1. Komplex Fourierserie:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

Grundvinkelfrekvensen $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = \underline{\underline{1}}$

$$\Rightarrow f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt}$$

$$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t) e^{-jkt}\,dt =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 3e^{-jkt}\,dt + \frac{1}{2\pi}\int_{0}^{\pi}(-3)e^{-jkt}\,dt =$$

$$= \frac{3}{2\pi}\left[e^{-jkt}\right]_{-\pi}^{0}\frac{1}{-jk} - \frac{1\cdot 3}{2\pi}\frac{1}{-jk}\left[e^{-jkt}\right]_{0}^{\pi} =$$

$$= \frac{3j}{2\pi k}\left[1-e^{jk\pi}\right] - \frac{3j}{2\pi k}\left[e^{-jk\pi}-1\right] =$$

$$= \frac{3j}{2\pi k}\left[2-(e^{jk\pi}+e^{-jk\pi})\right] = \frac{3j}{2\pi k}\left[2-2\cos(\pi k)\right] =$$

$$= \frac{3j}{\pi k}(1-\cos(\pi k)).$$

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Så för $k \neq 0$ är $c_k = \frac{3j}{\pi k}(1-\cos(\pi k))$.

För $k=0$ får vi

$$c_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)e^{0}\,dt = \frac{1}{2\pi}\int_{-\pi}^{0} 3\,dt + \frac{1}{2\pi}\int_{0}^{\pi}(-3)\,dt =$$

$$= \underline{\underline{0}}.$$

Så

$$\begin{cases} f(t) = \sum_{k\neq 0} c_k e^{jkt}, \\ \text{där} \quad c_k = \frac{3j}{k\pi}(1-\cos(k\pi)). \end{cases}$$

För att hitta den reella Fourierserien använder vi sambanden

$$a_k = 2\operatorname{Re}(c_k)$$
$$b_k = -2\operatorname{Im}(c_k).$$

Eftersom c_k är rent imaginär är

$$a_k = 0 \quad \forall k.$$

För b_k får vi

$$b_k = \frac{-6}{k\pi}(1-\cos(k\pi)).$$

Alltså är

$$f(t) = -\frac{6}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} (1 - \cos(k\pi)) \sin(kt).$$

2. Funktionen är udda eftersom

$$f(-t) = -f(t).$$

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(2) Rita functionen:

$f(t) = 3u(2t+2) - 4(t-4)u(t-4)$

$$u(\tau) = \sigma(\tau) = \begin{cases} 1 & \tau > 0 \\ 0 & \tau < 0 \end{cases}$$

Consider $f(t) = g(t) + h(t)$ where

$g(t) = 3u(2t+2)$ and $h(t) = -4(t-4)u(t-4)$

$$u(2t+2) = \begin{cases} 1 & 2t+2 > 0 \Rightarrow t > -1 \\ 0 & 2t+2 < 0 \Rightarrow t < -1 \end{cases}$$

$$u(t-4) = \begin{cases} 1 & t-4 > 0 \Rightarrow t > 4 \\ 0 & t-4 < 0 \Rightarrow t < 4 \end{cases}$$

Then $g(t) = 3u(2t+2) = \begin{cases} 3 & t > -1 \\ 0 & t < -1 \end{cases}$

$$h(t) = -4(t-4)u(t-4) = \begin{cases} -4(t-4) & t > 4 \\ 0 & t < 4 \end{cases}$$

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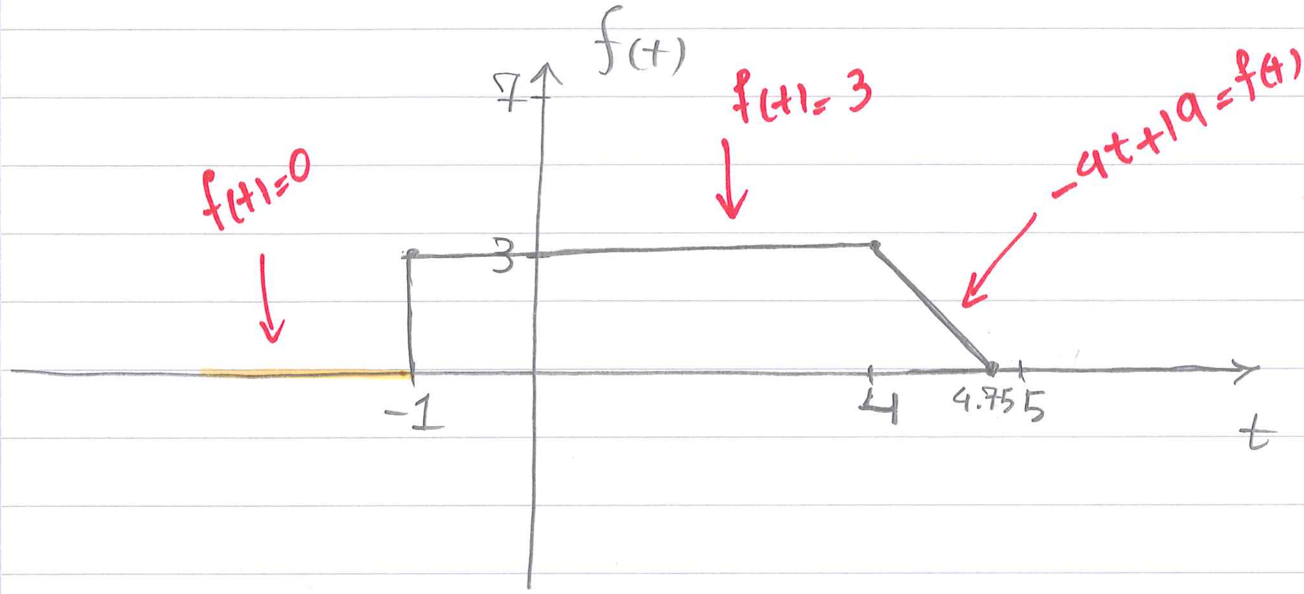
$$\text{So} \quad f(t) = g(t) + h(t) = \begin{cases} 0 & t < -1 \\ 3 + 0 = 3 & -1 < t < 4 \\ 3 - 4(t-4) = -4t + 19 & t > 4 \end{cases}$$



③ Fourier Transform of $f(t)$

$$f(t) = e^{-2t}\left[u(t-1) - u(t-3)\right]$$

$$f(t) = \begin{cases} e^{-2t}(0-0) = 0, & t<1 \\ e^{-2t}(1-0) = e^{-2t}, & 1<t<3 \\ e^{-2t}(1-1) = 0, & t>3 \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{1} \cancelto{0}{f(t) e^{-j\omega t}}\, dt + \dots$$

$f(t)=0$

FT of $f(t)$

$$\dots = \int_{1}^{3} f(t)\, e^{-j\omega t}\, dt + \int_{3}^{+\infty} \cancelto{0}{f(t) e^{-j\omega t}}\, dt = \int_{1}^{3} f(t) e^{-j\omega t}\, dt$$

$f(t)=0$

$$= \int_{1}^{3} e^{-2t} \cdot e^{-j\omega t}\, dt = \int_{1}^{3} e^{-(2+j\omega)t}\, dt = \frac{1}{-(2+j\omega)}\, e^{-(2+j\omega)t}\Big]_{1}^{3}$$

$$= \frac{-1}{-(2+j\omega)}\left[e^{-(6+3j\omega)} - e^{-(2+j\omega)}\right]$$

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(4) Laplace Transform of

(a) $f(t) = \dfrac{e^{-t} + 3e^{4t}}{5e^{5t}} = 0.2\,e^{-6t} + 0.6\,e^{-t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{0.2e^{-6t} + 0.6e^{-t}\right\}$

$= \dfrac{0.2}{s+6} + \dfrac{0.6}{s+1} = \underline{\dfrac{0.8s + 3.8}{s^2+7s+6}}$

(b) $f(t) = e^{-2t}\cos(2t - \pi/6)$

we note : $e^{-\alpha t} \cdot g(t) \xrightarrow{\mathcal{L}} G(s+\alpha)$

we let $g(t) = \cos(2t - \pi/6)$

$G(s) = \mathcal{L}\{g(t)\} = \mathcal{L}\{\cos(2t - \pi/6)\}$

$= \mathcal{L}\left\{\cos 2t \cos \pi/6 + \sin 2t \sin \pi/6\right\} = \left\{\begin{array}{l}\cos \pi/6 = \frac{\sqrt{3}}{2} \\ \sin \pi/6 = \frac{1}{2}\end{array}\right\}$

$= \mathcal{L}\left\{\frac{\sqrt{3}}{2}\cos 2t + \frac{1}{2}\sin 2t\right\} = \frac{\sqrt{3}}{2}\,\frac{s}{s^2+2^2} + \frac{1}{2}\,\frac{2}{s^2+2^2}$

$= \dfrac{\sqrt{3}s + 2}{2(s^2+4)} \longrightarrow G(s+2) = \dfrac{\sqrt{3}(s+2)+2}{2((s+2)^2+4)}$ #

4b)

Now $G(s+\alpha) = \{\alpha = 2\}$

$$= G(s+2) = \frac{\sqrt{3}(s+2)+2}{2\left((s+2)^2+4\right)}$$

$$= \frac{\sqrt{3}s + 2\sqrt{3} + 2}{2(s^2+4s+4+4)} = \frac{0.86s + \sqrt{3} + 1}{s^2+4s+8}$$

So for $f(t) = e^{-2t}\cos(2t - \pi/6)$

$$F(s) = G(s+2) = \frac{0.86s + \sqrt{3} + 1}{s^2+4s+8}$$

5a) $F(s) = \dfrac{2}{s^3+2s^2+s}$

$$F(s) = \frac{2}{s(s^2+2s+1)} = \frac{T(s)}{N(s)} \text{, where}$$

$N(s) = s(s^2+2s+1)$, roots of $N(s)=0 \rightarrow s=0=\alpha_1$, $\rightarrow s=-1=\alpha_2$

For $s=0 \rightsquigarrow f_1(t) = Ae^{\alpha_1 t} = Ae^{0t} = A$ where

$$A = \left[\frac{T(s)}{N'(s)}\right]_{s=\alpha=0} = \frac{2}{s^2+2s+1+2s^2+2s} = \left.\frac{2}{3s^2+4s+1}\right]_{s=0}$$

$$\boxed{A=2}$$

So for simple & Real Root $s=0 \rightsquigarrow \underline{f_1(t)=2}$

For Double real roots: $s=-1 \rightsquigarrow$

$$f_2(t) = \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{ds^{n-1}}\left\{(s-\alpha_2)^n \cdot F(s) \cdot e^{st}\right\}\right]_{s=\alpha_2=-1} =$$

$\boxed{n=2}$

$$f_2(t) = \frac{1}{(2-1)!}\left[\frac{d}{ds}\left\{\cancel{(s+1)^2} \cdot \frac{2}{s\cancel{(s+1)^2}} \cdot e^{st}\right\}\right]_{s=-1}$$

$$f_2(t) = \frac{d}{ds}\left\{\frac{2e^{st}}{s}\right\}_{s=-1} = \left.\frac{2te^{st} \times s - 2e^{st}}{s^2}\right]_{s=-1}$$

$$= -2te^{-t} - 2e^{-t} = 2e^{-t}(-1-t)$$

So $\mathcal{L}^{-1}\{F(s)\} = f_1(t) + f_2(t) = 2 + 2e^{-t}(-1-t)$ #

5b)

$$F(S) = \frac{1}{S(S^2+4)} = \frac{T(S)}{N(S)}$$

$$S(S^2+4) = 0 \longrightarrow S = 0 \quad \text{(reel)}$$
$$\searrow S^2 = -4 \quad S = \pm j2 \quad \text{(Komplex)}$$

$$f(t) = f_1(t) + f_2(t) = Ae^{0t} + f_2(t)$$

where $A = \left[\frac{T(s)}{N'(s)}\right]_{S=0} = \left.\frac{1}{3s^2+4}\right|_{S=0} = 0.25$

$$N'(s) = s^2+4+2s^2 = 3s^2+4$$

$$f_2(t) = Be^{\alpha t}\cos(\omega t + \varphi) \quad \text{where}$$

$$Be^{j\varphi} = 2\left[\frac{T(S)}{N'(s)}\right]_{S=\alpha+j\omega}$$

$$Be^{j\varphi} = \left.\frac{2}{3s^2+4}\right]_{s=j2} = \frac{2}{3(-4)+4} = -0.25$$
$$= -0.25e^{j0}$$

$$f_2(t) = -0.25 \cdot e^{0t} \cdot \cos(2t+0)$$

$$f(t) = f_1 + f_2 = 0.25 - 0.25\cos(2t) \quad \#$$