Lösningar Linjära system 23 okt 2010

1. $f_s = 10 kHz \Rightarrow T_s = 10^{-4} s \Rightarrow t = n \cdot 10^{-4} \Rightarrow$

$$x[n] = 4 \cdot e^{-800 \cdot n \cdot 10^{-4}} \cdot \cos[6500\pi \cdot n \cdot 10^{-4}] = \underline{\underline{4 \cdot 0{,}9231^n \cdot \cos[0{,}65\pi \cdot n]}}$$

2a. $f(t) = 2 \cdot t \cdot e^{-8{,}5t} \cdot e^{4{,}5t} = 2t \cdot e^{-4t} \supset 2 \cdot \frac{1}{(s+4)^2}$

2b. $x[n] = \left(\frac{0{,}3}{0{,}6}\right)^n + \left(\frac{0{,}72}{0{,}6}\right)^n = 0{,}5^n + 1{,}2^n \supset \underline{\underline{\frac{z}{z-0{,}5} + \frac{z}{z-1{,}2}}}$

3a. z-transformera $Y(z) = 2X(z) + 0{,}2z^{-1}Y(z) \Rightarrow$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1-0{,}2z^{-1}} = \underline{\underline{\frac{2z}{z-0{,}2}}}$$

3b. $H(z)$ = z-transform av impulssvar $= \frac{z}{z-0{,}4}$

Insignal $\sigma[n] \Rightarrow X(z) = \frac{z}{z-1} \Rightarrow$

$$Y(z) = H(z) \cdot X(z) = \frac{z}{z-0{,}4} \cdot \frac{z}{z-1} = z \cdot \frac{z}{(z-0{,}4)(z-1)} = z \cdot \frac{T}{N}$$

$N' = 1 \cdot (z-1) + (z-0{,}4) \cdot 1$, Poler $z = \begin{cases} 0{,}4 \\ 1 \end{cases} \Rightarrow$

$$y[n] = A \cdot 0{,}4^n + B \cdot 1^n = A \cdot 0{,}4^n + B$$

$$A = \left[\frac{T}{N'}\right]_{z=0{,}4} = \frac{0{,}4}{0{,}4-1} = -\frac{2}{3} \quad , \quad B = \left[\frac{T}{N'}\right]_{z=1} = \frac{1}{1-0{,}4} = \frac{5}{3} \Rightarrow$$

$$\underline{\underline{y[n] = \frac{5}{3} - \frac{2}{3} \cdot 0{,}4^n}}$$

(4.) Impolssvar = invers av överföringsfunktionen $H(s)$

$$H(s) = \frac{s}{s^2+120s+6100} = \frac{T}{N} \; ; \; N' = 2s+120$$

Poler $s = -60 \pm j50 \Rightarrow h(t) = A \cdot e^{-60t} \cos(50t+\alpha)$

$$Ae^{j\alpha} = 2 \cdot \left[\frac{T}{N'}\right]_{s=-60+j50} = 2 \cdot \frac{-60+j50}{-120+j100+120} =$$

$$= 2 \cdot \frac{-60+j50}{j100} = 2 \cdot \frac{-0{,}6+j0{,}5}{j} = 2 \cdot (0{,}5+j0{,}6) =$$

$$= 2 \cdot \sqrt{0{,}61}\, e^{j50{,}2^\circ} \Rightarrow$$

$$h(t) = 2\sqrt{0{,}61} \cdot e^{-60t} \cdot \cos(50t + 50{,}2^\circ) =$$

$$= \underline{\underline{1{,}56 \cdot e^{-60t} \cdot \cos(50t + 50{,}2^\circ)}}$$

(5.) Fyrkant: $T_1 = 0{,}004\,s \Rightarrow f_0 = 250\,Hz$

Sinus : $T_2 = \dfrac{0{,}004\,s}{4{,}5} \Rightarrow f = 1125\,Hz$ $\Bigg\} \Rightarrow$

Signalen innehåller frekvenserna $\begin{cases} 1125\,Hz \\ k \cdot 250\,Hz \quad k=1,2,3,\ldots \end{cases}$

(6.) $$Y(z) = \frac{z}{z-0{,}8} - \frac{z}{z-0{,}6} = \frac{0{,}2z}{z^2-1{,}4z+0{,}48}$$

$$X(z) = \frac{z}{z-1} \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\dfrac{0{,}2z}{z^2-1{,}4z+0{,}48}}{\dfrac{z}{z-1}} =$$

$$= \frac{0{,}2z-0{,}2}{z^2-1{,}4z+0{,}48} = \frac{0{,}2z^{-1}-0{,}2z^{-2}}{1-1{,}4z^{-1}+0{,}48z^{-2}} \Rightarrow$$

$$\underline{\underline{y[n] = 0{,}2x[n-1] - 0{,}2x[n-2] + 1{,}4y[n-1] - 0{,}48y[n-2]}}$$

7a.

$$U(s) = \frac{\frac{1}{sC}}{\frac{1}{sC} + R + sL} \cdot \frac{10}{s} = \frac{10}{s(1+sRC+s^2LC)} =$$

$$= \frac{10}{s(0{,}2s^2+1{,}2s+1)} = \frac{50}{s(s^2+6s+5)} = \frac{T}{N}$$

$$N' = 1\cdot(s^2+6s+5) + s\cdot(2s+6)$$

Poler: $s=0$, $s=-1$, $s=-5$ $\Rightarrow$

$$u(t) = A + B\cdot e^{-t} + C\cdot e^{-5t}$$

$$A = \left[\frac{T}{N'}\right]_{s=0} = \frac{50}{5} = 10 \qquad B = \left[\frac{T}{N'}\right]_{s=-1} = \frac{50}{-1\cdot 4} = -12{,}5$$

$$C = \left[\frac{T}{N'}\right]_{s=-5} = \frac{50}{-5\cdot(-4)} = 2{,}5 \Rightarrow$$

$$\underline{\underline{u(t) = 10 - 12{,}5\cdot e^{-t} + 2{,}5 e^{-5t}}}$$

7b.

$$U(s) = \frac{10}{s(1+sRC+s^2LC)} = \frac{10}{s(1+s\cdot R\cdot 0{,}2+s^2\cdot 0{,}2)} =$$

$$= \frac{50}{s(s^2+sR+5)}$$ Poler: $s^2+sR+5=0 \Rightarrow$

$$s = -\frac{R}{2} \pm \sqrt{\frac{R^2}{4}-5}\,.$$

$u(t)$ blir svängande om polerna är komplexa, dvs om $\frac{R^2}{4}-5<0$ $\quad R^2<20$ $\quad R<\sqrt{20}\,\Omega \approx \underline{\underline{4{,}47\,\Omega}}$

8a. $T = 5 \Rightarrow \omega_0 = \frac{2\pi}{5} = 0{,}4\pi \quad \omega_0 T = 2\pi$

$$c_0 = \frac{1}{T}\int_{1\,\text{period}} x(t)\,dt = \frac{1}{5}\int_{-1}^{4} x(t)\,dt = \frac{1}{5}\cdot\left(2\cdot 4 + 3\cdot(-1)\right) = \underline{\underline{1}}$$

$c_k,\ k \neq 0$: Betrakta $x(t)+1$ som har samma c_k.

$$c_k = \frac{1}{T}\int_{1\,\text{period}} x(t)\,e^{-jk\omega_0 t}\,dt = \frac{1}{T}\int_{-1}^{1} 5\cdot e^{-jk\omega_0 t}\,dt = \frac{5}{T}\left[\frac{1}{-jk\omega_0}e^{-jk\omega_0 t}\right]_{-1}^{1} =$$

$$= \frac{5}{-jk\omega_0 T}\cdot\left(e^{-jk\omega_0} - e^{jk\omega_0}\right) = \frac{5}{-jk2\pi}\cdot\left(-2j\sin(k\cdot 0{,}4\pi)\right) =$$

$$= \frac{5\cdot\sin(k\cdot 0{,}4\pi)}{k\pi} \Rightarrow \underline{\underline{x(t) = 1 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{5\sin(k\cdot 0{,}4\pi)}{k\pi}\cdot e^{jk0{,}4\pi t}}}$$

Reell serie

$$a_0 = c_0 = 1\ ;\ a_k = 2\,\mathrm{Re}\,c_k = \frac{10\sin(k\cdot 0{,}4\pi)}{k\pi}\ ;\ b_k = -2\,\mathrm{Im}\,c_k = 0$$

$$\Rightarrow \underline{\underline{x(t) = 1 + \sum_{k=1}^{\infty}\frac{10\sin(k\cdot 0{,}4\pi)}{k\pi}\cdot\cos(k\cdot 0{,}4\pi t)}}$$

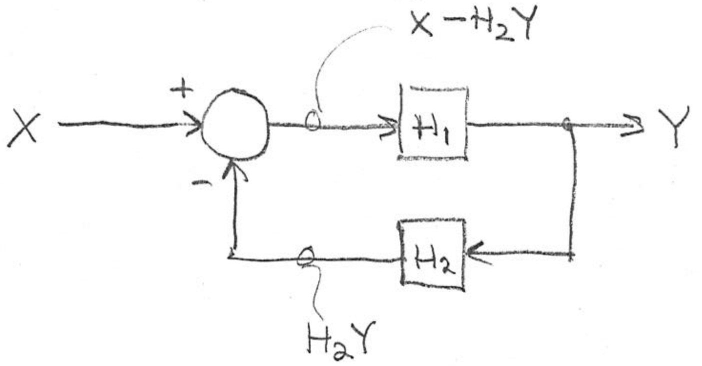
8b. $x(t) = 6\cos 8630t \Rightarrow \omega = 8630\ \text{rad/s} = \frac{2\pi\ \text{rad}}{T} \Rightarrow$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8630}\,\text{s} = 728\,\mu\text{s}.$$

25 sampel/period $\Rightarrow T_s = \frac{728\,\mu\text{s}}{25} = \underline{\underline{29\,\mu\text{s}}} \Rightarrow$

$$f_s = \frac{1}{T_s} = \underline{\underline{34{,}34\ \text{kHz}}}$$

9.



$$H_1(s)=\frac{a}{s+2}$$

$$H_2(s)=\frac{3}{s+4}$$

$$\Rightarrow \quad Y = H_1\cdot(X-H_2Y) = H_1X - H_1H_2\cdot Y \Rightarrow$$

$$H=\frac{Y}{X}=\frac{H_1}{1+H_1\cdot H_2}=\frac{\frac{a}{s+2}}{1+\frac{a}{s+2}\cdot\frac{3}{s+4}}=\frac{a}{s+2+a\cdot\frac{3}{s+4}}=$$

$$=\frac{a(s+4)}{(s+2)(s+4)+3a}=\frac{a(s+4)}{s^2+6s+8+3a}$$

Stegsvaret blir icke-svängande och stabilt om $H(s)$ har reella negativa poler.

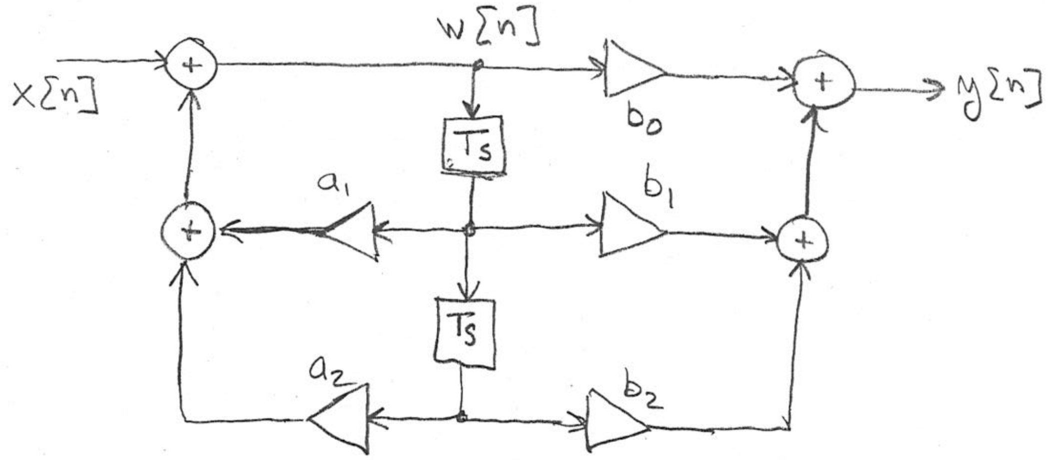
$$s^2+6s+8+3a=0 \Rightarrow s=-3\pm\sqrt{9-(8+3a)}=-3\pm\sqrt{1-3a}$$

Reella: $1-3a\geq 0 \Leftrightarrow a\leq\frac{1}{3}$

Negativa: $\sqrt{1-3a}<3 \Leftrightarrow 1-3a<9 \Leftrightarrow a>-\frac{8}{3}$

Alltså $-\frac{8}{3}<a\leq\frac{1}{3}$

10.



z-transformera och sätt upp sambanden mellan $X(z)$, $W(z)$ och $Y(z)$:

$$W(z) = X(z) + a_1 z^{-1} W(z) + a_2 z^{-2} W(z) \qquad (1)$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) \qquad (2)$$

(1) ger $W(z) = \dfrac{1}{1 - a_1 z^{-1} - a_2 z^{-2}} \cdot X(z)$

(2) ger $Y(z) = (b_0 + b_1 z^{-1} + b_2 z^{-2}) \cdot W(z)$ $\Rightarrow$

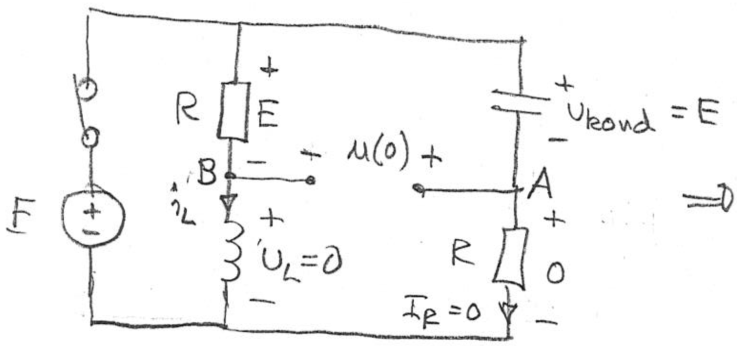
$$Y(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} \cdot X(z) \qquad (3) \Rightarrow$$

$$Y(z)(1 - a_1 z^{-1} - a_2 z^{-2}) = (b_0 + b_1 z^{-1} + b_2 z^{-2}) X(z) \Rightarrow$$

$$\underline{\underline{y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + a_1 y[n-1] + a_2 y[n-2]}}$$

(3) $\Rightarrow$ $$\underline{\underline{H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}}}$$

11a. Stationär DC: spolens spänning = kondens ström = 0 ⇒

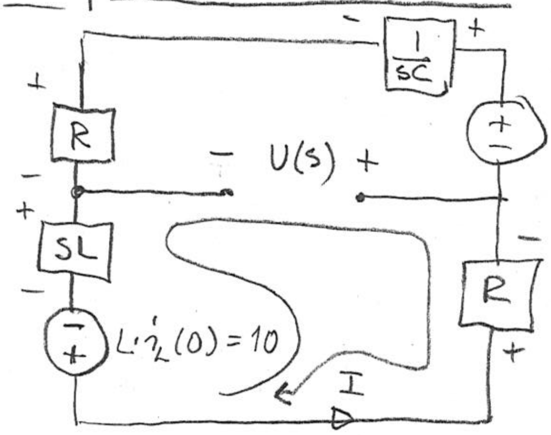


$$\Rightarrow \underline{\underline{u(0) = V_A - V_B = 0}}$$

11b. Enligt schemat i 11a gäller

$$u_{kond.}(0) = E = 10\,V \; ; \quad i_L(0) = \frac{E}{R} = \frac{10\,V}{1\,\Omega} = 10\,A$$

<u>komplext schema $t>0$</u>



$$\frac{u_{kond}(0)}{s} = \frac{10}{s}$$

$$I = \frac{10 + \frac{10}{s}}{2R + sL + \frac{1}{sC}} = \frac{10s+10}{\frac{1}{C} + s^2L + 2sR} = \frac{10s+10}{\frac{1}{0{,}5} + s^2 + 2s}$$

$$= \frac{10s+10}{s^2+2s+2} \,,$$ Kirchhoff i nedre maskan ger

$$-10 + sL\cdot I + U(s) + R\cdot I = 0 \Rightarrow U(s) = +10 - (R+sL)\cdot I =$$

$$= 10 - (1+s)\cdot\frac{10s+10}{s^2+2s+2} = \frac{10s^2+20s+20-10s-10-10s^2-10s}{s^2+2s+2}$$

$$= \frac{10}{s^2+2s+2} = \frac{T}{N} \; ; \; N' = 2s+2 \; ; \text{ poler } s = -1 \pm j \Rightarrow$$

$$u(t) = Ae^{-t}\cdot\cos(t+\alpha)$$

$$Ae^{j\alpha} = 2\cdot\left[\frac{T}{N'}\right]_{s=-1+j} = \frac{2\cdot 10}{-2+j2+2} = \frac{10}{j} = 10e^{-j90^\circ} \Rightarrow$$

$$A = 10 \; ; \; \alpha = -90^\circ \Rightarrow \underline{\underline{u(t) = 10e^{-t}\cos(t-90^\circ)}} = \underline{\underline{10e^{-t}\sin t}}$$

12. Filtret släcker frekvenser som ges av dess nollställen.

I detta fall skall nollstället ha beloppet 1 och vinkeln $\Omega = 2\pi \frac{f}{f_s} = 2\pi \cdot \frac{150\,Hz}{800\,Hz}$ rad $= \underline{1{,}178\text{ rad}} = \underline{67{,}5^\circ} \Rightarrow$

$$z_{noll} = e^{\pm j67{,}5^\circ} = \cos 67{,}5^\circ \pm j67{,}5^\circ \Rightarrow$$

$$H(z) = K \cdot \frac{(z - \cos 67{,}5^\circ + j\sin 67{,}5^\circ)(z - \cos 67{,}5^\circ - j\sin 67{,}5^\circ)}{z^2 + 0{,}95az + 0{,}9025b} =$$

$$= K \cdot \frac{(z - \cos 67{,}5^\circ)^2 + \sin^2 67{,}5^\circ}{z^2 + 0{,}95az + 0{,}9025b} = K \cdot \frac{z^2 - 0{,}7654z + 1}{z^2 + 0{,}95az + 0{,}9025b} \Rightarrow$$

$\underline{K = 1}$ $\quad \underline{\underline{a = -0{,}7654}}$ $\quad \underline{\underline{b = 1}}$