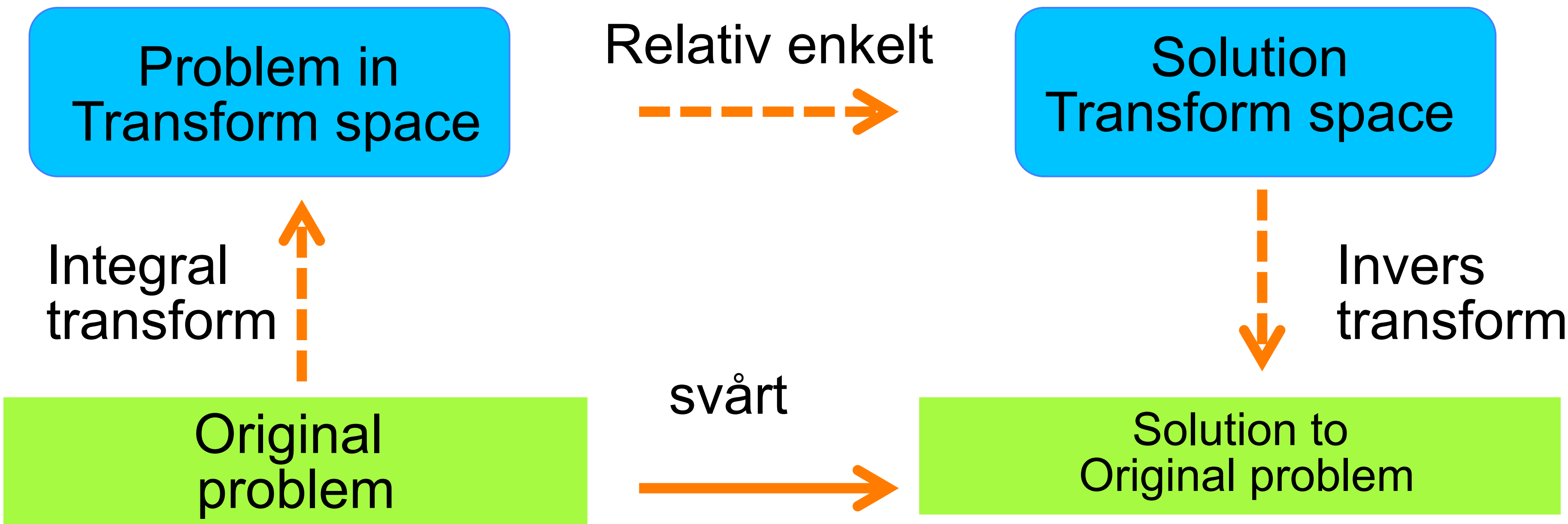
Idag

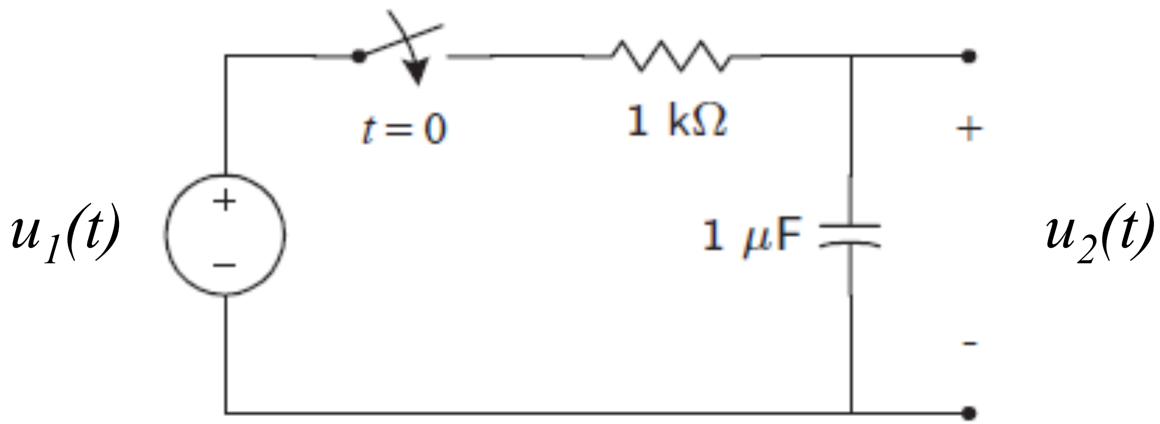
1. System och dess överföringsfunktion
2. S-planet & sammanhang med frekvensfunktionen
3. Impulssvar
4. Stegsvar

Varför är (Fourier/Laplace) transformer bra att ha

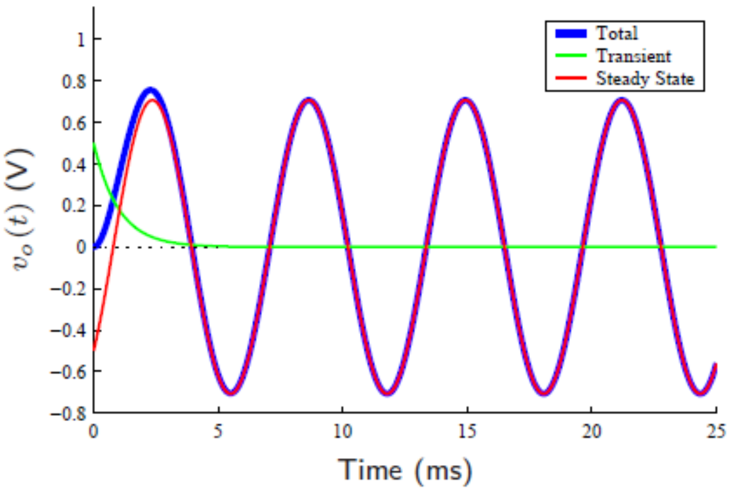
(1) Förenklar livet



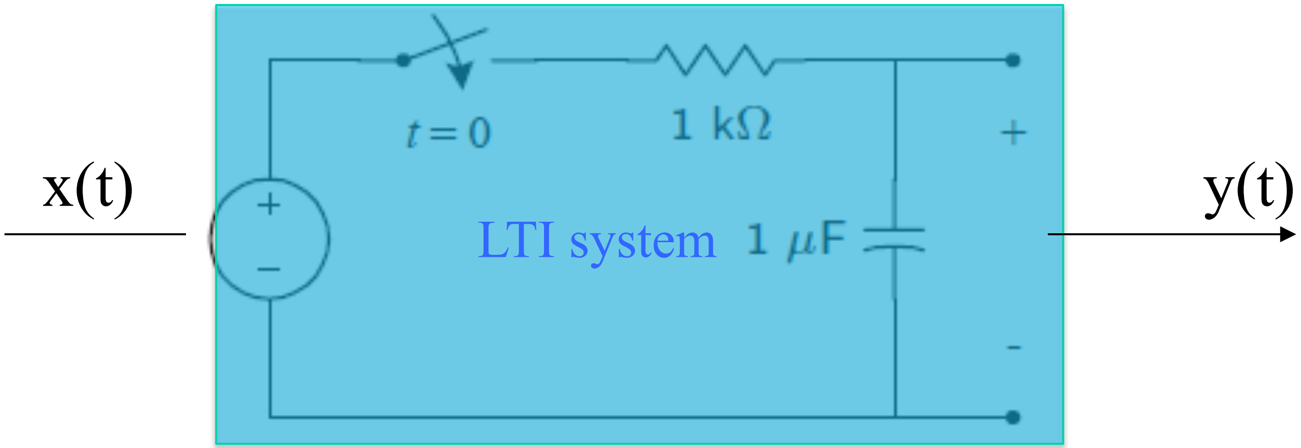
Krets



$$
\begin{aligned}
v_o(t) &= \tfrac{1}{2}\mathrm{e}^{-t/0.001} + \tfrac{1}{\sqrt{2}}\sin(1000t - 45^\circ) \\
&= v_{\mathsf{tr}}(t) + v_{\mathsf{ss}}(t) \\
v_{\mathsf{tr}}(t) &= \tfrac{1}{2}\mathrm{e}^{-t/0.001} \\
v_{\mathsf{ss}}(t) &= \tfrac{1}{\sqrt{2}}\sin(1000t - 45^\circ)
\end{aligned}
$$



Krets ➔ System

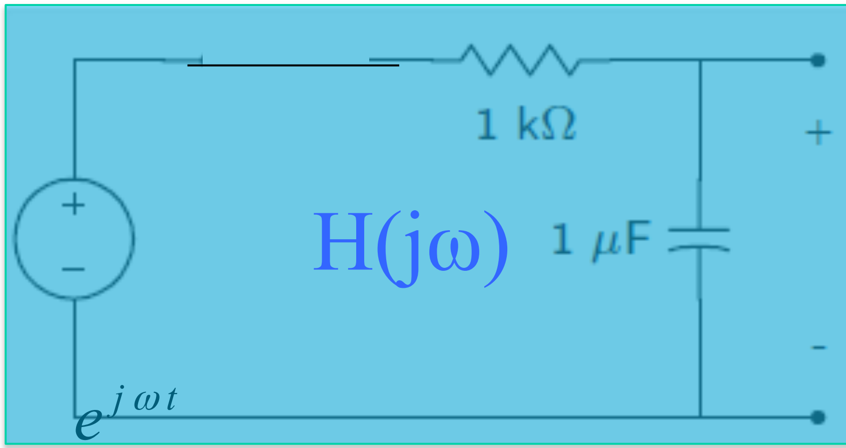


System representering: Frekvenssvår/Stationärtsvår

LTI system

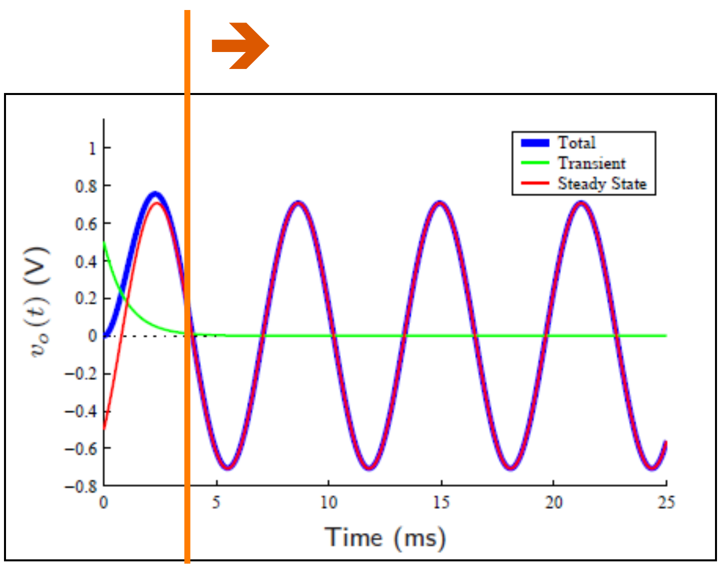
$x(t)$

$\cos(\omega_0 t)$



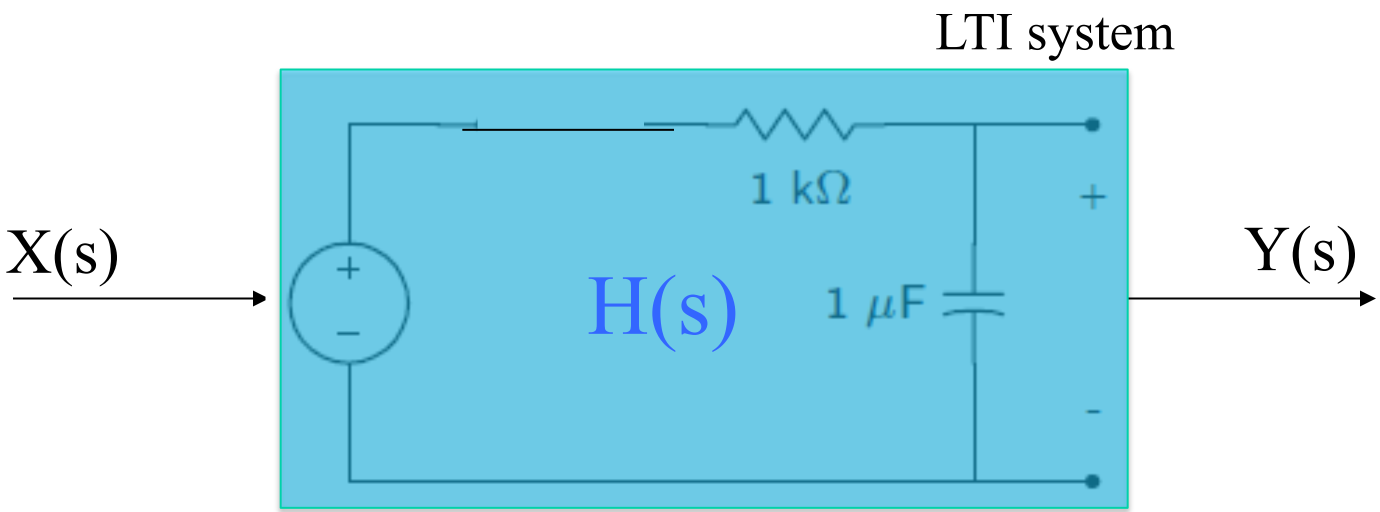
$y(t)$

$|H(\omega)|\cos(\omega_0 t + \angle H(\omega))$



$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

System representering: Överföringsfunktion

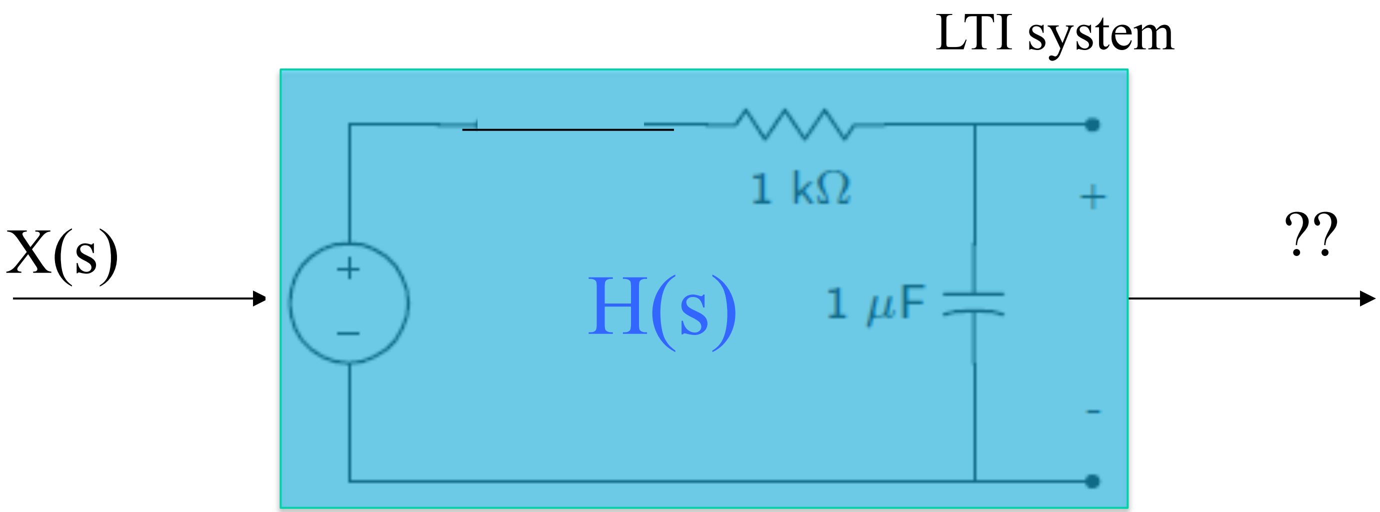


$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

!Obs!

$$y(0) = y'(0) = y''(0) = y^{(3)}(0) = \ldots = y^{(n-1)}(0) = 0$$
$$x(0) = x'(0) = x''(0) = x^{(3)}(0) = \ldots = x^{(n-1)}(0) = 0$$

System representering: Överföringsfunktion



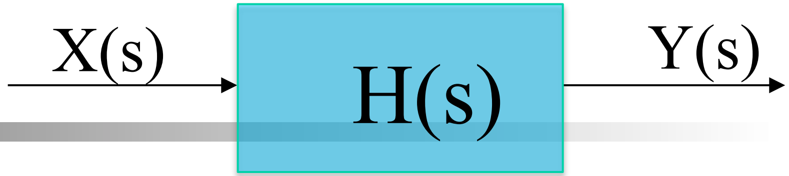
$$Y(s) = H(s)X(s)$$

Exempel

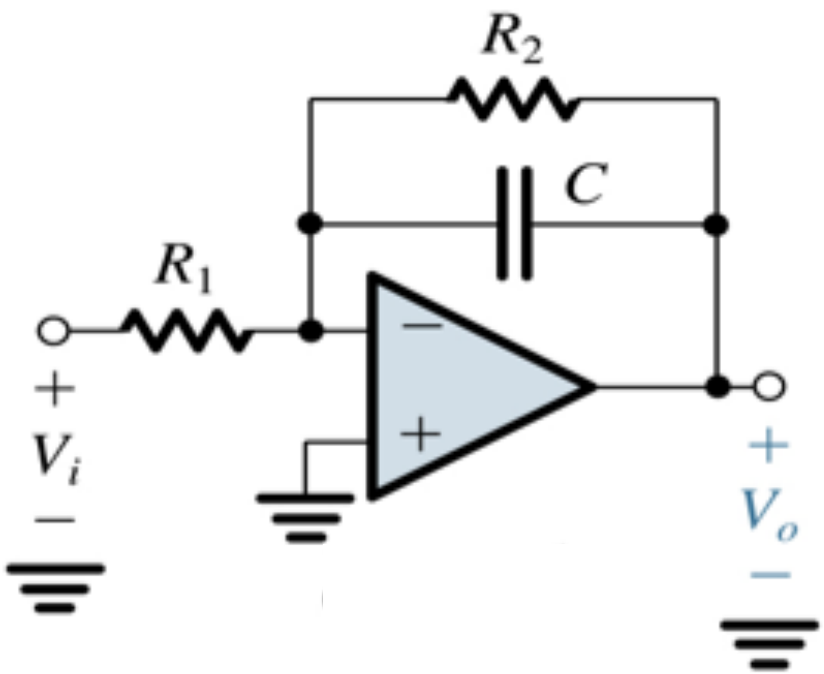
Bestäm H(s) för system som beskriv av differentialekvationen:

$$y''(t) + 4y'(t) + 10y(t) = x'(t) + 2x(t)$$

Exempel (2)



Bestäm H(s) för följande krets (filter):



Överförningsfunktion: Hur den ser ut

Om systemet beskrivs av en differentialekvation av typen

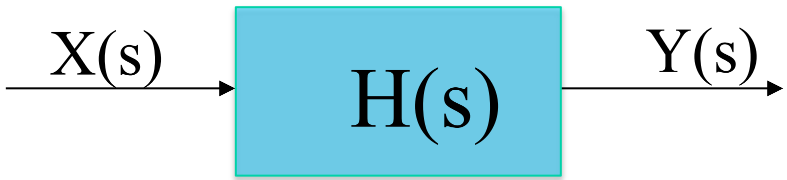
$$a_n x^{(n)} + a_{n-1} x^{(n-1)} + \ldots + a_0 x = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \ldots + b_0 u$$

Så kommer överföringsfunktionen att vara en rationell funktion i *s*

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

Den är ett viktigt verktyg för att studera systemets egenskaper.

Undersökning av S-plan



$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

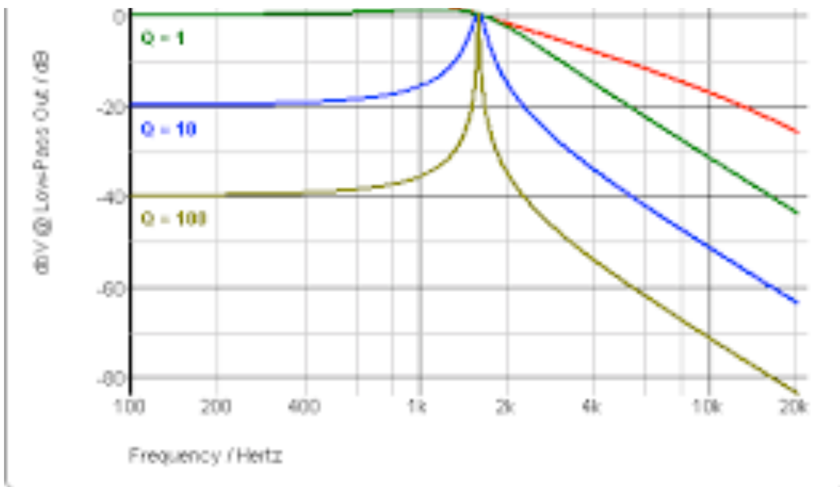
nollställe

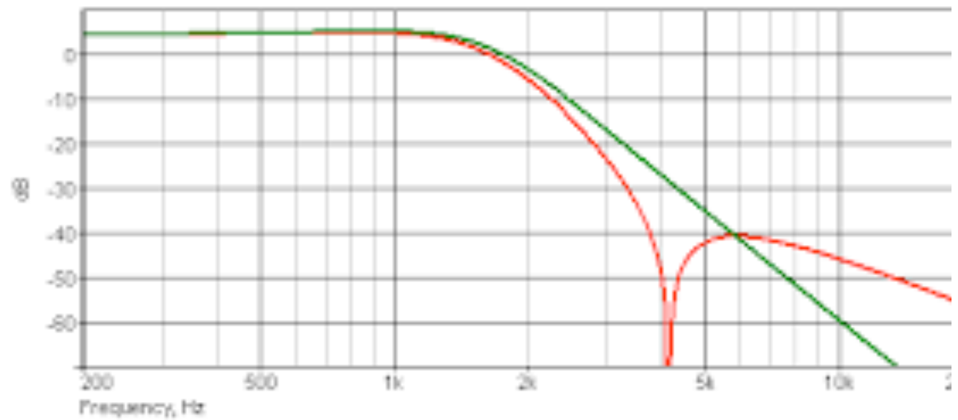
$$H(s) = \frac{K(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ \ldots\ (s - p_n)}$$

pol

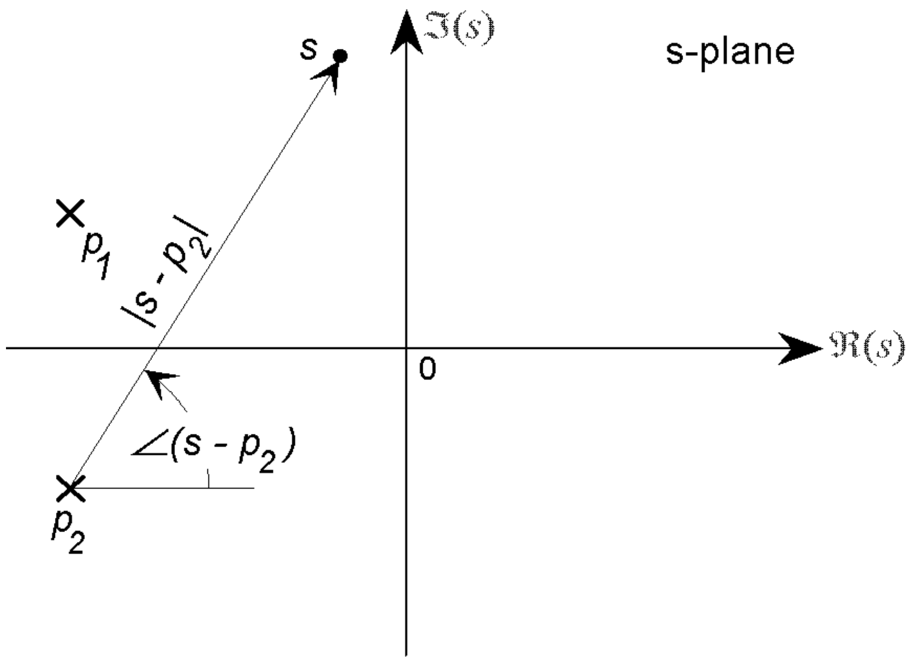
Frekvensfunktion & Bode

H(jw) av en 1:a ordning system





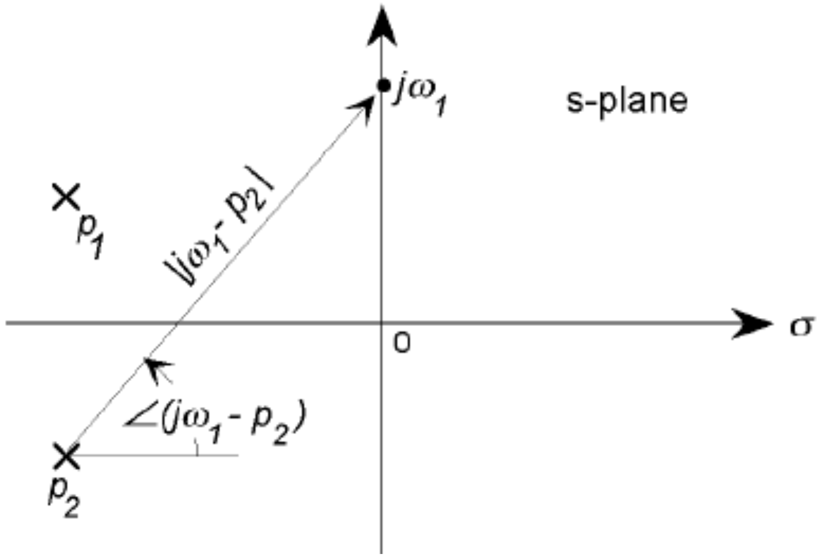
Beloppsytta & fasytta

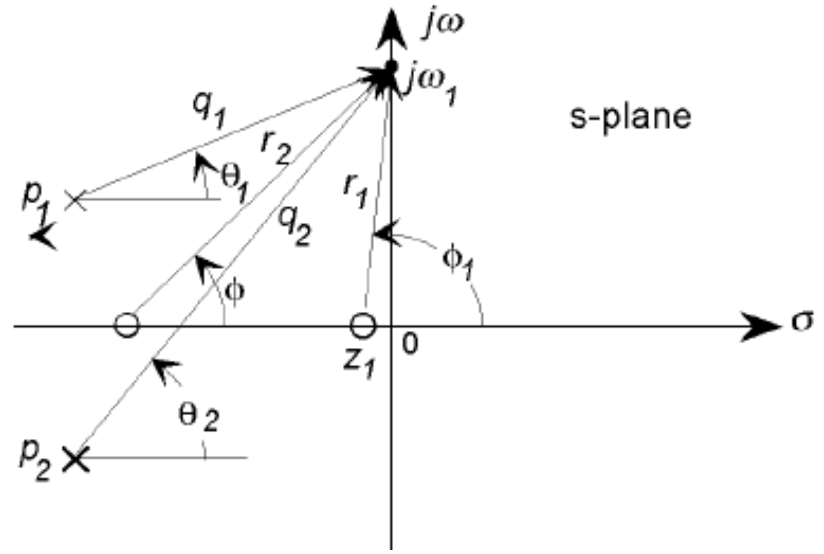


$$
\begin{aligned}
|s - p_i| &= \sqrt{(\sigma - \sigma_i)^2 + (\omega - \omega_i)^2}, \\
\angle(s - p_i) &= \tan^{-1}\left(\frac{\omega - \omega_i}{\sigma - \sigma_i}\right)
\end{aligned}
$$

$$
\begin{aligned}
|H(s)| &= K \frac{\prod_{i=1}^{m} |(s - z_i)|}{\prod_{i=1}^{n} |(s - p_i)|} \\
\angle H(s) &= \sum_{i=1}^{m} \angle(s - z_i) - \sum_{i=1}^{n} \angle(s - p_i)
\end{aligned}
$$

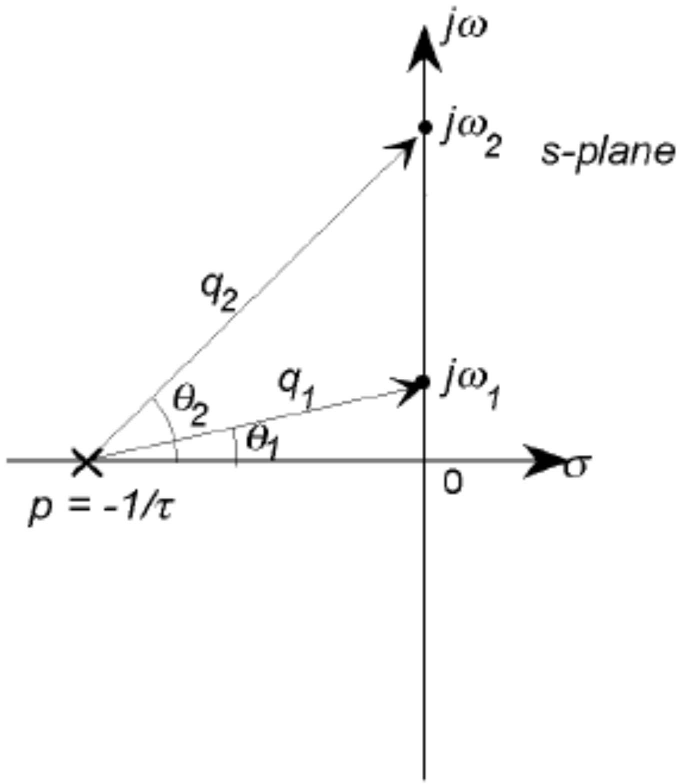
S-plan & Frekvensfunktion

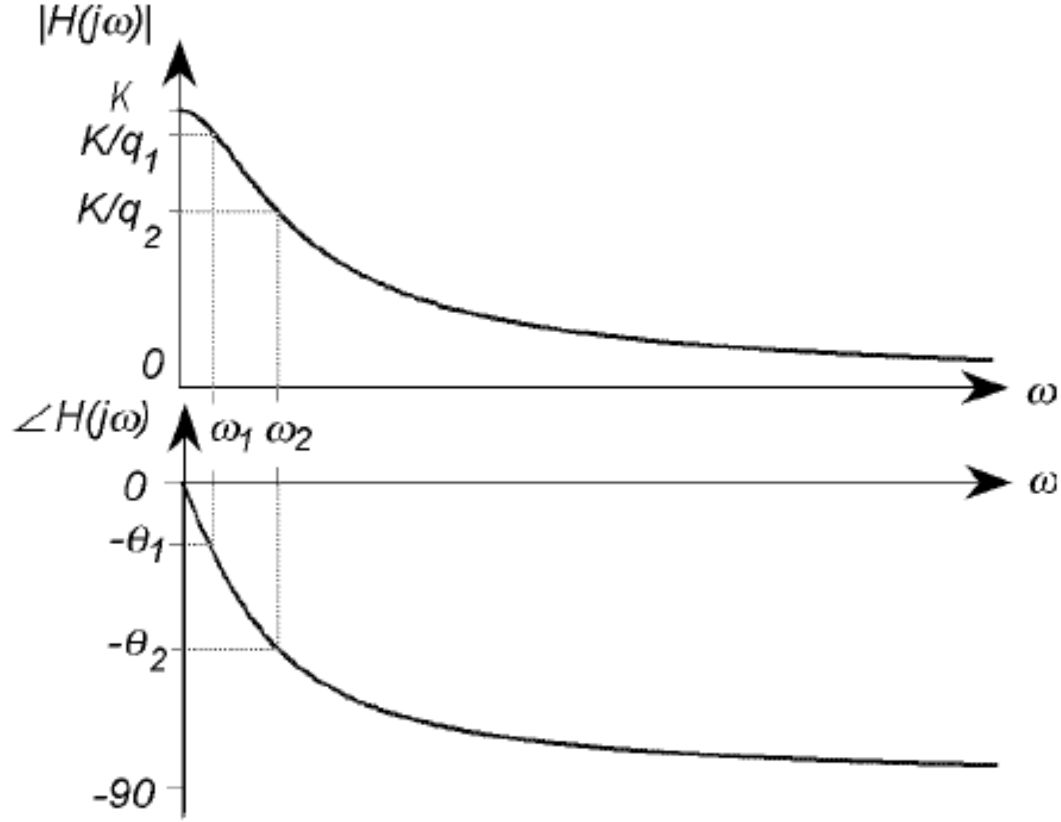




$$
\begin{aligned}
|H(j\omega)| &= K\frac{r_1 \dots r_m}{q_1 \dots q_n} \\
\angle H(j\omega) &= (\phi_1 + \dots + \phi_m) - (\theta_1 + \dots + \theta_n)
\end{aligned}
$$

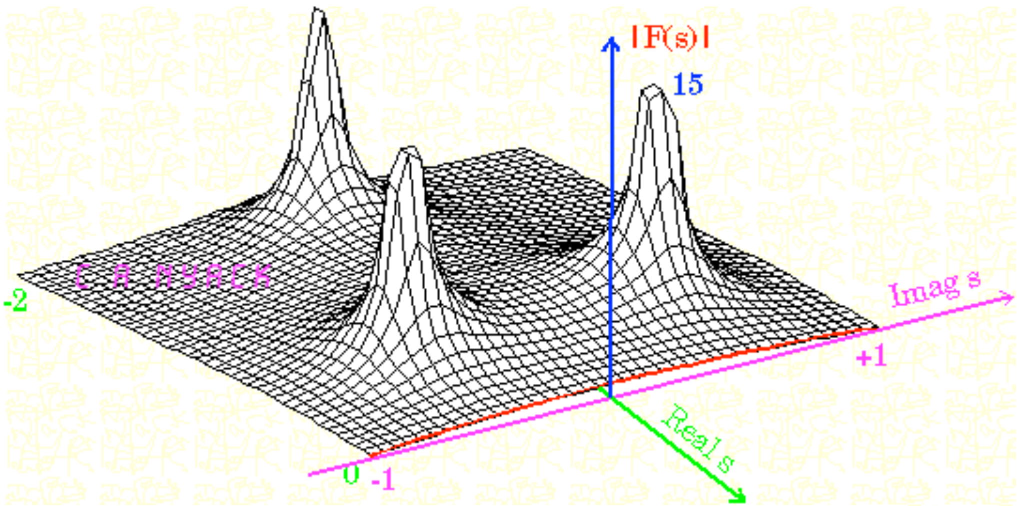
S-plan & Frekvensfunktion



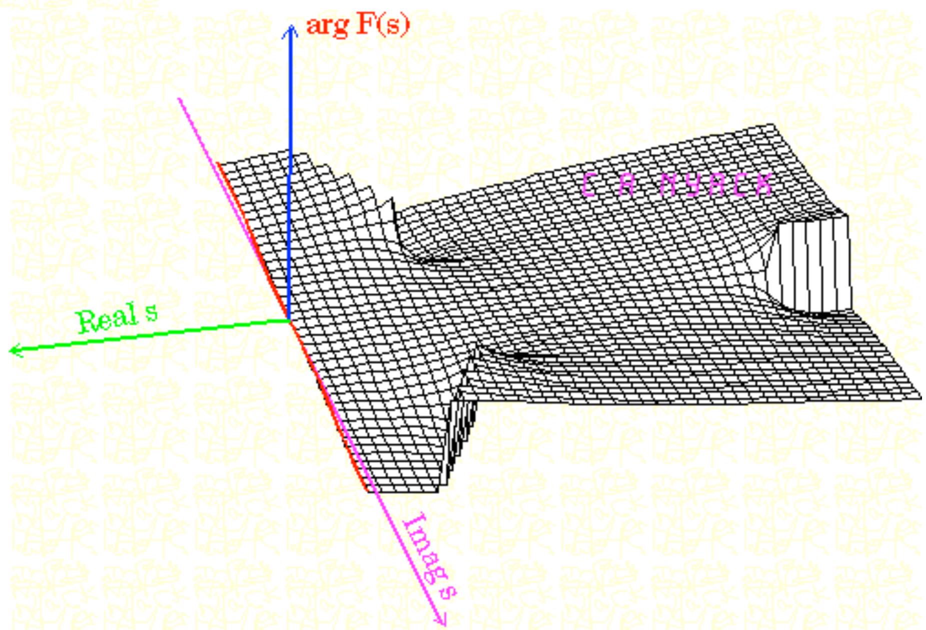


Poler & nolor i s-plan

Beloppsyttan

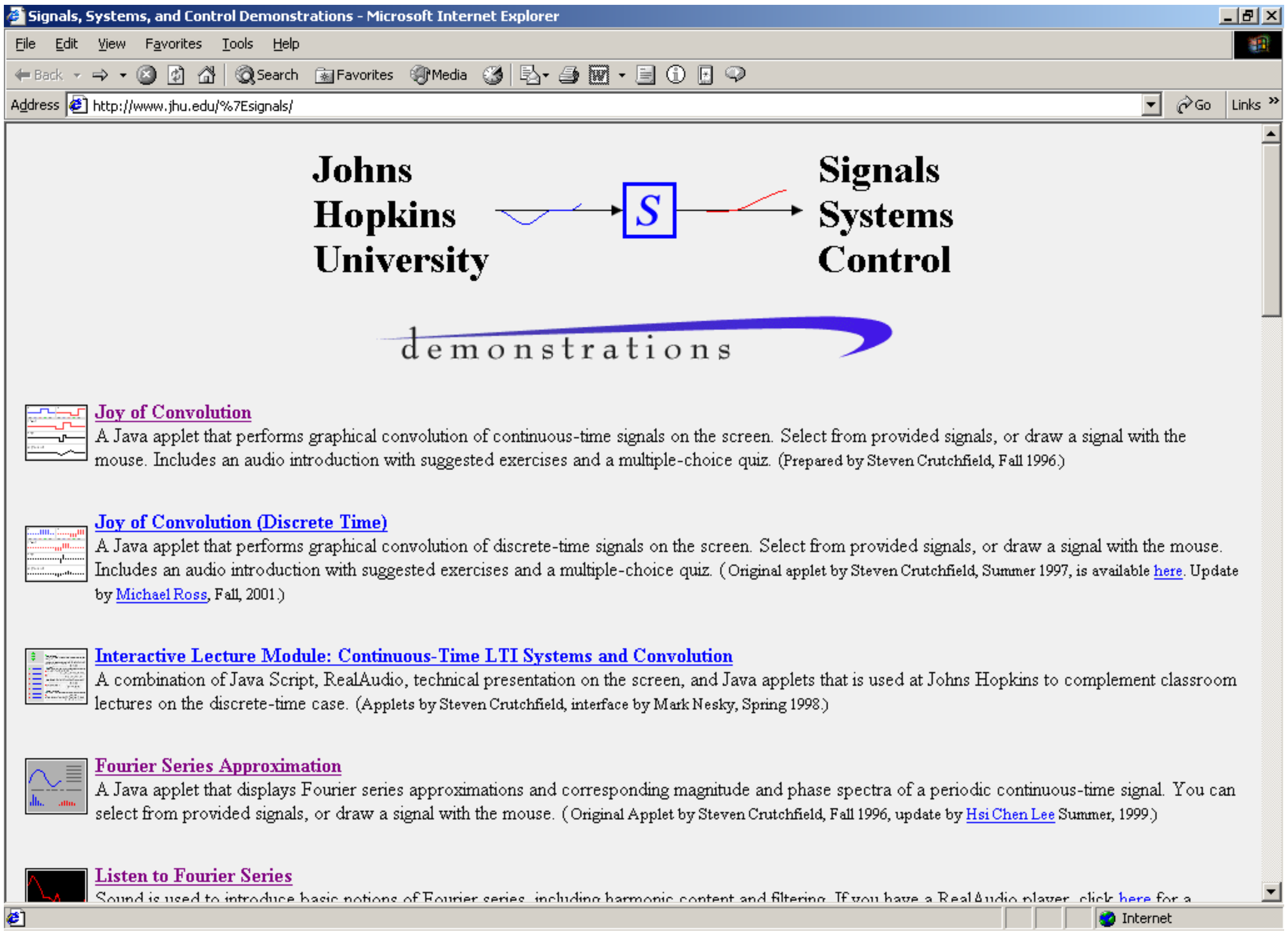


Fasyttan



1 reel och 2 komplexa poler

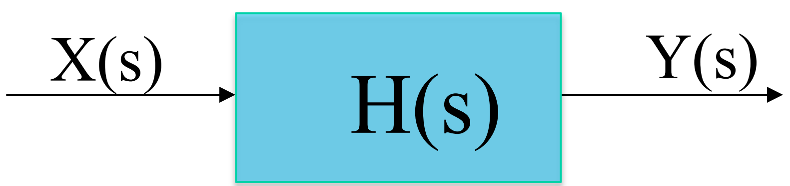
http://www.jhu.edu/%7Esignals/



Analys → **Design**

Varför är (Fourier/Laplace) transformer bra att ha

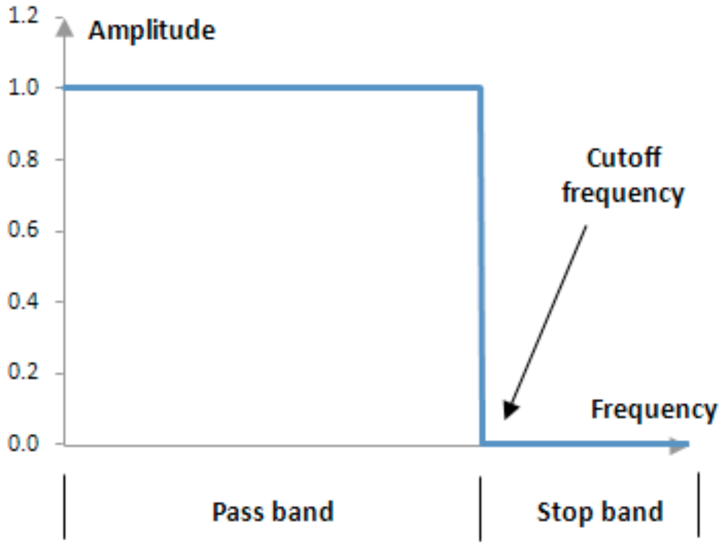
(2) Tillåter systemdesign för en 'fågel'perspektiv



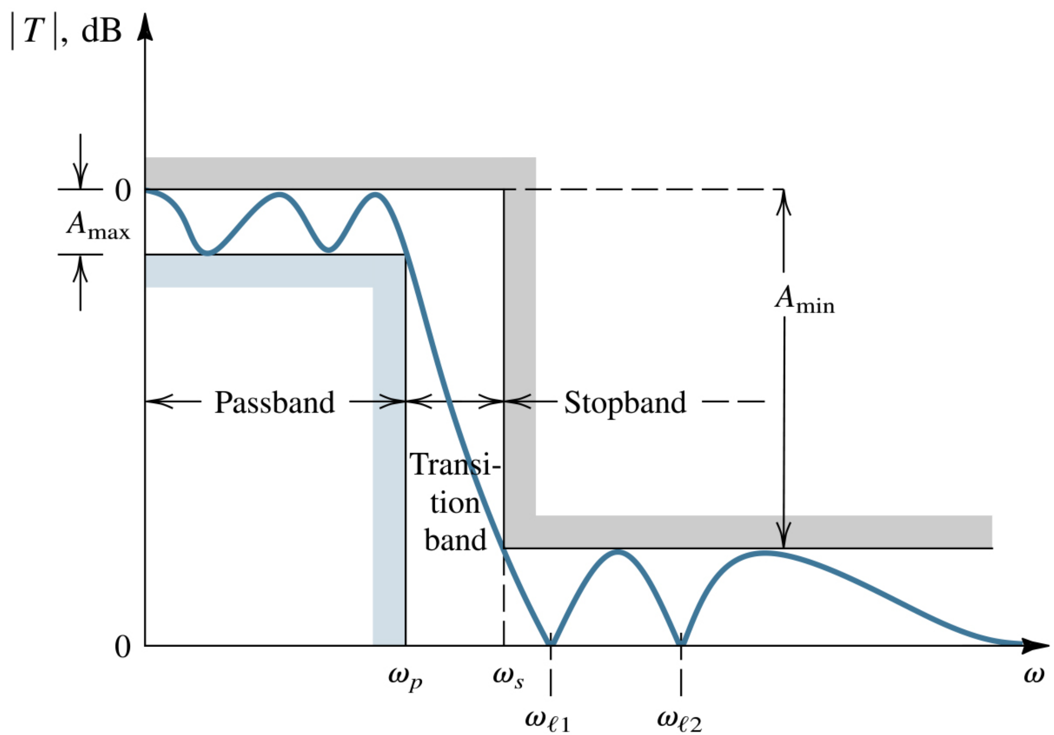
Jag vill ha en krets som filtrerar bort alla frekvenser över 100KHz
(a) Jag tillåter ingen översväng
(b) Liten översväng är ok, men redan f=101kHz måste filtreras bort

Filter

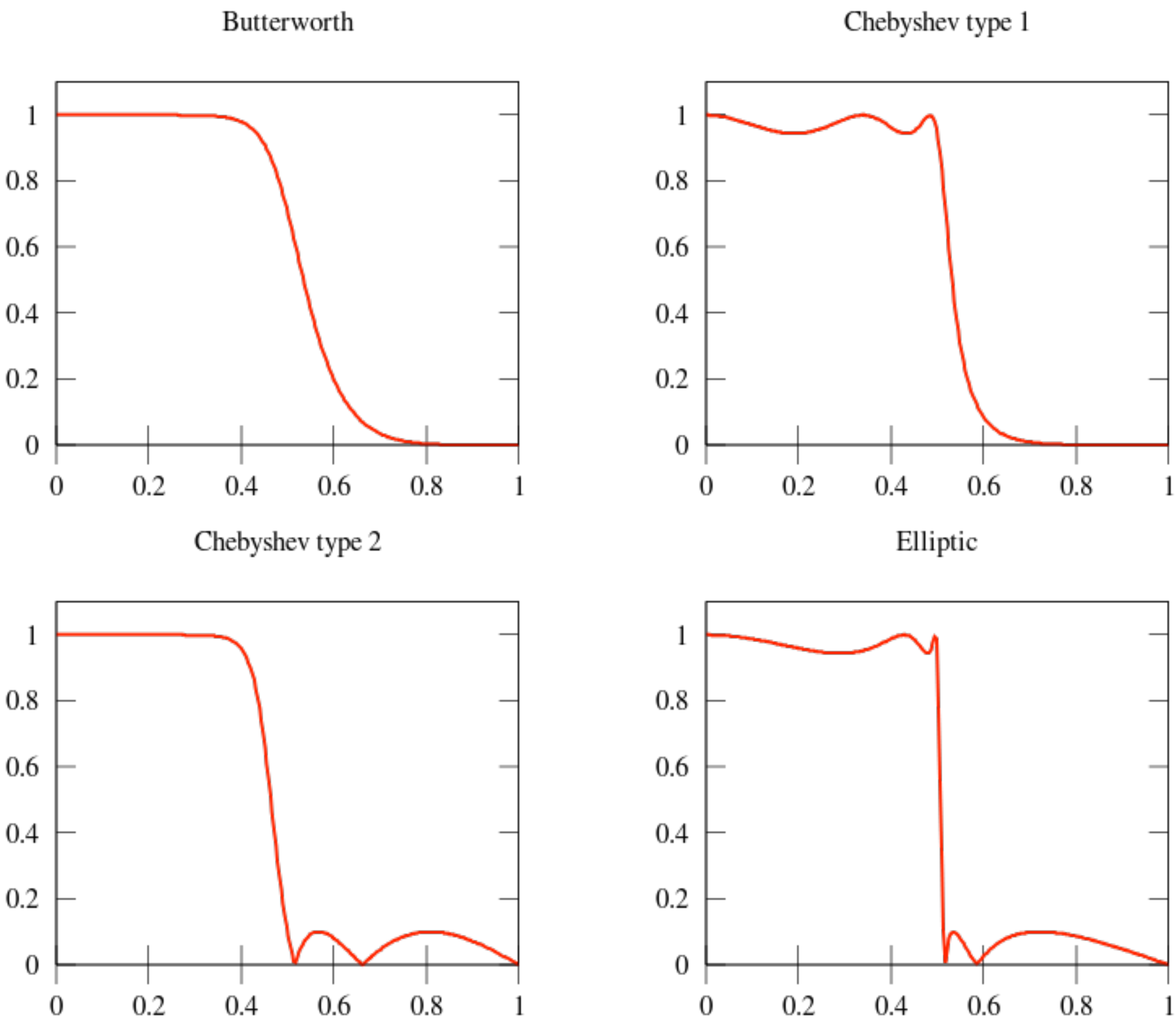
Ideel filter



Reel filter

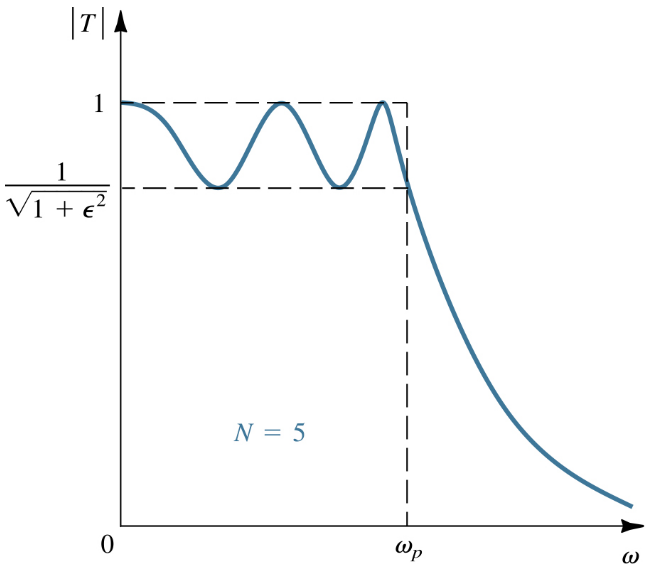


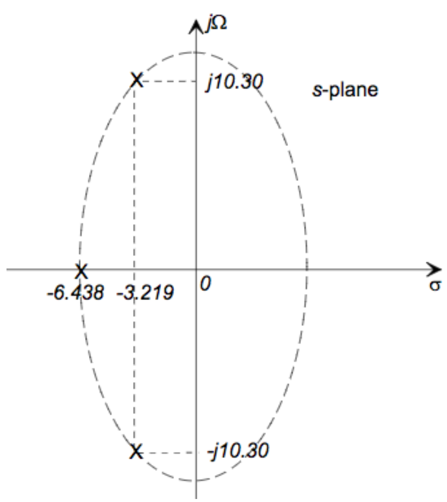
Filter


Butterworth
Chebyshev type 1
Chebyshev type 2
Elliptic
0
0.2
0.4
0.6
0.8
1

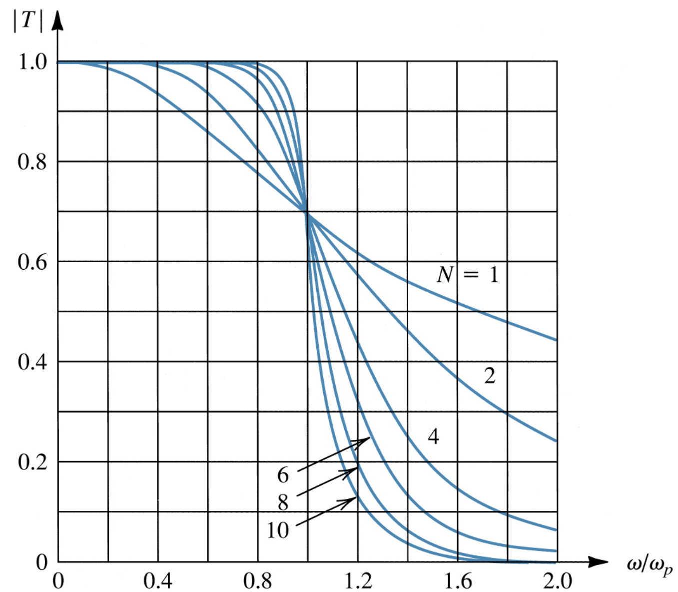
Filter design: polplacering

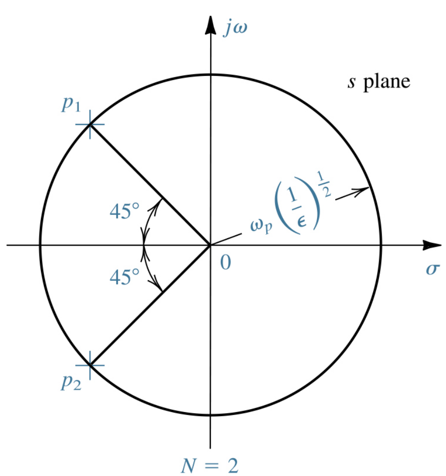
Chebyshev

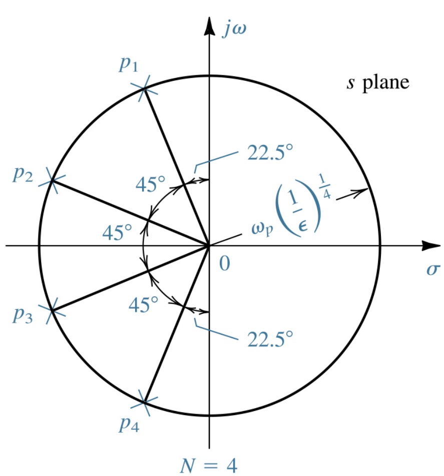




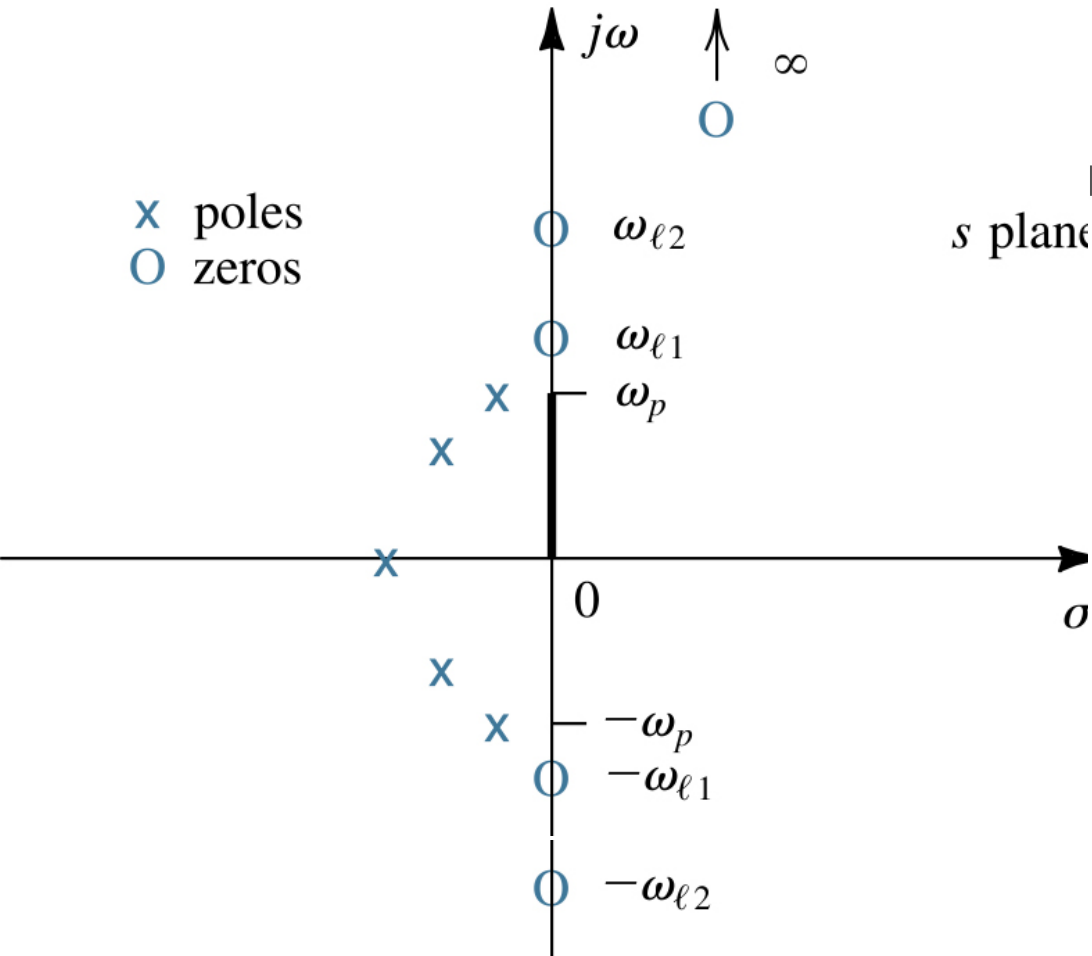
Butterworth

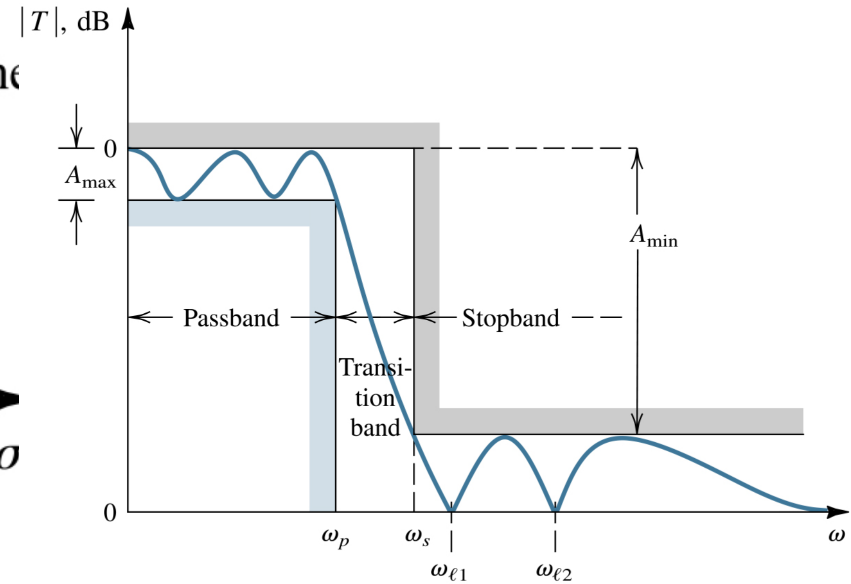






Filter



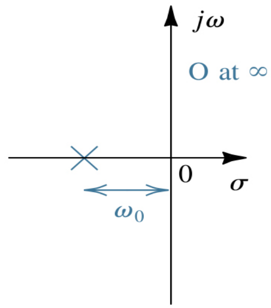
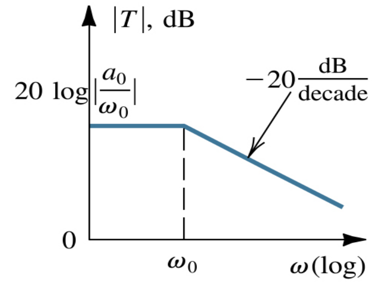
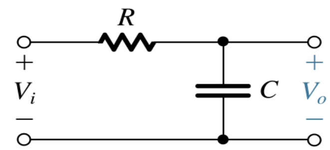
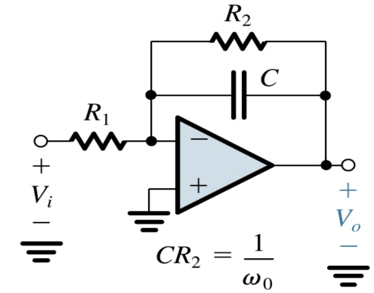
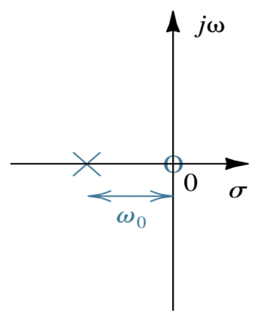
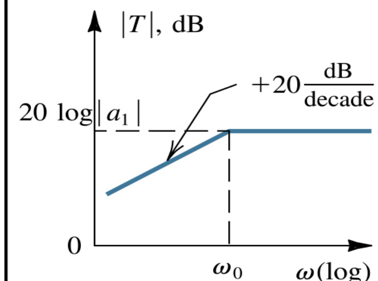
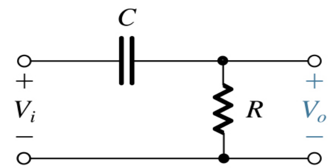
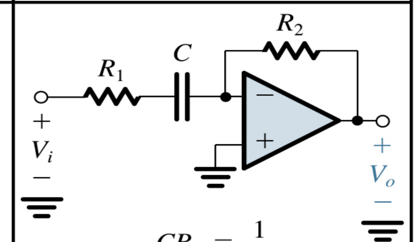
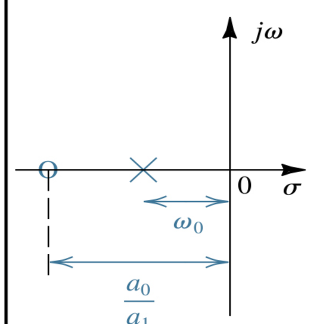
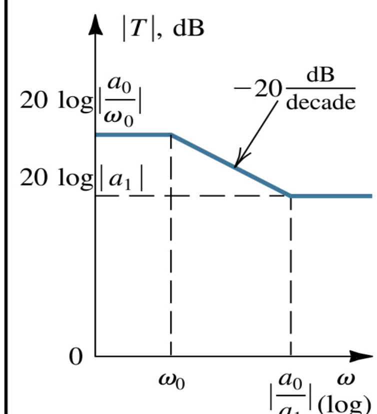
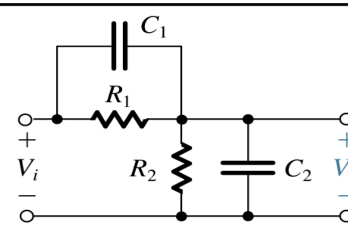
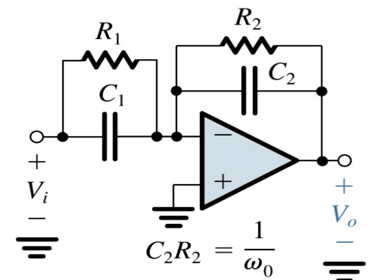


transfer function zeros or transmission zeros

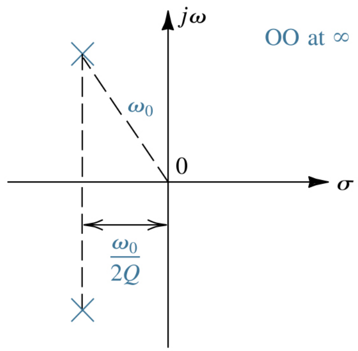
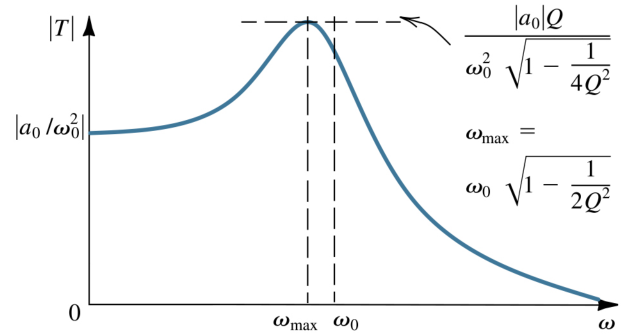
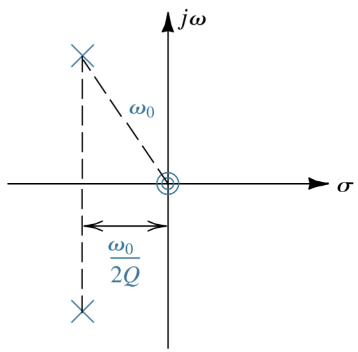
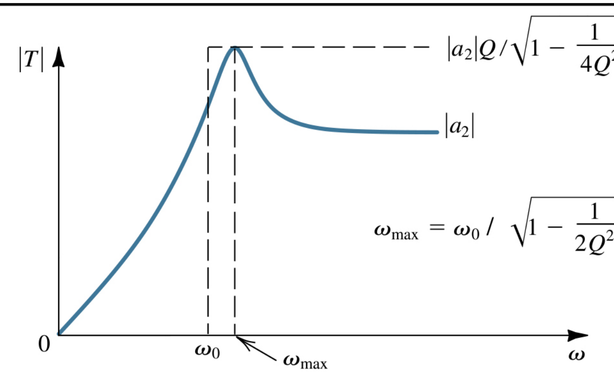
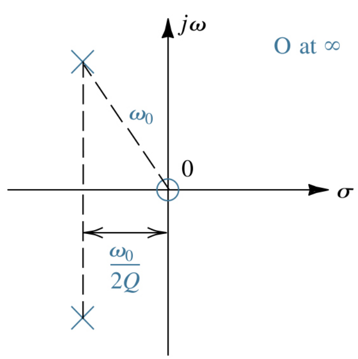
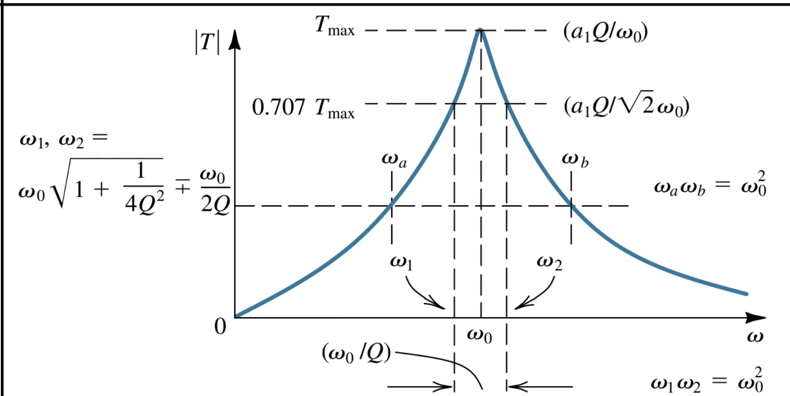
$$T(s) = \frac{a_M \cdot (s - z_1) \cdot (s - z_2) \cdot (s - z_3) \cdots (s - z_M)}{(s - p_1) \cdot (s - p_2) \cdot (s - p_3) \cdots (s - p_N)}$$

transfer function poles or the natural poles

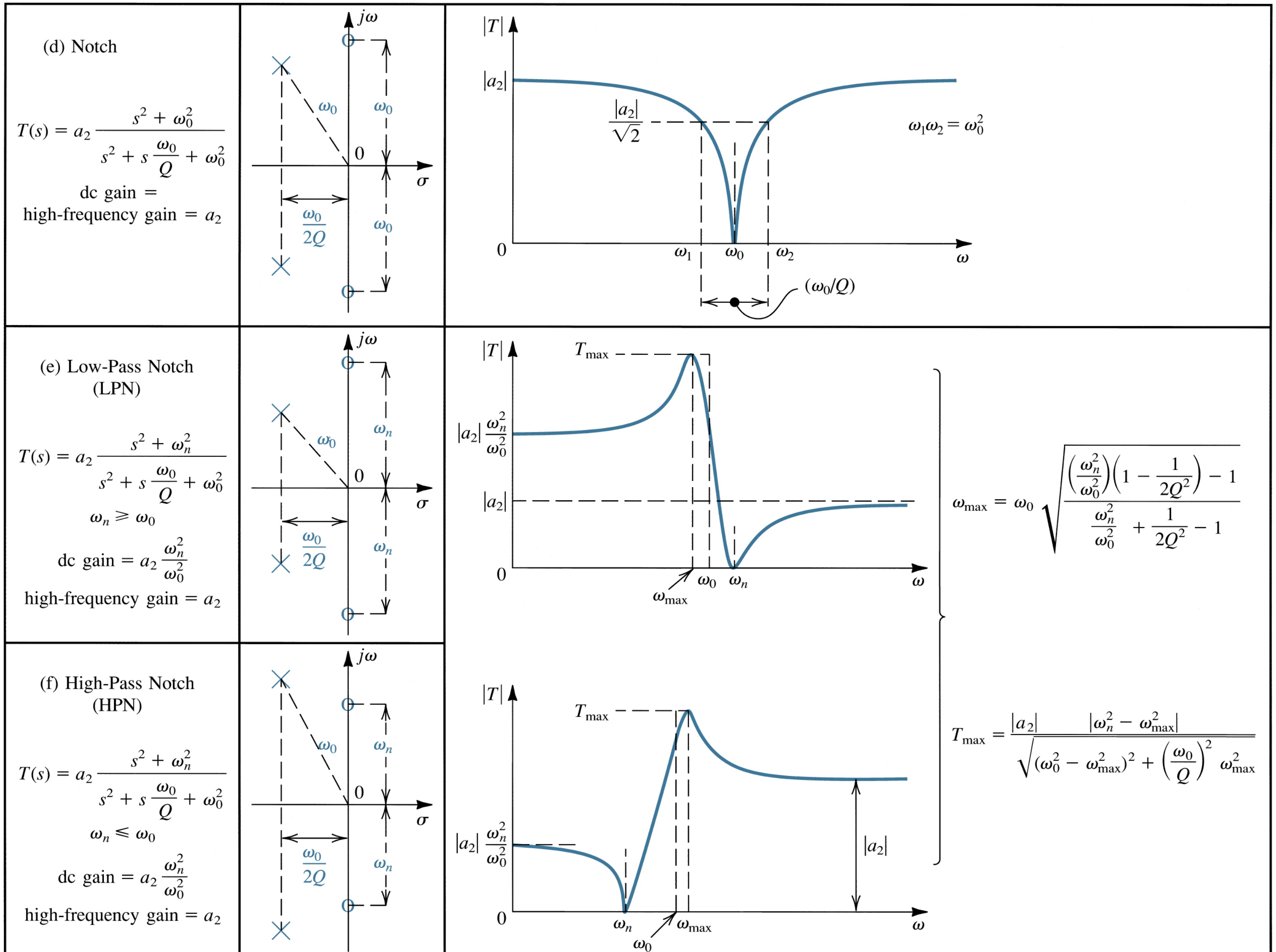
Filter design: exempel på 1:a ordning filtrar

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T \rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ dc gain $= 1$	 $CR_2 = \dfrac{1}{\omega_0}$ dc gain $= -\dfrac{R_2}{R_1}$
(b) High-Pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_0}{a_1}$ dc gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ dc gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

Filterdesign: exempel på 2:a ordning filter

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

Filterdesign: exempel på 2:a ordning filter forts…



LTI system Representering

Varje LTI system kan unikt charakterizeras med sitt

Impulssvar h(t)

svar av systemet till en impulsfunktionen

Frekvenssvar H(jω):

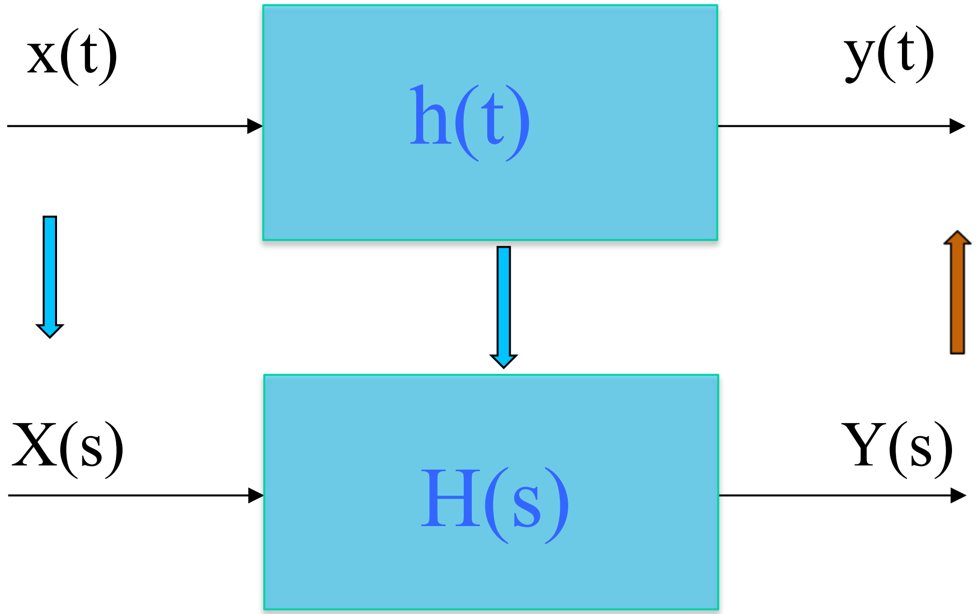
svar av systemet till en komplex exponentiella $e^{j\omega}$ för alla frekvenser f.

Överföringsfunktion H(s):

Laplacetransformen av impulssvar

Alla tre representeringar är likvärdiga
Om vi vet en representering, kan vi hitta de andra två under förutsättning att de finns

System representering



Hur bestämmer vi H(s)?

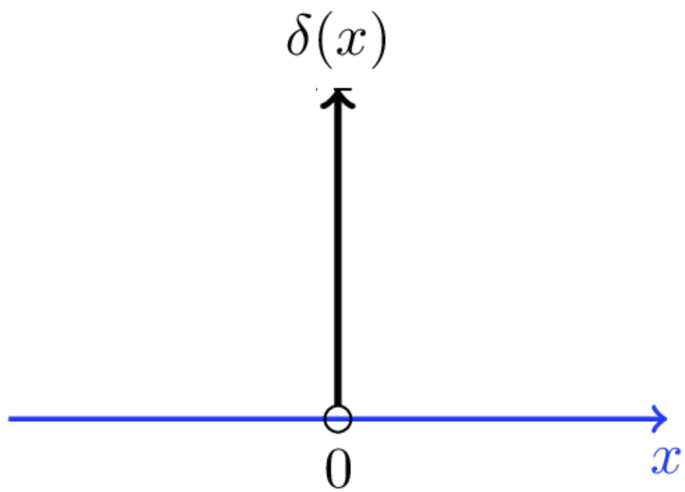
$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

Vad är det ‘enklaste’ signal att skicka in som ingång?

Singularity funktioner

Implusfunktion (Delta-funktionen)

$$x(t) = \delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$



Två viktiga egenskaper

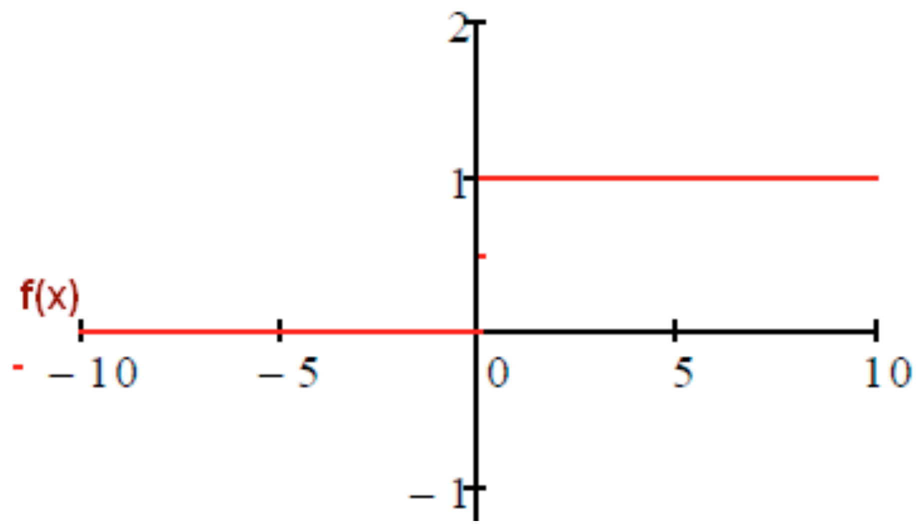
$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0)$$

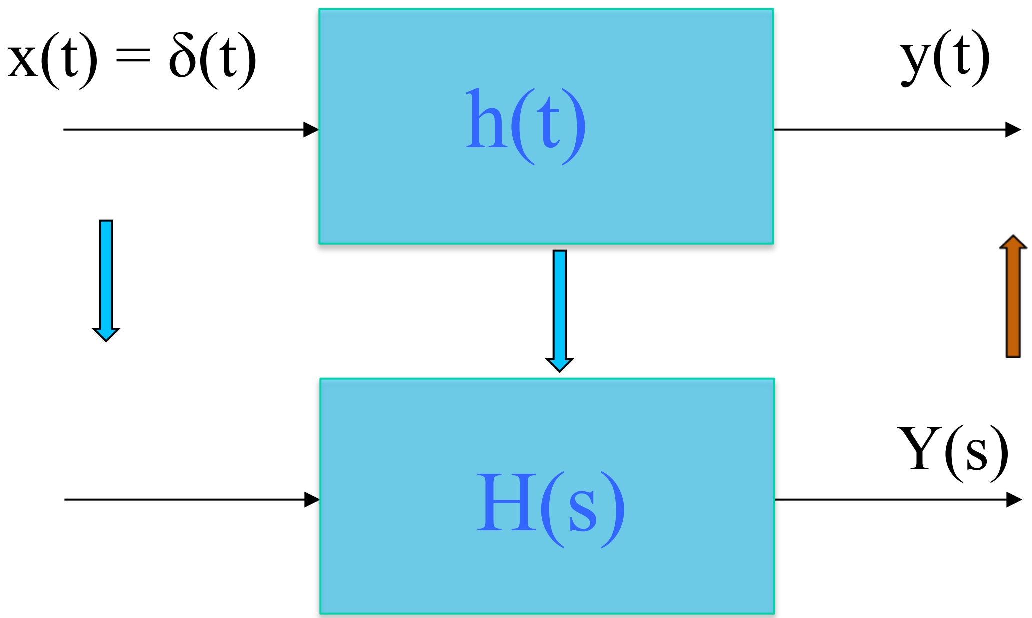
Stegfunktion (Trappfunktion)

$$x(t) = u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

$$x(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



System representering: Impulssvår



Laplacetransform av impulsfunktion

För att ta Laplacetransformen av impulsfunktionen $\delta(t)$ så modifierar vi definitionen så att

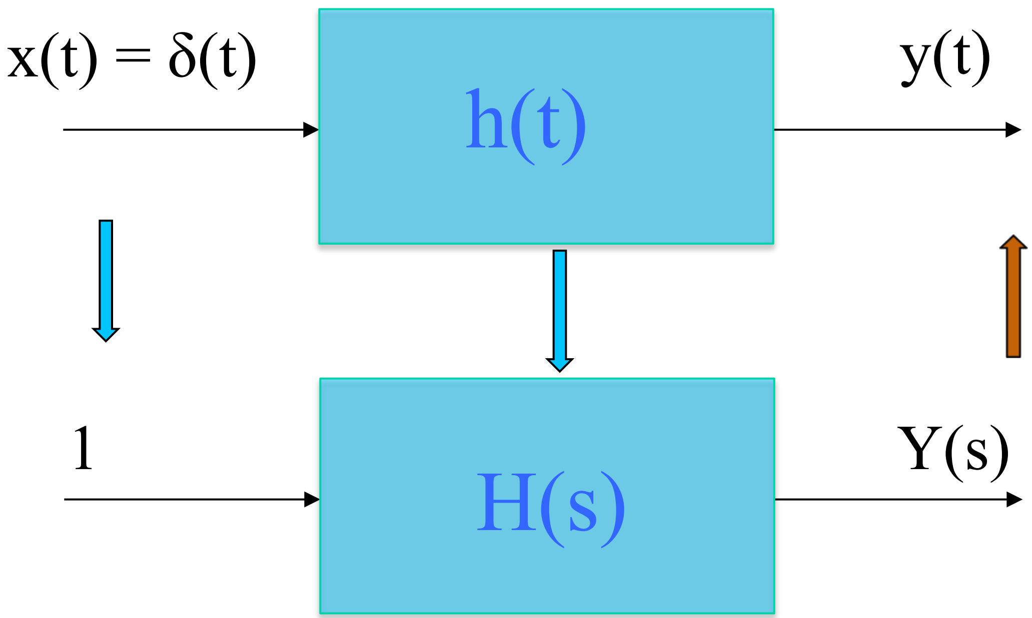
$$f(t) \quad \rightarrow \quad \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

där 0^- betyder att man skall ta gränsvärdet då från vänster (dvs för $t \rightarrow 0$)

Egenskaperna hos impulsfunktionen ger då att

$$\delta(t) \quad \rightarrow \quad \int_{0^-}^{\infty} \delta(t)e^{-st}\,dt = \int_{0^-}^{\infty} \delta(t)\,dt = 1$$

System representering: Impulssvår

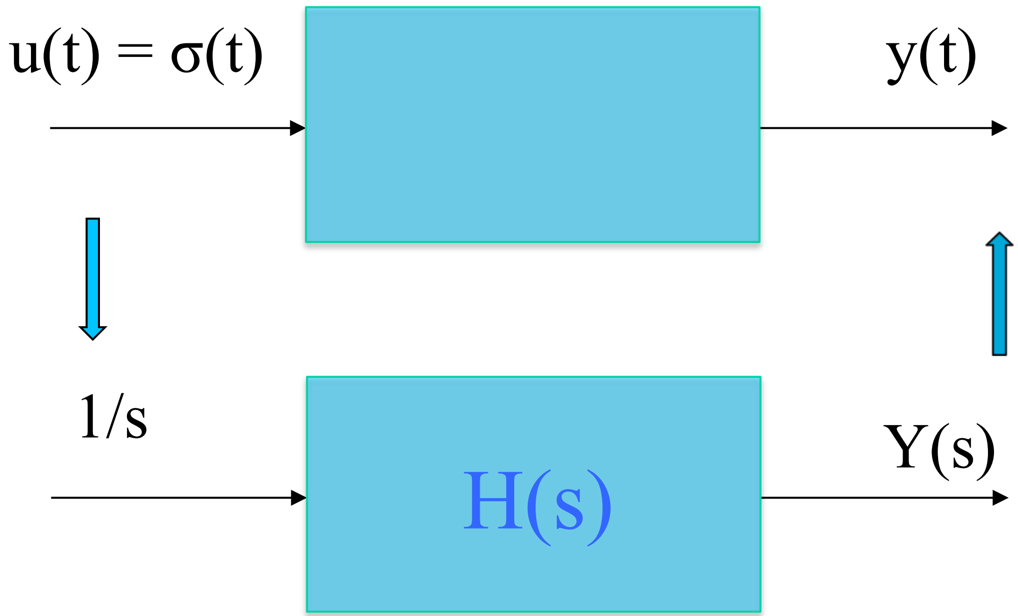


$$\boxed{Y(s) = 1H(s)}$$

Alternativ definition av H(s)

Beskriver hur en linjärsystem svårar på en impuls

System representering: Stegsvår



$$\boxed{Y(s) = \frac{1}{s} H(s)}$$

Beskriver systemets egenskaper vid plötsliga stegvisa förändringar hos signalen