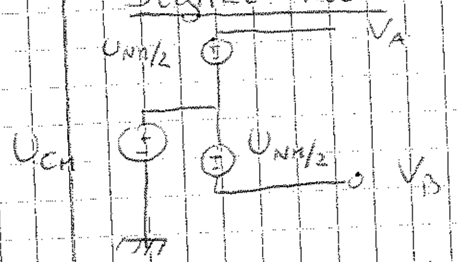
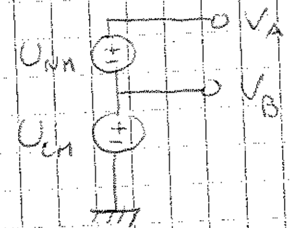
Signal model



Book



not correct!

$$\frac{V_A + V_B}{2} \neq U_{CM}$$

but quicker to draw; so we will use it

$$U_{out} = U_{out,NM} + U_{out,CM}$$

$$F_{NM} = \frac{U_{out,NM}}{U_{NM}} \quad ; \quad F_{CM} = \frac{U_{out,CM}}{U_{CM}}$$

- common mode rejection ratio (CMRR)

$$CMRR = 20 \log_{10} \frac{F_{NM}}{F_{CM}} \quad [dB]$$

- Example

$F_{NM} = 100$; $CMRR = 80$ dB $\qquad F_{CM} = ?$

$$CMRR = 20 \log_{10} \frac{F_{NM}}{F_{CM}}$$

$$\Rightarrow \quad 10^{CMRR/20} = \frac{F_{NM}}{F_{CM}} \Rightarrow F_{CM} = \frac{F_{NM}}{10^{CMRR/20}} = \frac{100}{10^4}$$

$$= 0{,}01$$

- Interpretation

$$\text{"SNR"} = \frac{\text{useful signal}}{\text{disturbance}}$$

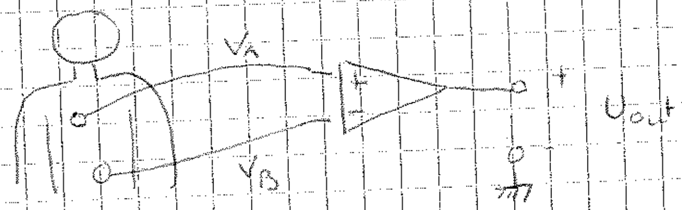
$$\text{"SNR}_{in}\text{"} = \frac{U_{NM}}{U_{CM}} \qquad \text{"SNR}_{out}\text{"} = \frac{U_{out,NM}}{U_{out,CM}} = \frac{U_{NM} \cdot F_{NM}}{U_{CM} \cdot F_{CM}}$$

$$= \text{"SNR}_{in}\text{"} \cdot \underbrace{CMRR}_{\text{linear scale}}$$

$$\Rightarrow \text{"SNR}_{out}\text{"} [dB] = \text{"SNR"}_{in} [dB] + CMRR [dB]$$

Example ECG

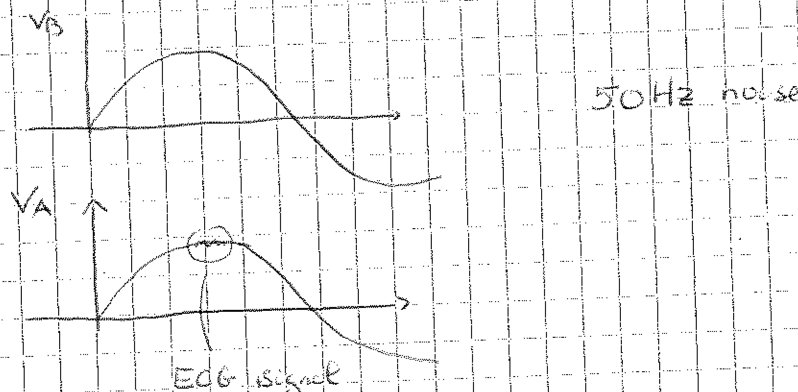
electrode disks



- 50 Hz interference from power line on the cables

$U_{CM} = 4V$

$U_{NM} = 20\,mV$



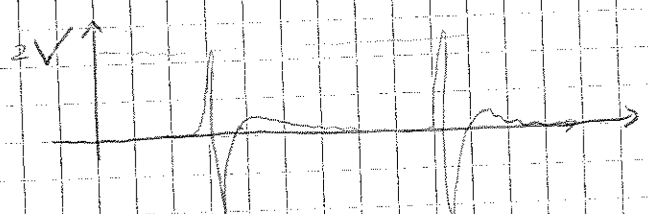
- instrumentation amplifier

$F_{NM} = 100 \qquad CMRR = 80\,dB$

$\Rightarrow U_{NM,out} = 100 \cdot 20\,mV = 2{,}0\,V$

$CMRR = 20 \log_{10} \frac{F_{NM}}{F_{CM}} \Rightarrow F_{CM} = 0.01$

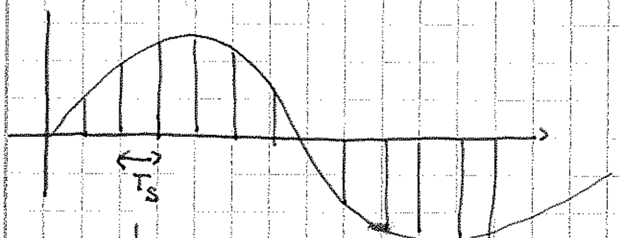
$U_{CM,out} = F_{CM} \cdot U_{CM} = 0{,}04\,V$



sampling and AD converters [cheat sheet]

Slide -> block diagram | analog $\xrightarrow[\text{sampling, quantization}]{}$ digital

- Sampling (repetition)



$f_s = \frac{1}{T_s}$ sampling freq.

$[Hz] \longleftrightarrow \left[\frac{\text{samples}}{s}\right]$

$X[n] = X(nT_s)$ ← nth sample

- $\boxed{\text{Sampling theorem for sin. functions}}$

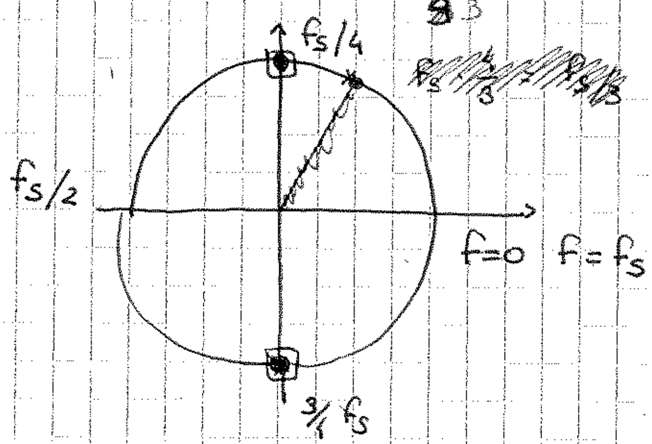
To recover a sin function at f Hz one has to sample at a frequency $f_s > 2f$

Equivalently, $f < f_s/2$.

- If we sample at $f_s < 2f$ we get $\boxed{\text{aliasing}}$: we reconstruct a sin of frequency lower than f !

- slide: aliasing;
- Recall $\Omega = 2\pi \frac{f}{f_s}$: normalized freq.

- In our example: $f_s = \frac{4}{3}$; $f = 1$ $\Rightarrow$ $\Omega = 2\pi \frac{3}{4}$



$$\boxed{f = \frac{3}{4} f_s \Rightarrow \frac{1}{4} f_s = \frac{1}{3} \text{ Hz}}$$

$\frac{3}{4} = -\frac{1}{4} = \frac{1}{4}$

slide: second example

rule: all frequencies of the form

$$\boxed{f = \pm f_1 + m f_s} \qquad m \in \mathbb{N}$$

are mapped to f_1.

input signal

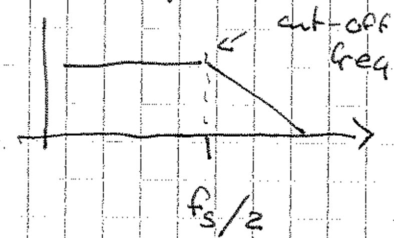
(In) Example : $f_1 = 1\,\text{kHz}$ $\quad f = \cancel{10}\ 9\,\text{kHz} = -f_1 + f_s$ 10

That's why we get 1 kHz.

- **Sampling theorem for bandlimited signals:**

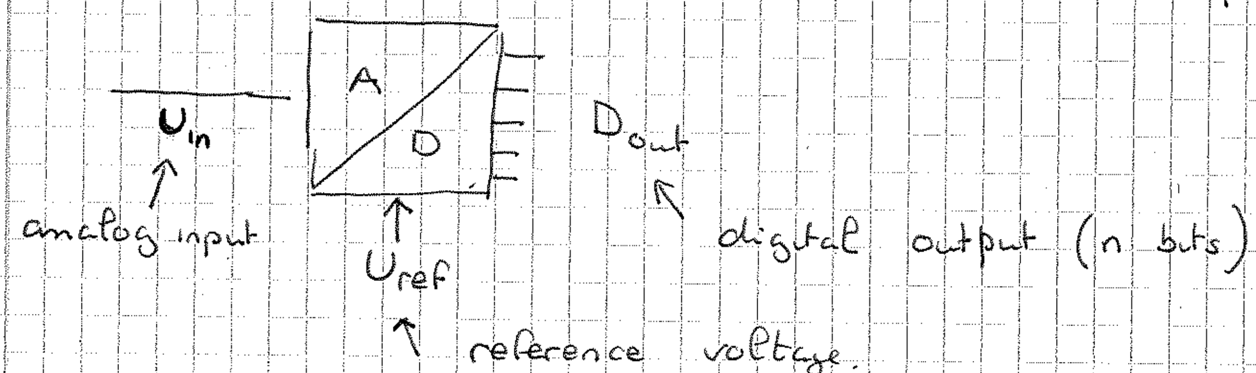
A bandlimited signal can be perfectly reconstructed by its → samples taken at twice the largest freq.

- To avoid aliasing: use lpf ~~filter~~



Analog to digital converters ADC

- perform quantization : real-valued samples → integer-valued samples



- U_{ref} divided into 2^n levels
  $0, 1, \ldots, 2^n - 1$; Compare U_{in} with levels

- Resolution $\Delta U = \dfrac{U_{ref}}{2^n}$: minimum variation of U_{in} that determines a variation in D

Example : slide

- $D_{out} = \left[\frac{U_{in}}{\Delta U} \right]$ → round to nearest integer.

- Given D_{out}, $U_{in} \in \left[D_{out} \cdot \Delta U - \frac{\Delta U}{2}, D_{out} \cdot \Delta U + \frac{\Delta U}{2} \right]$

- $\frac{\Delta}{2}$: maximum quantization error

- $U_Q = U_{in} - D_{out} \cdot \Delta U$ → quantization error

---

- $\boxed{\text{successive approximation}}$: fast (200 MS/s)
  accurate (24 bit)
  cheap

- $\boxed{\text{dual slope}}$: slow (10 S/s)
  very accurate

- $\boxed{\text{flash ADC}}$: very fast (20 GS/s)
  expensive
  low resolution (8 bit)

TPS

- Successive approximation ADC

Y/N game

integer number between 1 and 32

- SLIDE : SAR: successive approximation register
  EOC : end of conversion
  DAC ↔ resistive ladder → need n comparison.
  digital-to-analog converter

- SLIDE . $n=4$ bit

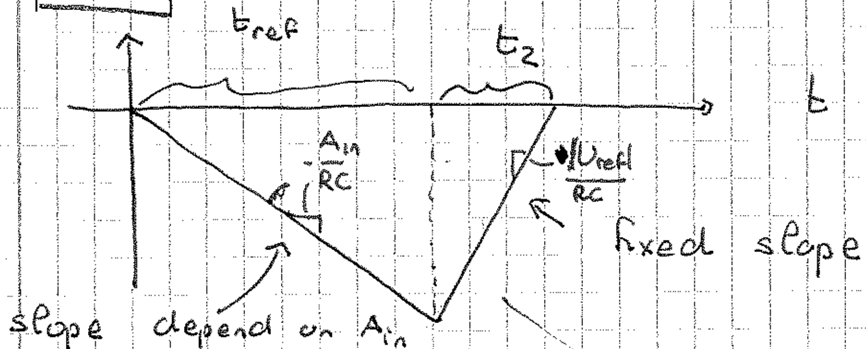
$\Delta U = \frac{5}{\ } = 0,3125$ V

Flash ADC

- slide : general → $2^n - 1$ comparators
- slide : 3 bits
- # bits limited by size, power consumption costs

Dual slope ADC

- **slide**



$\frac{A_{in}}{\cancel{RC}} t_{ref} = \frac{|U_{ref}|}{\cancel{RC}} t_2$

maybe skip!

$$U_1(t) = -\frac{1}{RC} A_{in} \cdot t$$

$$U_2(t) = U_1\Big|_{t_{ref}} + \underbrace{\frac{1}{RC} |U_{ref}|}_{\text{fixed slope.}} \cdot t \qquad t > t_{ref}$$

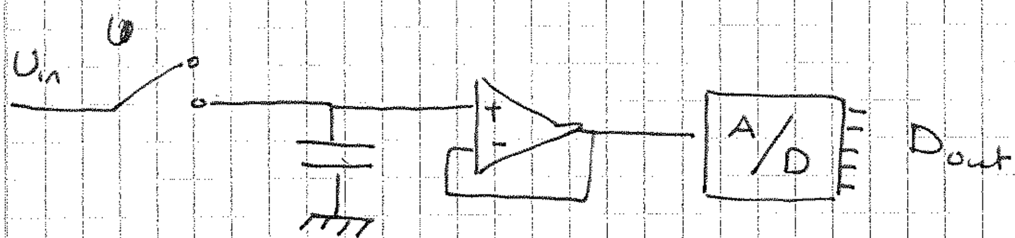
$$\rightarrow A_{in} = \frac{t_2}{t_{ref}} |U_{ref}| \quad \rightarrow \text{[see book for details]}$$

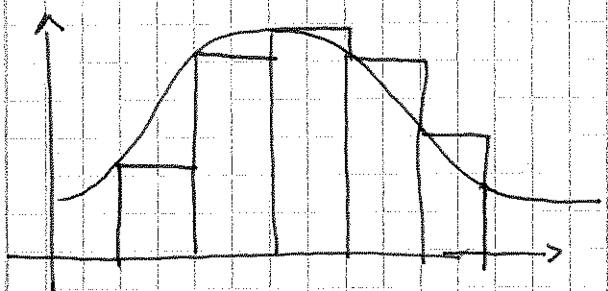
- accurate but slow
- used in **digital voltmeters**
- $t_{ref} = 100$ ms (5 periods @ 50 Hz, 6 periods @ 60 Hz) to avoid power-line disturbances.

Sample & hold

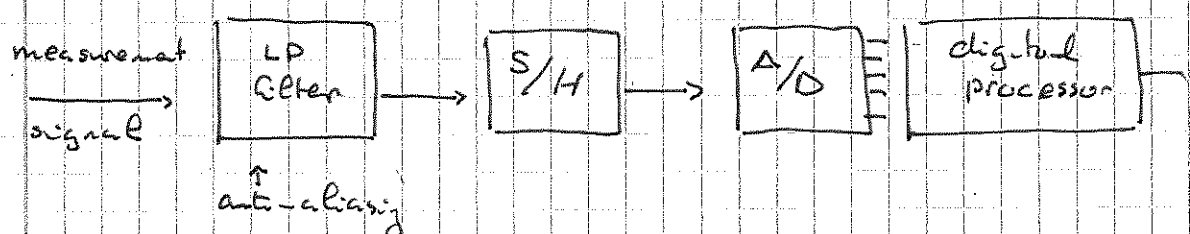
- ADCs need constant signal over an interval

- SH circuitry





- Block schema measurement system



- Oscilloscope

skip

- voltmeter (digital)
- 8 bit vertical resolution, 1000 samples recordable
- max sampling rate: 500 MS/s $\Rightarrow$ $f \leq 250$ MHz

- 3 basic settings
- vertical: attenuation / amplification of signal [V / div]
- horizontal: amount of time per division:

Note larger time/div $\Leftrightarrow$ lower sampling rate

Noise and interference
in electrical measurement systems

cheat sheet
Lecture 5

- measurement signal superimposed to unwanted signal
- extractable with SIG. PROC. methods
- First : prevent disturbances from entering the system
- Related to EMC

system → immune to EM fields ← our goal
system → radiate limited amount of EM

- Disturbances occur if
  - ~~exists~~ a noise source exists
  - noise freq. is within bandwidth of meas. sys.
  - coupling between noise and meas. sys.

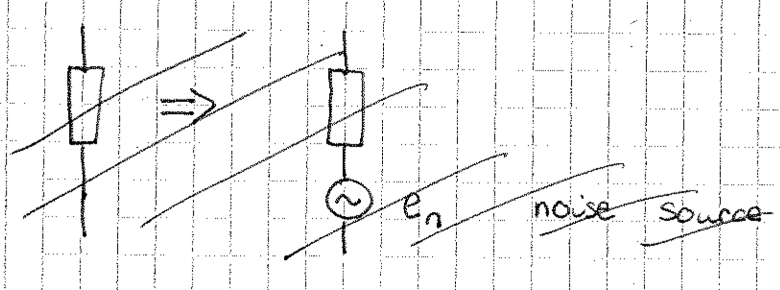
often the only thing we can ~~to~~ work on →

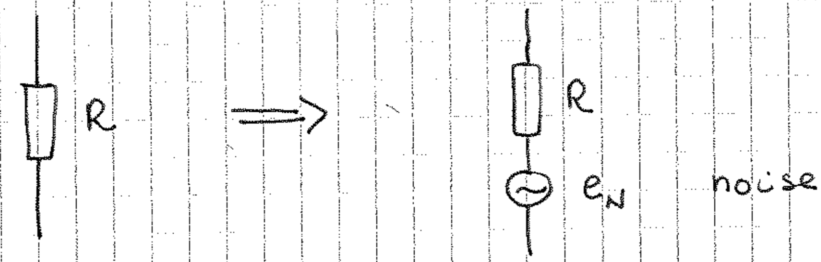
- disturbances can be NM or CM

- Coupling
  - capacitive
  - inductive
  - conductive
  - radiative
  - via ground loops
  - via internally generated noise

Thermal noise

- internally generated ⇒ { thermal agitation of electrons in conductors





Error in book (no π).

- RMS voltage

$$e_N = \sqrt{4 k_B T R B}$$

Statistics:
- standard deviation of (Gaussian) random variable

$k_B = 1,38 \cdot 10^{-23}$ J/K [Boltzman constant]

T = temperature °K

B = bandwidth [Hz] over which noise is measured.

- $e_N \uparrow$ if $T \uparrow$, $R \uparrow$ or $B \uparrow$

or impedance

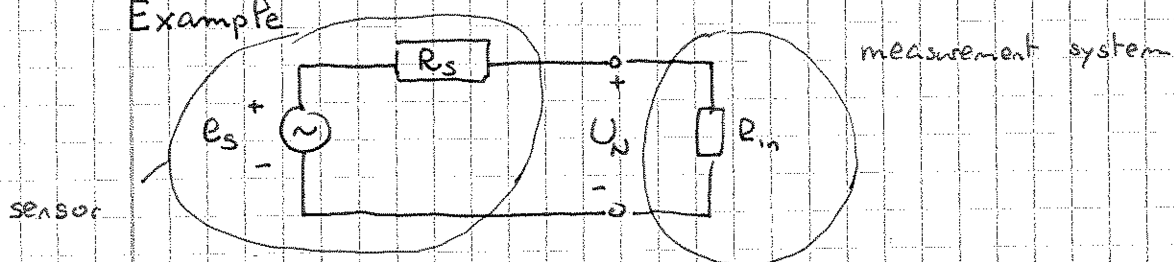
Don't use larger bandwidth values than actually needed.

- independent noise add quadratically:

$$e_{N_1}, \; e_{N_2} \Rightarrow e_{tot} = \sqrt{e_{N_1}^2 + e_{N_2}^2}$$

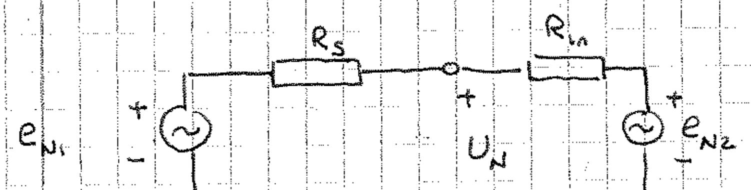
- Think of noise as (Gaussian) r.v. with variance e_N^2 (more later)

Example



- ~~imp.~~ thermal noise impact on U_N?
- Superposition: e_s off + noise sources on

$$e_{N1} = \sqrt{4 k_B T R_s B} \; ; \quad e_{N2} = \sqrt{4 k_B T R_{in} B}$$



- noise from R_s : $U_{N1} = \dfrac{R_{in}}{R_{in} + R_s} e_{N1}$

- noise from R_{in} : $U_{N2} = \dfrac{R_s}{R_{in} + R_s} e_{N2}$

Total noise

$$U_N^2 = U_{N1}^2 + U_{N2}^2 = \frac{R_{in}^2}{(R_{in}+R_s)^2} 4TBk_b R_s + \frac{R_s^2}{(R_{in}+R_s)^2} 4TBk_b R_{in}$$

$$= 4TBk_b \frac{R_{in} R_s \cancel{(R_{in}+R_s)}}{(R_{in}+R_s)^2}$$

$$= 4TBk_b \underbrace{(R_{in} /\!/ R_s)}_{R_p} \Rightarrow U_N = \sqrt{4TBk_b R_p}$$

- Example

$R_s = 100\,\Omega$, $R_{in} = 1\,M\Omega$, $T = 300\,K$, $B = 50\,MHz$

$$R_p = R_s /\!/ R_{in} \approx 100\,\Omega \qquad U_N = \sqrt{4 k_B T R_p B}$$

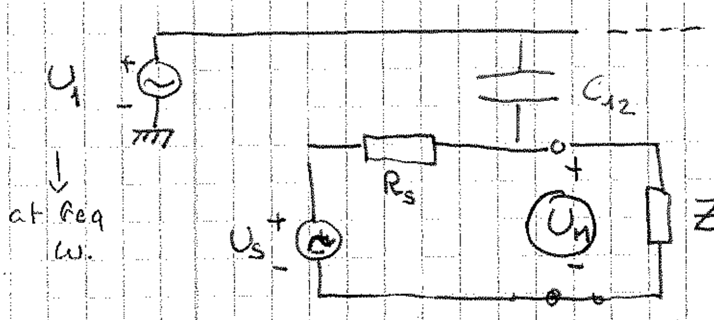
$$\approx 9{,}1\,\mu V$$

- If $R_s = 1\,M\Omega$, however

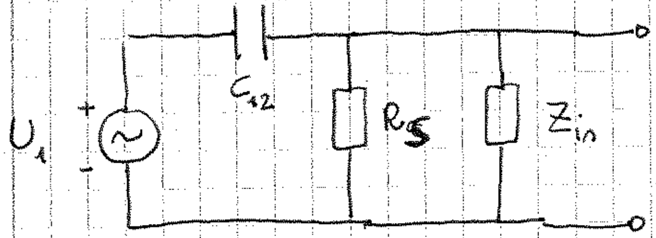
$\Rightarrow R_p = R_s /\!/ R_{in} = 500\,k\Omega \Rightarrow U_N = 0{,}643\,mV$.

- $\boxed{\text{Capacitive coupling}}$

- two ~~wire~~ cables form a condensator!



Impact on U_M

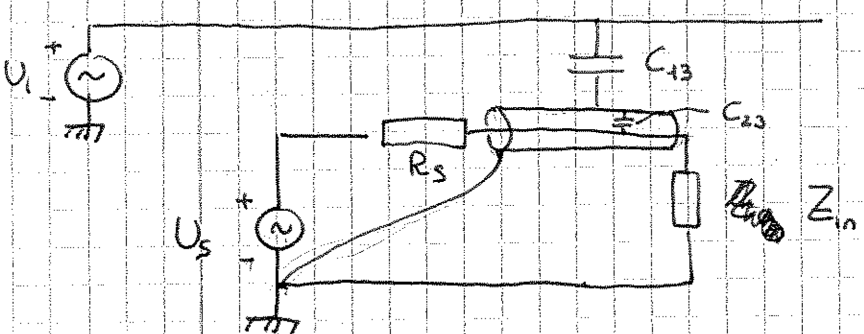


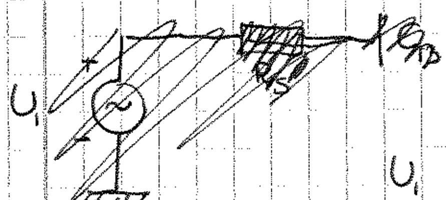
$$Z_p = R_S \parallel Z_{in} \Rightarrow U_M = \frac{Z_p}{Z_p + \frac{1}{j\omega C_{12}}} U_1 \approx j\omega C_{12} Z_p U_1$$

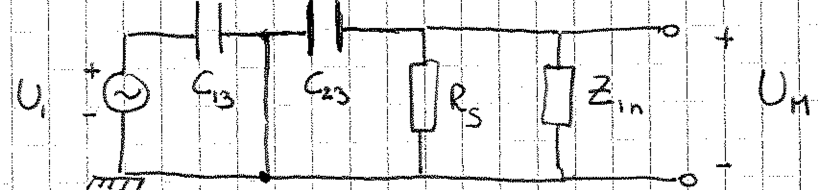
$$|Z_p| \ll \frac{1}{\omega C_{12}}$$

because R_s is small

- Countermeasures:
  - reduce ω or U_1 (often not possible)
  - make $|Z_p|$ small (choose R_s small)
  - use shielded cables that are grounded.



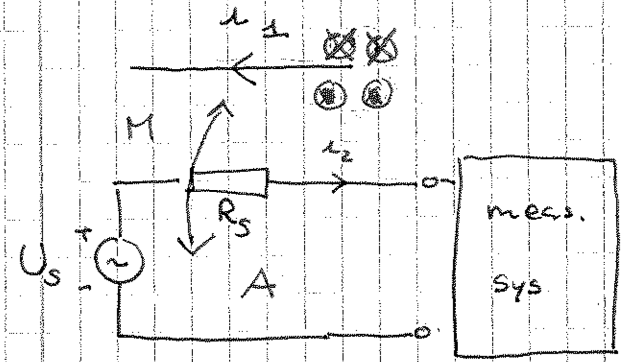




$U_M = \emptyset$ if good connection to ground.

Inductive coupling

- mutual ~~impedence~~ inductance between two cables: they act as transformers



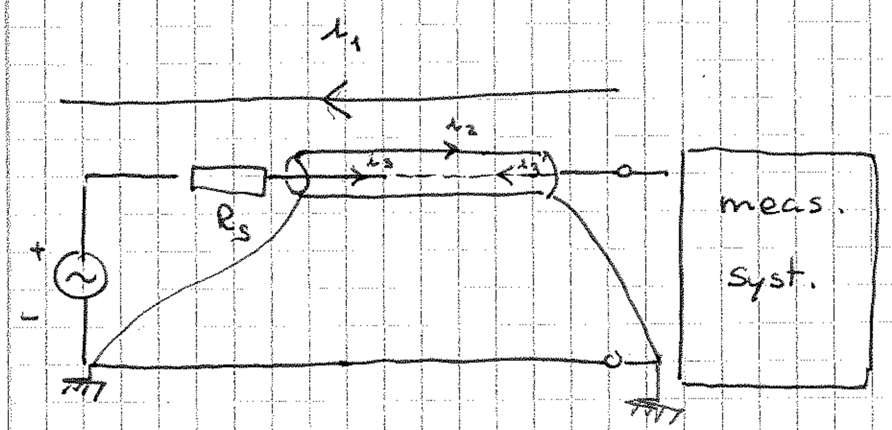
$i_1 \longmapsto$ magnetic field

$\rightarrow$ tension

$$U_{ind} = A \frac{dB}{dt} = M \frac{di_1}{dt}$$

mutual inductance

- Countermeasures
  - reduce $\frac{di_1}{dt} \Rightarrow$ amplitude ↓ freq. ↓
  - reduce M
  - use shielded cable

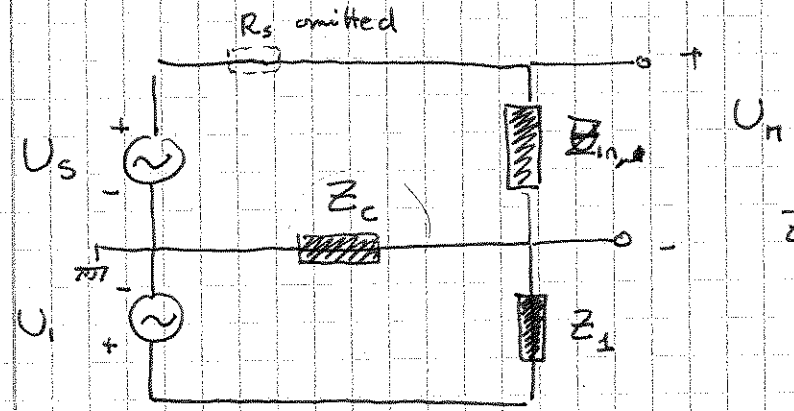


ground at both end to

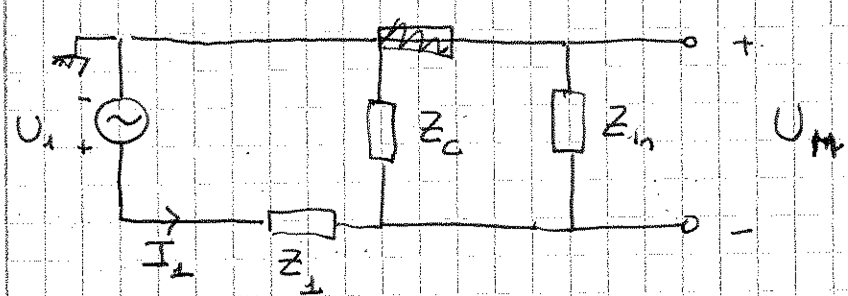
$\Rightarrow$ close the circuit !

Conductive coupling

- multiwire cable with single return wire
- direct contact with conductive body



Z_c: impedance of common return wire

- 

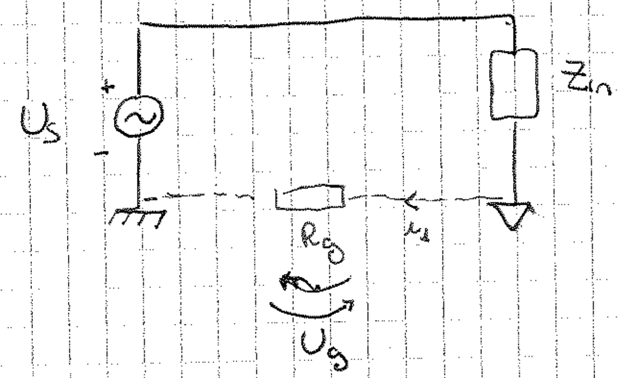
Typically $|Z_c| \ll |Z_{in}|$ and $|Z_c| \ll |Z_1|$

$$U_M \approx -\frac{Z_c}{Z_c + Z_1} U_1 \approx -\frac{Z_c}{Z_1} U_1 = -Z_c I_1$$

- Countermeasures
  - reduce I_1 or Z_c
  - use separate return wires
  - dedicate one layer for ground only in ~~PBCs~~ PBCs.

printed circuit boards

Grounding



$U_M + U_g$

↑ problematic when U_M is small

· ~~Do not use shield as return wire~~

SLIDE · Ground only at one end (but then inductive coupling)

SLIDE · do not use shield as return wire

⇒ need sensor output to be $\boxed{\text{differential ended (i.e., not relative to ground)}}$

- active sensors ok

- passive sensors → make it independent of ground!

single-ended ⇒ differentially-ended

SLIDES about this!

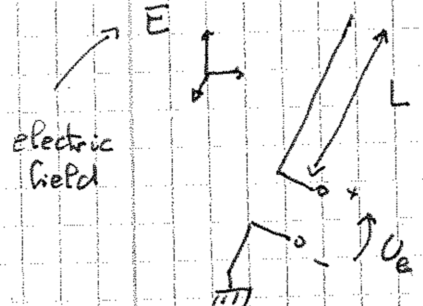
coax cable ⇒ shielded cable where the shield is used as return wire

shielded-pair cable ⇒ shield is used as shield only!

Radiation coupling

- EM radiation sources:
  - radio ~~wave~~ + TV + wireless
  - any electronic device containing AC circuit
  - How EM enters a meas. system

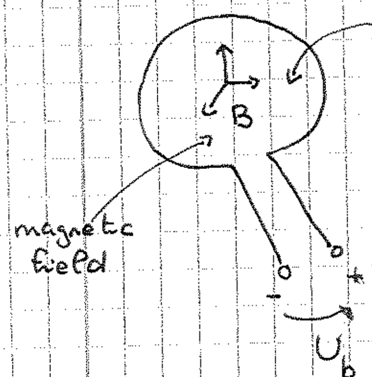
electric dipole



$$U_e = E_{\parallel} \cdot L$$

($E_{\parallel}$: parallel to dipole)

magnetic dipole

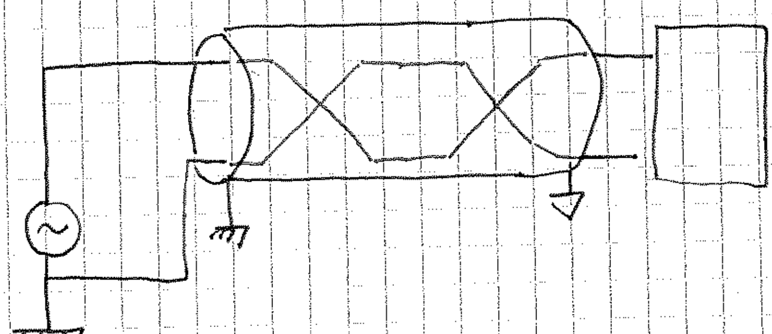


A : area

$B_{\perp}$: perpendicular to dipole

$$U_b = A \cdot \frac{dB_{\perp}}{dt}$$

- Countermeasures

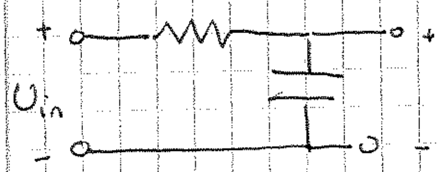


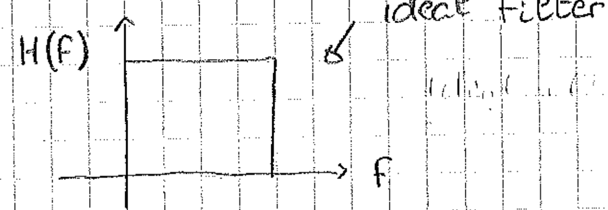
shield : protect agai[nst] E-field (Faraday cage)

twisted cable : protect against M field.

Bandwidth of measurement systems

- bandwidth of signal vs bandwidth of instrument
- Instrument: low-pass filter of first order;
  cut-off freq $f_g = B$





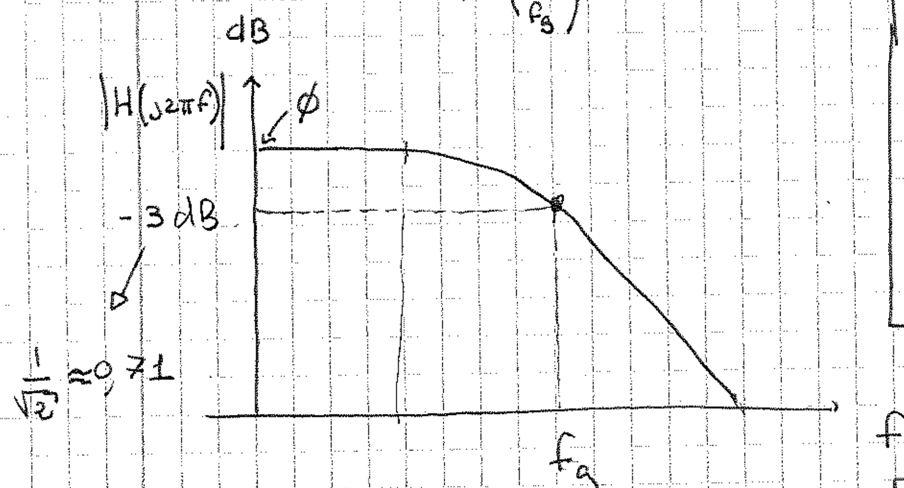
- Transfer function filter

$$H(j\omega) = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j2\pi f\, CR}$$

$$= \frac{1}{1 + j\frac{f}{f_g}} \,, \quad f_g = \frac{1}{2\pi CR}$$

- Amplitude

$$\left| H\left(j2\pi f\right) \right| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_g}\right)^2}}$$



f	$\lvert H(j2\pi f)\rvert$	amplitude error
f_g	0,71	29 %
$f_g/2$	0,89	11 %
$f_g/5$	0,98	2 %
$f_g/10$	0,995	0,5 %

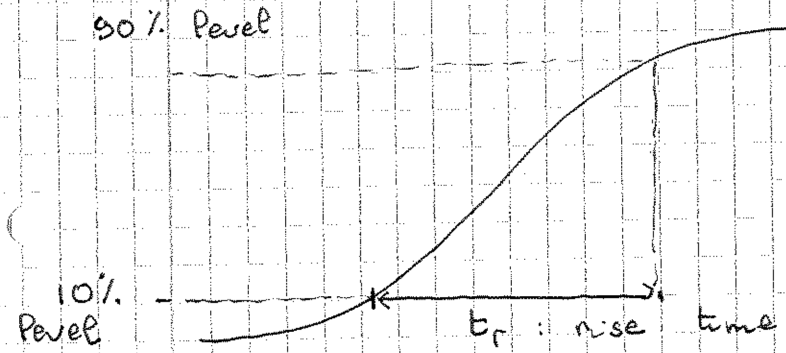
- Bandwidth of a signal

  $\Rightarrow$ largest harmonic component present in the signal

- Fast variations $\Leftrightarrow$ large bandwidth

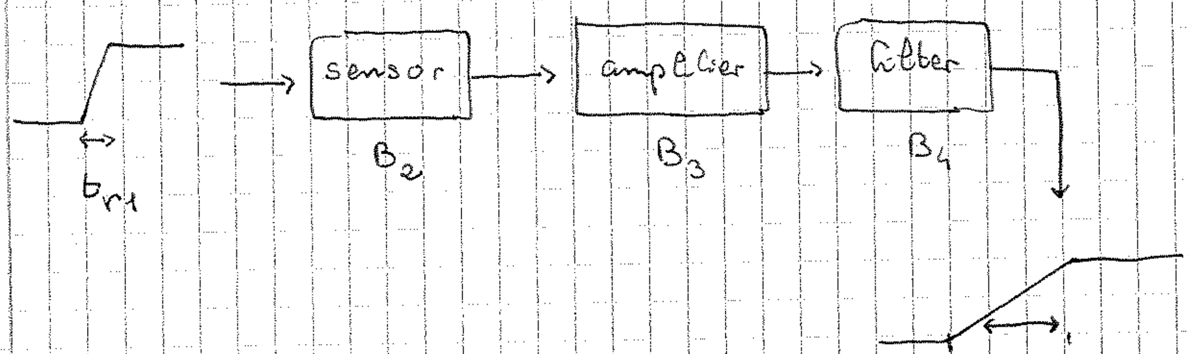
  $\downarrow$

  rise time



- Rule : if a signal as bandwidth B, its rise time cannot be smaller than

$$t_r \approx \frac{0{,}35}{B}$$



$t_{out} > t_{r1}$?

- Reaction time of block k :

$$t_{rk} = \frac{0{,}35}{B_k}$$

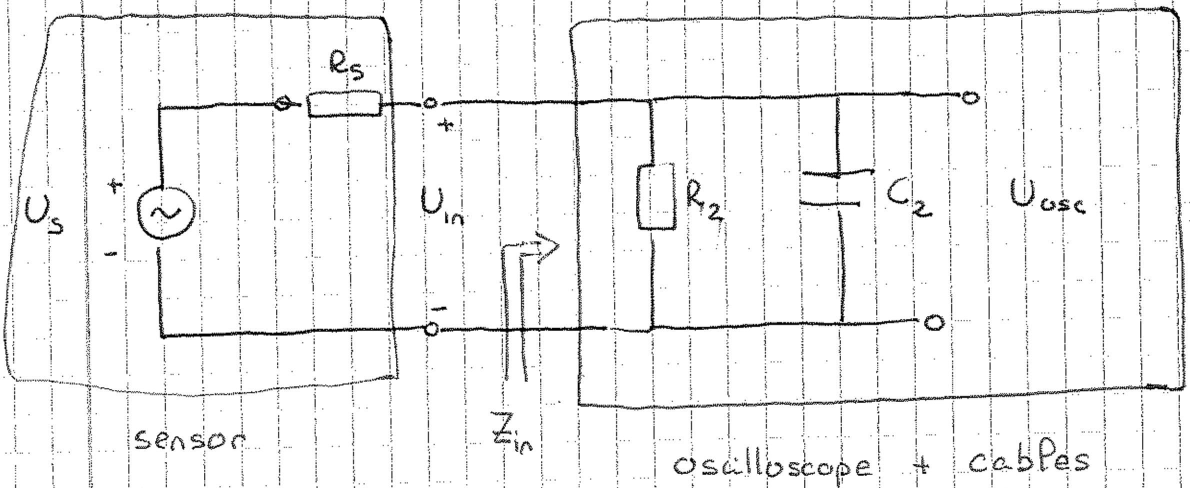
- It can be shown that

$$t_{out} \approx \sqrt{t_{r1}^2 + t_{r2}^2 + t_{r3}^2 + t_{r4}^2}$$

Oscilloscope probes

$R_2 = 1\,M\Omega \qquad C_2 = 20\,pF + 80\,pF = 100\,pF$

cables



- input impedance $\quad Z_{in} = R_2 \,//\, C_2 = \dfrac{R_2 \dfrac{1}{j\omega C_2}}{R_2 + \dfrac{1}{j\omega C_2}} = \dfrac{R_2}{j\omega C_2 R_2 + 1}$

- Want $U_{osc} \approx U_s$

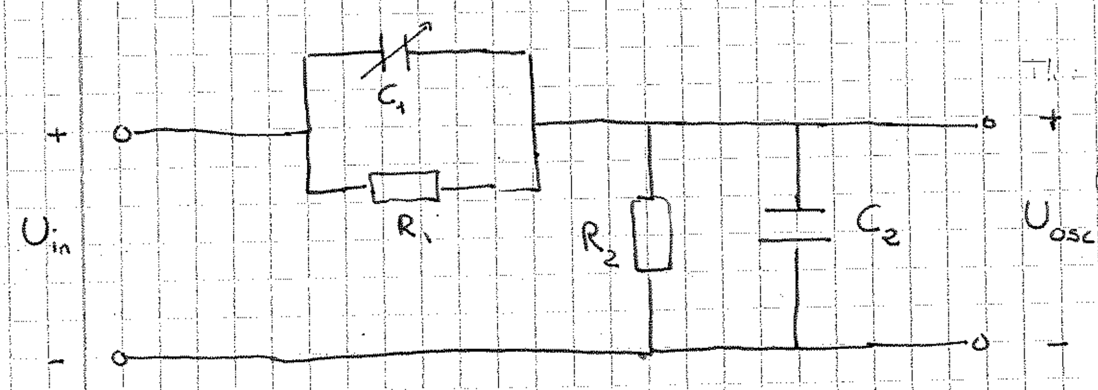
$$\Rightarrow \quad |Z_{in}| \gg R_s$$

- Problems if
  - $R_s \approx R_2$ [does not typically occur]
  - or if ~~$\frac{1}{\omega C_2} \gg R_2$~~

$$\omega C_2 R_2 \gg 1 \quad \Rightarrow \quad |Z_{in}| \approx \frac{1}{\omega C_2} \quad \Leftarrow \text{ comparable with } R_s$$

$\Rightarrow$ big freq. dep. attenuation.

- Solution: increase Z_{in} using probes



- Problem: gain reduction (may be frequency dependent)

$$\frac{U_{osc}}{U_{in}} = \frac{\dfrac{R_2}{j\omega R_2 C_2 + 1}}{\dfrac{R_1}{j\omega R_1 C_1 + 1} + \dfrac{R_2}{j\omega R_2 C_2 + 1}}$$

- at $\omega = \emptyset$ (DC component)

$$\frac{U_{osc}}{U_{in}} = \frac{R_2}{R_1 + R_2}$$

TPS $\rightarrow$ how do I make gain reduction independent of ω and, hence, constant over frequency?

- Gain reduction independent of ω if

$$R_1 C_1 = R_2 C_2 \quad \longrightarrow \text{ obtained by adjusting } C_1$$

$$\Rightarrow \frac{U_{osc}}{U_{in}} = \frac{R_2}{R_1 + R} \quad \text{for all } \omega \text{ if } \quad R_1 C_1 = R_2 C_2$$

- New input impedance

$$Z_{in}' = R_1 // C_1 + R_2 // C_2 = \frac{R_1}{j\omega R_1 C_1 + 1} + \frac{R_2}{j\omega R_2 C_2 + 1}$$

$$\overset{R_1 C_1 = R_2 C_2}{=} \frac{R_1 + R_2}{j\omega R_2 C_2 + 1} = \frac{R_1 + R_2}{R_2} Z_{in}$$

- Summary:
  - o amplitude reduction by $\dfrac{R_2}{R_1 + R_2}$
  - o input impedence increase by $\dfrac{R_1 + R_2}{\ldots}$

Example : 10x probe oscilloscope

$\Rightarrow$ 10 x impedence increase $\Rightarrow$ 10 x gain reduction

$$\frac{R_2}{R_1+R_2} = \frac{1}{10} \Rightarrow \frac{R_1}{R_2} + 1 = 10 \Rightarrow \frac{R_1}{R_2} = 9$$

Typically, $R_2 = 1\,M\Omega$

$C_2 = 20\,pF + 80\,pF = 100\,pF$

$\Rightarrow R_1 = 9\,M\Omega$

- To choose C_1,

$$R_2 C_2 = R_1 C_1 \Rightarrow C_1 = \frac{R_2}{R_1} C_2 = 100 \frac{1}{9}\,pF =$$

$$= 11{,}11\,pF$$

$$Z'_{in} \Rightarrow \begin{cases} R'_{in} = R_1 + R_2 = 10\,M\Omega \\ C'_{in} = C_1 \mathbin{/\!/} C_2 = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{C_2}{\cancel{\frac{C_1}{C_2}} + 1 + \frac{C_2}{C_1}} = 10\,pF \end{cases}$$

$$\frac{C_2}{C_1} = \frac{R_1}{R_2} = 9$$