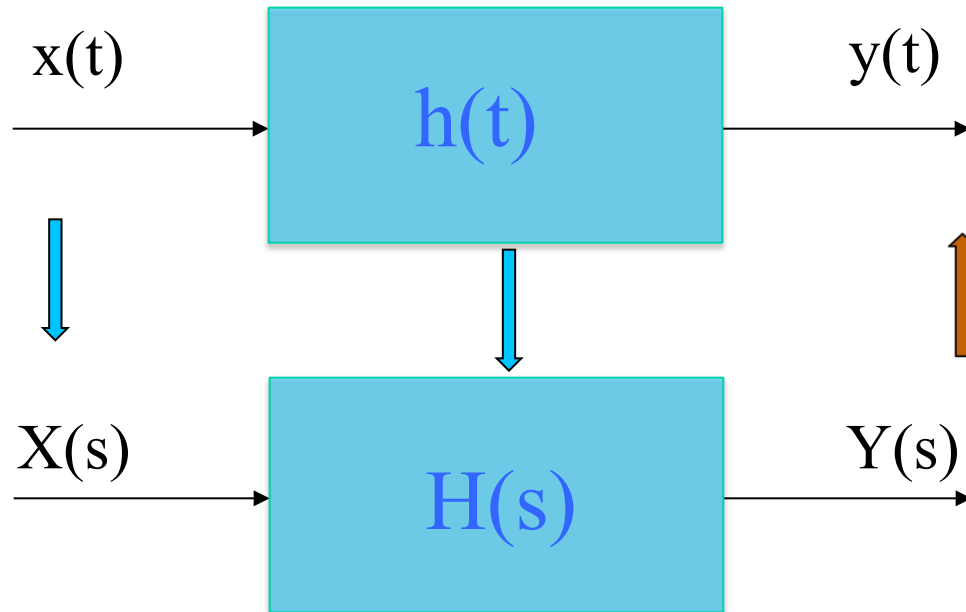
Överföringsfunktion $H(s)$:

Laplaceformen av impulssvar

Alla tre representeringar är likvärdiga

Om vi vet en representering, kan vi hitta de andra två under förutsättning att de finns

System representering



Hur bestämmer vi $H(s)$?

$$H(s) = \frac{Y(s)}{X(s)}$$

Vad är det 'enklaste' signal att skicka in som ingång?

Singularity funktioner

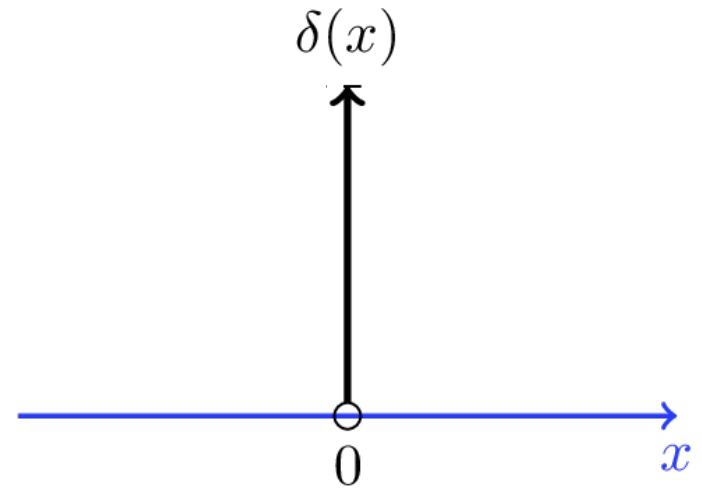
Implusfunktion (Delta-funktionen)

$$x(t) = \delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

Två viktiga egenskaper

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$

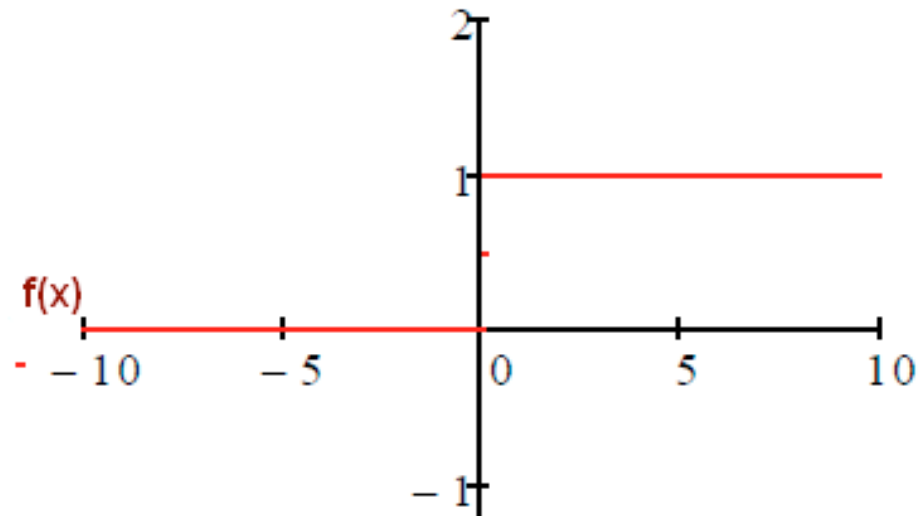
$$\int_{-\infty}^{\infty} f(t) \delta(t - t_0) dt = f(t_0)$$



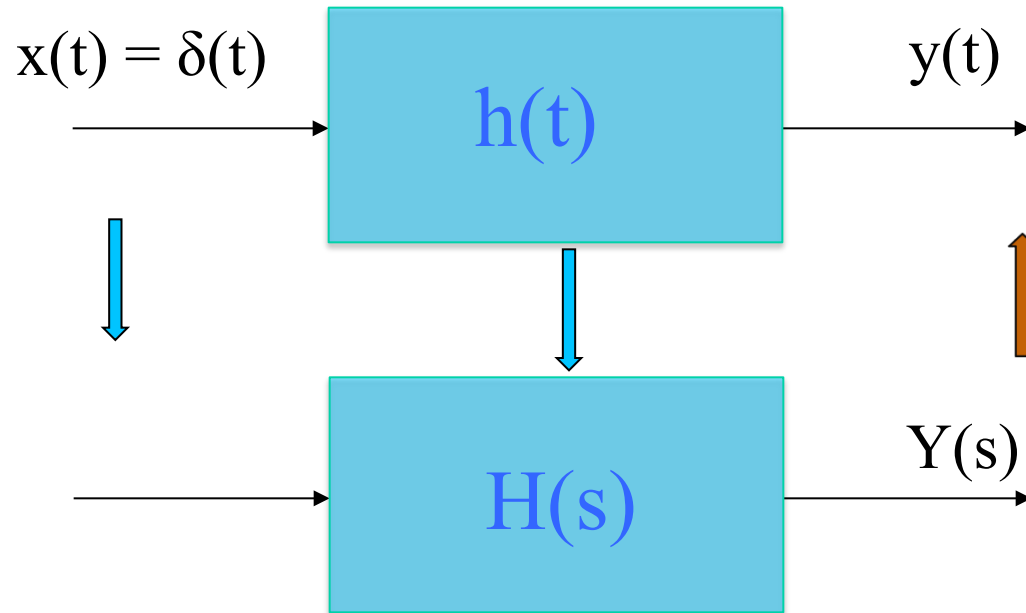
Stegfunktion (Trappfunktion)

$$x(t) = u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$x(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



System representering: Impulssvår



Laplacetransform av impulsfunktion

För att ta Laplacetransformen av impulsfunktionen $\delta(t)$ så modifierar vi definitionen så att

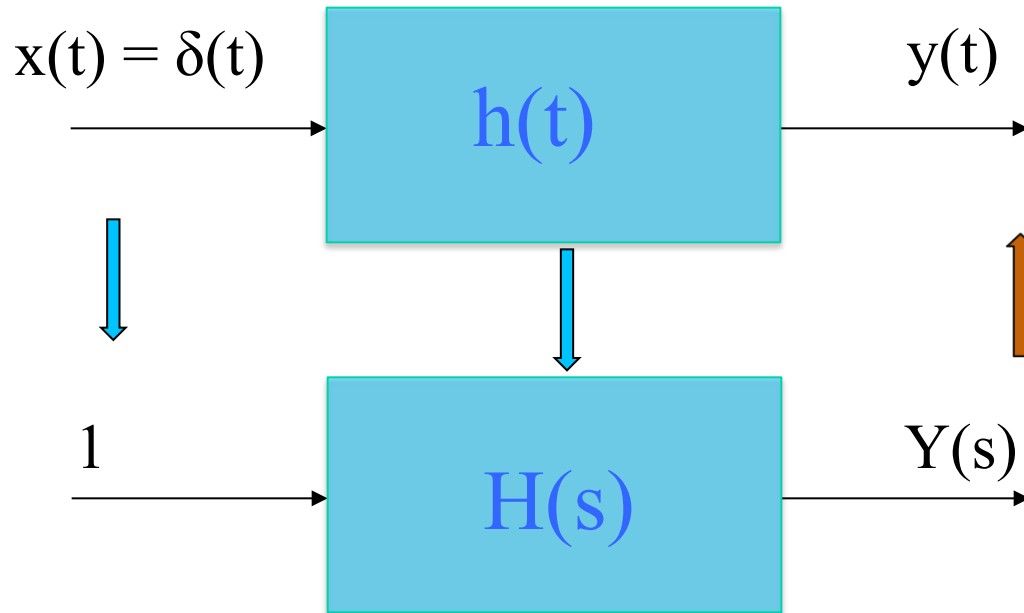
$$f(t) \rightarrow \int_{0^-}^{\infty} f(t)e^{-st} dt$$

där 0^- betyder att man skall ta gränsvärdet då från vänster (dvs för $t \rightarrow 0$)

Egenskaperna hos impulsfunktionen ger då att

$$\delta(t) \rightarrow \int_{0^-}^{\infty} \delta(t)e^{-st} dt = \int_{0^-}^{\infty} \delta(t) dt = 1$$

System representering: Impulssvår

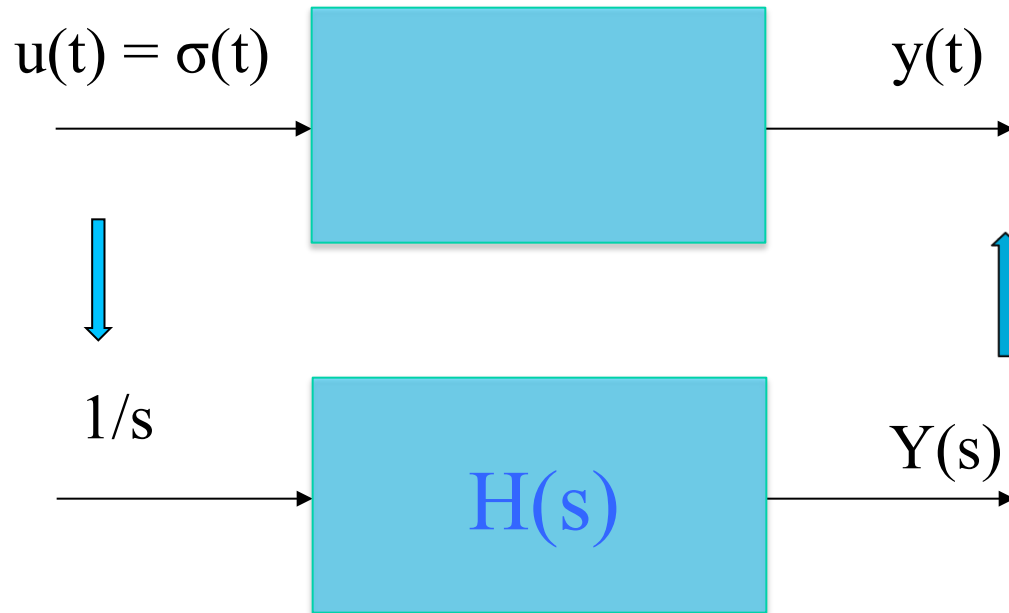


$$Y(s) = 1H(s)$$

Alternativ definition av $H(s)$

Beskriver hur en linjärsystem svarar på en impuls

System representering: Stegsvår



$$Y(s) = \frac{1}{s} H(s)$$

Beskriver systemets egenskaper vid plötsliga stegvisa förändringar hos signalen