Lösningar Linjära system aug 2013

1a. $f_S = 25\,kHz \Rightarrow T_S = \frac{1}{f_S} = 4\cdot 10^{-5}\,s \Rightarrow$

$$x[n] = x(n\cdot T_S) = 12\cdot e^{-12000\cdot 4\cdot 10^{-5}\cdot n}\cdot \cos(2400\pi\cdot 4\cdot 10^{-5} n) =$$

$$= 12\cdot\left(e^{-12000\cdot 4\cdot 10^{-5}}\right)^n\cdot \cos(9600\cdot 10^{-5} n\cdot\pi) =$$

$$= 12\cdot 0{,}6188^n\cdot \cos[n\cdot 0{,}096\pi] \Rightarrow$$

$A = 12$ $\quad b = 0{,}6188$ $\quad \Omega = 0{,}096\pi$

1b. $x[n] = \frac{2-0{,}8^n}{2^n} = 2\cdot\left(\frac{1}{2}\right)^n - \left(\frac{0{,}8}{2}\right)^n = 2\cdot 0{,}5^n - 0{,}4^n \Rightarrow$

$$X(z) = \frac{2z}{z-0{,}5} - \frac{z}{z-0{,}4}$$

1c. $X(s) = \frac{2}{2s+1} - \frac{2}{s} = \frac{1}{s+0{,}5} - \frac{2}{s} \Rightarrow$

$$x(t) = e^{-0{,}5t} - 2$$

2a. $a=-8 \Rightarrow H(s) = \dfrac{s+10}{(s+3)(2s^2+8s-24)}$

Poler $s+3=0 \Rightarrow s=-3$

$2s^2+8s-24=0 \Rightarrow s^2+4s-12=0 \Rightarrow$

$s=-2\pm\sqrt{16} = -2\pm 4 = \begin{cases} -6 \\ 2 \end{cases} \Rightarrow$

Instabilt, ty en positiv pol.

2b. $b=0{,}8 \Rightarrow H(z) = \dfrac{z+1{,}2}{z^2+0{,}8z+0{,}8}$

Poler: $z^2+0{,}8z+0{,}8=0 \Rightarrow z=-0{,}4\pm\sqrt{0{,}16-0{,}8} =$

$= -0{,}4\pm j0{,}8 = \sqrt{0{,}8}\cdot e^{\pm j116{,}6^\circ} = r\cdot e^{\pm j\phi}$

$H(z) = z\cdot\dfrac{1{,}2}{z^2+0{,}8z+0{,}8} + z^{-1}\cdot z\cdot\dfrac{1}{z^2+0{,}8z+0{,}8} \Rightarrow$

$h[n] = A\cdot r^n\cdot\cos[n\cdot\phi+\alpha] + A\cdot r^{n-1}\cdot\cos[(n-1)\phi+\alpha]\cdot\sigma[n-1]$

$A\cdot e^{j\alpha} = 2\cdot\left[\dfrac{1{,}2}{2z+0{,}8}\right]_{z=-0{,}4+j0{,}8} =$

$= 2\cdot\dfrac{1{,}2}{-0{,}8+j1{,}6+0{,}8} = -j1{,}5 = 1{,}5\,e^{-j90^\circ} \Rightarrow$

$h[n] = 1{,}5\cdot\sqrt{0{,}8}^{\,n}\cdot\cos[n\cdot 116{,}6^\circ-90^\circ] +$

$+1{,}5\sqrt{0{,}8}^{\,n-1}\cdot\cos[(n-1)\cdot 116{,}6^\circ-90^\circ]\cdot\sigma[n-1]$

(3.) $x[n] = \sigma[n]$; $y[n] = 1 + (-0{,}8)^n \Longrightarrow$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{z}{z-1} + \frac{z}{z+0{,}8}}{\frac{z}{z-1}} = 1 + \frac{\cancel{z}}{z+0{,}8} \cdot \frac{z-1}{\cancel{z}} =$$

$$= \frac{z+0{,}8+z-1}{z+0{,}8} = \frac{2z-0{,}2}{z+0{,}8} = 2 \cdot \frac{z-0{,}1}{z+0{,}8} \Longrightarrow$$

$\underline{\underline{A = 2}}$ $\underline{\underline{B = -0{,}1}}$ $\underline{\underline{C = 0{,}8}}$

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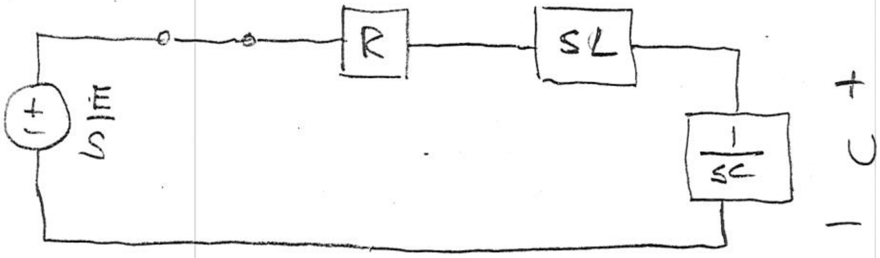
(4) $y[n] = b\,x[n] + x[n-1] + y[n-1] + a\,y[n-2] \Longrightarrow$

$$Y(z) = bX(z) + z^{-1}X(z) + z^{-1}Y(z) + a z^{-2} Y(z) \Longrightarrow$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b + z^{-1}}{1 - z^{-1} - a z^{-2}} = \underline{\underline{\frac{bz^2 + z}{z^2 - z - a}}}$$

(5a.) "Laplace-schema" för $t>0$



$$U = \frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} \cdot \frac{E}{s} = \frac{E}{s\left(s^2LC + sRC + 1\right)} =$$

$$= \frac{\frac{E}{LC}}{s\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)} = \frac{\frac{20}{2\cdot 10^{-12}}}{s\left(s^2 + s\cdot\frac{1000}{10^{-3}} + \frac{1}{2\cdot 10^{-12}}\right)} =$$

$$= \frac{10\cdot 10^{12}}{s\left(s^2+10^6 s + 0{,}5\cdot 10^{12}\right)} = \frac{T(s)}{N(s)}$$

Poler: $s=0 \quad s = -0{,}5\cdot 10^6 \pm \sqrt{0{,}25\cdot 10^{12} - 0{,}5\cdot 10^{12}} =$

$$= \left(-0{,}5 \pm j0{,}5\right)\cdot 10^6 = \left(-5 \pm j5\right)\cdot 10^5 \Longrightarrow$$

$$u(t) = A + B\cdot e^{-5\cdot 10^5 t}\cdot\cos\left(5\cdot 10^5 t + \beta\right)$$

$$N' = 1\cdot\left(s^2+10^6 s + 5\cdot 10^{11}\right) + s\left(2s+10^6\right) \Longrightarrow$$

$$A = \left[\frac{T}{N'}\right]_{s=0} = \frac{10\cdot 10^{12}}{0+0+5\cdot 10^{11}+0} = \underline{20}$$

Forts. $\Longrightarrow$

$$B \cdot e^{j\beta} = 2 \cdot \left[\frac{T}{N'} \right]_{s=(-5+j5)\cdot 10^5} =$$

$$= 2 \cdot \frac{10 \cdot 10^{12}}{0 + (-5+j5) \cdot 10^5 \left(-10 \cdot 10^5 + j10 \cdot 10^5 + 10^6\right)} =$$

$$= 2 \cdot \frac{10 \cdot 10^{12}}{(-5+j5)\, 10^5 \cdot j\, 10^6} = \frac{40}{-1-j} =$$

$$= -\frac{40}{1+j} = -\frac{40}{\sqrt{2}}\, e^{-j45^\circ} \Rightarrow$$

$$u(t) = 20 - \frac{40}{\sqrt{2}} \cdot e^{-5 \cdot 10^5 t} \cdot \cos\left(5 \cdot 10^5 t - 45^\circ\right)$$

5b. $U = \dfrac{E}{s(s^2+sRC+LC)}$

$u(t)$ är icke-svängande om polerna är reella.

$$s^2+sRC+LC=0 \Rightarrow s=-\frac{RC}{2} \pm \sqrt{\frac{R^2C^2}{4}-LC}$$

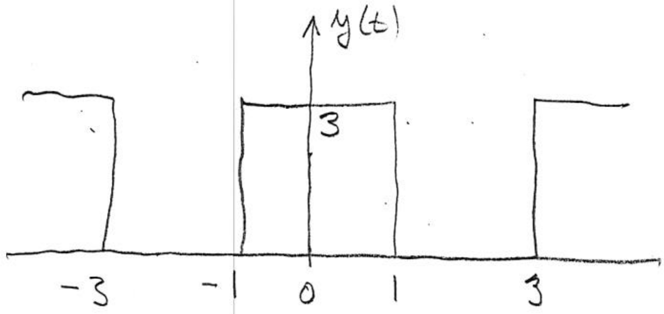
reellt om

$$\frac{R^2C^2}{4} \geq LC \Rightarrow R^2 > \frac{4LC}{C^2} = \frac{4L}{C} =$$

$$= \frac{4\cdot 10^{-3}}{2\cdot 10^{-9}} = 2\cdot 10^{6} \Rightarrow R \geq \sqrt{2}\cdot 1000\,\Omega = \underline{\underline{1414\,\Omega}}$$

6a. Betrakta $y(t) = x(t) + 1 \Rightarrow$

$C_{0x} = C_{0y} - 1 \qquad C_{kx} = C_{ky}, \quad k \neq 0$



Välj perioden $-2 \leq t \leq 2$

$T = 2-(-2) = 4 \Rightarrow \underline{\underline{\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}}} \Rightarrow \omega_0 T = 2\pi$

$\underline{k=0}$

$C_{0y} = \frac{1}{T}\int_{-2}^{2} y(t)\,dt = \frac{1}{4}\int_{-1}^{1} 3\,dt = 1{,}5 \Rightarrow$

$\underline{\underline{C_{0x}}} = 1{,}5 - 1 = \underline{\underline{0{,}5}}$

$\underline{k \neq 0}$

$C_{kx} = C_{ky} = \frac{1}{T}\int_{-2}^{2} y(t)\, e^{-jk\omega_0 t}\,dt = \frac{1}{T}\int_{-1}^{1} 3 \cdot e^{-jk\omega_0 t}\,dt =$

$= \frac{3}{-jk\omega_0 T}\left[e^{-jk\omega_0 t}\right]_{-1}^{1} = \frac{3}{-jk2\pi}\left(e^{-jk\omega_0} - e^{+jk\omega_0}\right) =$

$= \frac{3 \cdot (-2\cancel{j} \sin k\omega_0)}{-\cancel{j}k2\pi} = \frac{3 \sin k \cdot \frac{\pi}{2}}{k\pi} \Rightarrow$

$C_{-2} = \frac{3 \cdot \sin(-\pi)}{-2\pi} = \underline{\underline{0}} \qquad C_1 = \frac{3 \cdot \sin\frac{\pi}{2}}{\pi} = \underline{\underline{\frac{3}{\pi}}}$

$C_{-1} = \frac{3 \cdot \sin(-\frac{\pi}{2})}{-\pi} = \underline{\underline{\frac{3}{\pi}}} \qquad C_2 = \frac{3 \cdot \sin\pi}{2\pi} = \underline{\underline{0}}$

<u>6b</u>. Sampla med $f_s > 2 \cdot f_{max}$

$$f_1 = \frac{12800}{2\pi} = 2037\,Hz$$

$$f_2 = \frac{8990\pi}{2\pi} = 4495\,Hz$$

$$f_3 = \frac{4{,}5 \cdot 10^4}{2\pi} = 7162\,Hz \Longrightarrow$$

$$f_s \geq 2 \cdot 7162\,Hz = \underline{\underline{14{,}32\,kHz}}$$

7a. $Y(s) = H_1(s)\left(X(s) - H_2(s)\cdot Y(s)\right) \Rightarrow$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 + H_1(s)\cdot H_2(s)} =$$

$$= \frac{\frac{3}{s+1}}{1 + \frac{3}{s+1}\cdot\frac{b}{s+2}} = \frac{3}{s+1+\frac{3b}{s+2}} =$$

$$= \frac{3(s+2)}{s^2+3s+2+3b} \Rightarrow$$

Poler $s = -1{,}5 \pm \sqrt{0{,}25-3b}$ (1)

Icke-svängande stegsvar: reella poler $\Rightarrow$

$$0{,}25 \geq 3b \Rightarrow b \leq \frac{0{,}25}{3} = \frac{1}{12}$$

Stabilt system: Poler $> 0 \Rightarrow$

$$0{,}25 - 3b < 1{,}5^2 = 2{,}25 \Rightarrow 3b > -2 \Rightarrow b > -\frac{2}{3}$$

$$\Rightarrow -\frac{2}{3} < b \leq \frac{1}{12}$$

7b. Stegsvarets poler skall vara

$s = 0$ $s = -2{,}5$ $s = -0{,}5$ Kan det erhållas ur (1) ovan

$s = -2{,}5$ $s = -1{,}5 \pm \sqrt{0{,}25-3b} = -2{,}5 \Rightarrow$

$\sqrt{0{,}25-3b} = 1 \Rightarrow 0{,}25 - 3b = 1$ $b = -0{,}25$

Ger detta $s = -0{,}5$?

$s = -1{,}5 + \sqrt{0{,}25 - 3\cdot(-0{,}25)} = -0{,}5$ Ja!

(8.) Filtret skall släcka $f = 150\,Hz$.

$f_s = 800\,Hz \implies$

Filtrets nollställen skall

vara $Z = e^{\pm j\Omega} = \cos\Omega \pm j\sin\Omega$, där

$$\Omega = 2\pi \cdot \frac{f}{f_s}\ (\text{rad}) = 2\pi \cdot \frac{150}{800}\ \text{rad} = 0{,}375\pi\ \text{rad} = 67{,}5^\circ$$

$$\implies Z = \cos 67{,}5^\circ \pm j\sin 67{,}5^\circ \implies$$

$$H(Z) = \frac{(Z - \cos 67{,}5^\circ - j\sin 67{,}5^\circ)(Z - \cos 67{,}5^\circ + j\sin 67{,}5^\circ)}{Z^2 + 0{,}95aZ + 0{,}9025b} =$$

$$= \frac{Z^2 - 2\cos 67{,}5^\circ Z + \sin^2 67{,}5^\circ + \cos^2 67{,}5^\circ}{(\qquad)} =$$

$$= \frac{Z^2 - 0{,}7654Z + 1}{(\qquad)} \implies$$

$$\underline{\underline{a = -0{,}7654}}$$

$$\underline{\underline{b = 1}}$$

9.

$$u(t) = 5 \cdot e^{-10 \cdot |t|} = \begin{cases} 5 \cdot e^{-10 \cdot (-t)} = 5 \cdot e^{10t}, & t < 0 \\ 5 \cdot e^{-10t}, & t \geq 0 \end{cases} \Rightarrow$$

$$U(\omega) = \int_{-\infty}^{\infty} u(t) e^{-j\omega t} dt =$$

$$= \int_{-\infty}^{0} 5 \cdot e^{10t} \cdot e^{-j\omega t} dt + \int_{0}^{\infty} 5 \cdot e^{-10t} \cdot e^{-j\omega t} dt =$$

$$= 5 \cdot \int_{-\infty}^{0} e^{(10-j\omega)t} dt + 5 \int_{0}^{\infty} e^{-(10+j\omega)t} dt =$$

$$= \frac{5}{10-j\omega} \left[e^{(10-j\omega)t} \right]_{-\infty}^{0} + \frac{5}{-(10+j\omega)} \left[e^{-(10+j\omega)t} \right]_{0}^{\infty} =$$

$$= \frac{5}{10-j\omega} (1-0) + \frac{5}{-(10+j\omega)} (0-1) =$$

$$= 5 \cdot \left(\frac{1}{10-j\omega} + \frac{1}{10+j\omega} \right) = \frac{5(10+j\omega+10-j\omega)}{(10-j\omega)(10+j\omega)} =$$

$$= \underline{\underline{\frac{100}{100+\omega^2}}}$$