

Differensekvation

$$\sum_{k=0}^N a_k y[n-k] = \sum_{m=0}^M b_m x[n-m]$$

Exempel

Bestäm $y[n]$ av systemet representerad med differensekvation

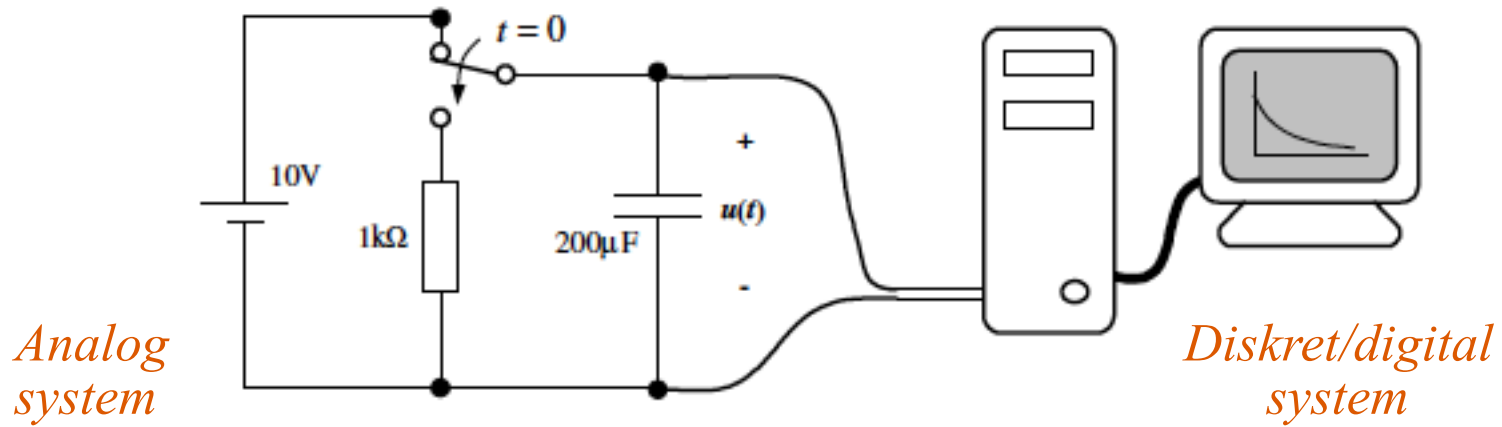
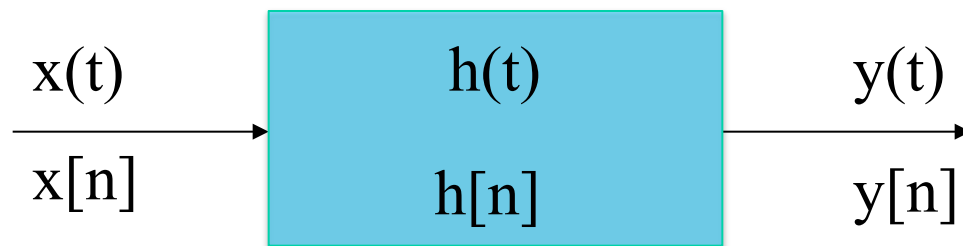
$$y[n] = 0.8 y[n-1] + x[n]$$

när $x[n] = \delta[n]$.

Idag

1. **Differensekvationer och deras lösning**
2. **Blockschema**
3. **Z-transformen** *(början blev det...)*

Från differential till differensekvationer



Differential vs Differensekvation

Differentialekvation

$$a_n y^n + a_{n-1} y^{n-1} + \dots + a_1 y' + a_0 y = b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x' + b_0 x$$

$$y(t) = y_p(t) + y_h(t)$$

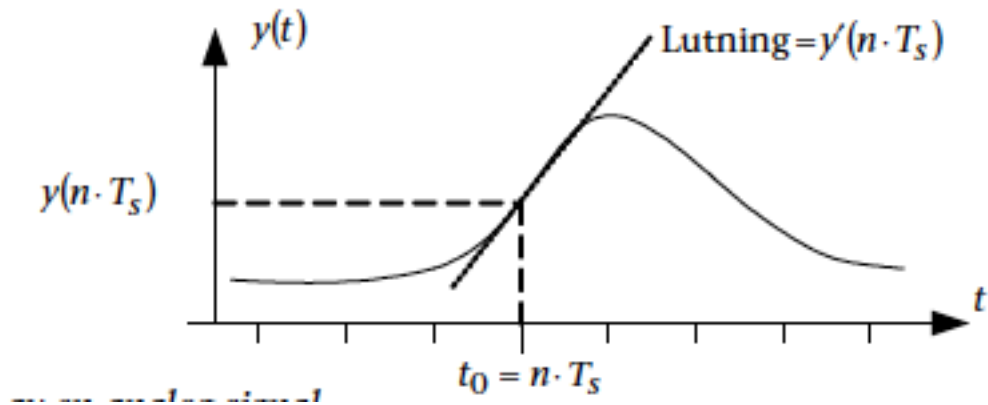
partikulär lösning homogen lösning

Differensekvation

$$\sum_{k=0}^N a_k y[n-k] = \sum_{m=0}^M b_m x[n-m]$$

$$y[n] = y_p[n] + y_h[n]$$

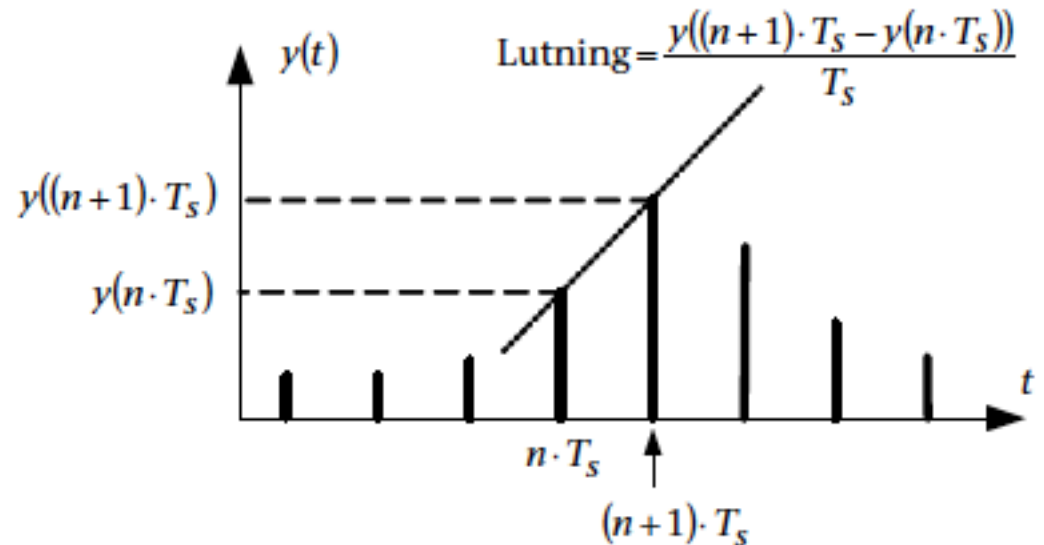
Differential vs Differensekvation



Derivata

Differens

‘Derivata av en diskret signal’



Exempel

Systemet representerad med differential ekvation

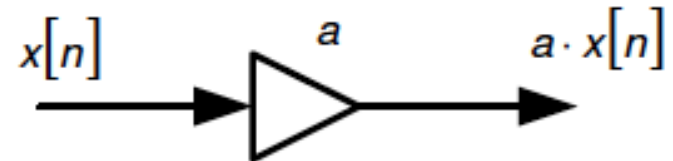
$$y'(t) - 2y(t) = \sigma(t), y(0) = 0,$$

samplas med samplingsfrekvens 10Hz. Lös approximativt.

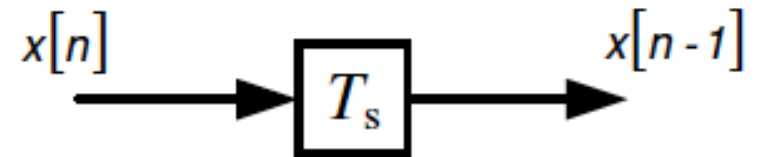
Blockschema för diskreta system

Transversellt system

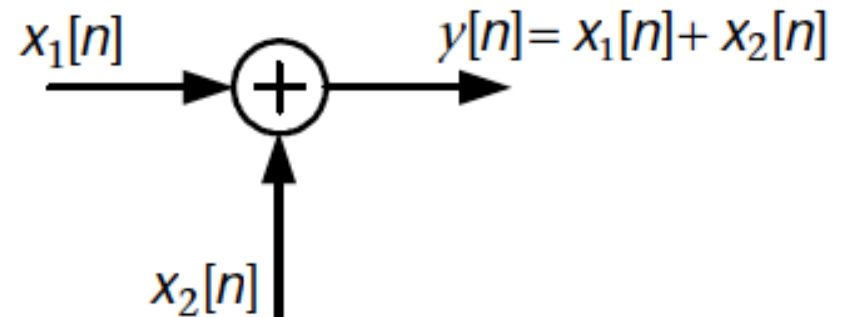
Multiplikation av signal
med konstant



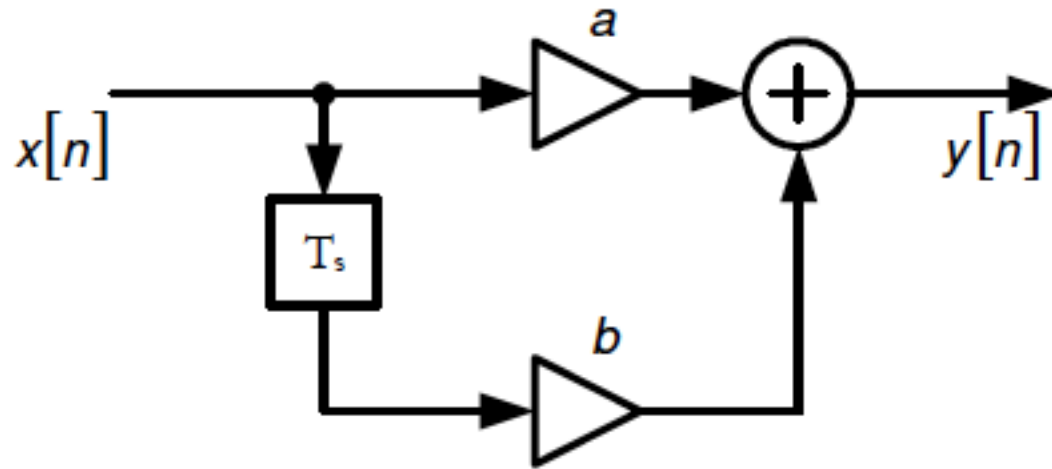
Fördröjning av signal



Addition av signaler

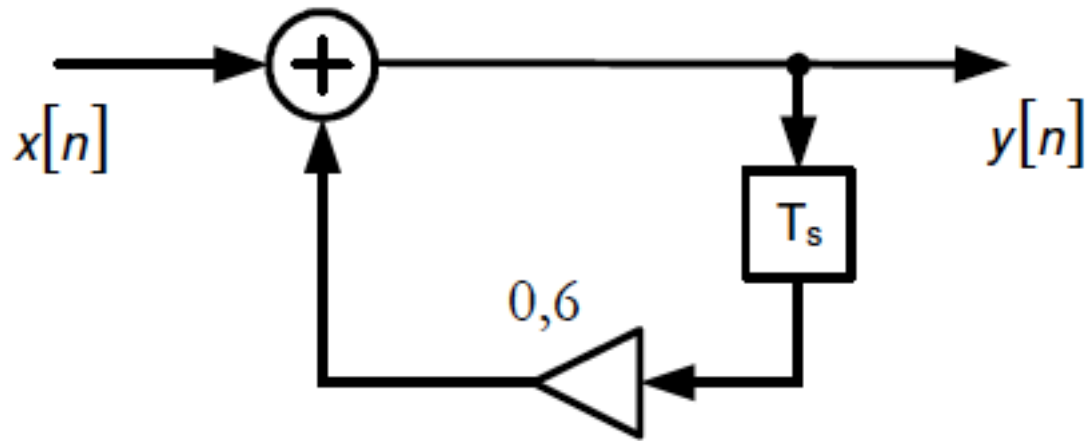


Transverselt system



$$y[n] = ax[n] + bx[n-1]$$

Rekursivt system



$$y[n] = x[n] + 0.6y[n-1]$$

Exempel: Blockdiagram

Rita blockschema för systemet representerade differensekvation:

a) $2y[n] - y[n-1] + 4y[n-2] = 5x[n]$

b) $y[n] - 5y[n-1] + y[n-2] = 3x[n] - 4x[n-2]$

Z-transformen

Definition

Mest generellt koncept för tiddiskreta signaler

*Z-transformen
(dubbelsidigt)*

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

*Z-transformen
(enkelsidigt)*

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Jämnför med Laplace

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

Ett par

$$x[n] \overset{Z}{\leftrightarrow} X(z)$$

Exempel: Z-transformen

Rita följande sekvens och bestäm dess Z-transformen

$$x = \{2, -3, 7, 4, 0, 0, \dots\}$$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]z^{-n} = 2 - 3z^{-1} + 7z^{-2} + 4z^{-3} \\ &= \frac{2z^3 - 3z^2 + 7z + 4}{z^3}, \quad |z| > 0 \end{aligned}$$