



# ENM061 - Power Electronic Converters 7.5 ECTS, 2017

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## Lecture outline

### **Converter Control and Improvements**

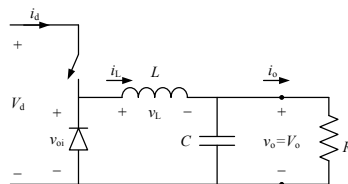
- Control of DC/DC Converters
- Dynamic modeling of a step-down converter
- Controller design for the step-down converter
- Digital Control
- Synchronous rectification in the step-down converter
- Interleaved step-down converters
- Summary

## Learning outcomes

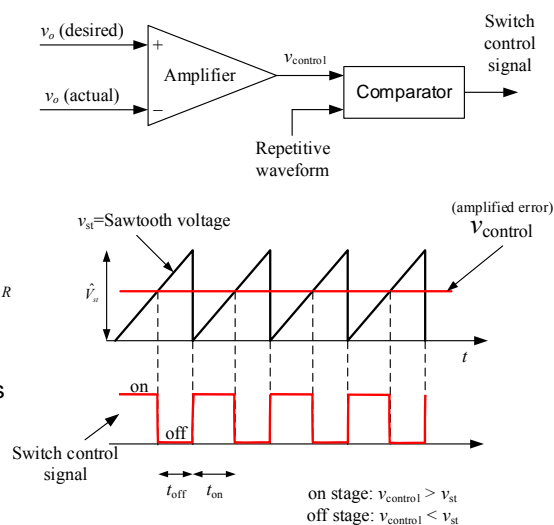
- Fourier components and total harmonic distortion (THD) for basic waveforms.
- Operating principles of the most common active components (e.g. diode, thyristor, IGBT, and MOSFET) and passive components (e.g. capacitors, transformers and inductors).
- Operation of a pulse width modulation (PWM), the purpose of controlling the desired quantity and the need for a controller circuit within the power electronic converter.
- Analysis of ideal DC/DC converters (e.g. buck, boost, buck-boost, flyback, the forward, the push-pull, half-bridge and full-bridge converters) in CCM and DCM operation.
- Operating principles of single-phase and three-phase AC/DC inverters with different modulation strategies (e.g. PWM and square wave operation).
- Operation of multilevel converters (e.g. NPC, flying capacitor and MMC topologies) using current and voltage waveform analysis. Pros and Cons of the converter in terms of harmonics and losses.
- Operation of single- and three-phase diode rectifiers operating with voltage-stiff and current-stiff DC-side. Investigating the impact of line impedance within the converter circuit for current commutation.
- Operation of single- and three-phase thyristor rectifiers operating with a current-stiff DC-side and the impact of line impedance for current commutation. Investigating the use of 6/12-pulse configurations.
- Loss calculation in passive and active components. Evaluating the temperature rise in the active components and choosing an appropriate heat-sink. Gaining a basic understanding of component life time.
- Identify simple power electronic converter schematics. Recognizing the different parts in a physical circuit on which basic wave-shape and efficiency measurements is performed.
- Utilizing the software Cadence PSpice to simulate basic power electronic circuits and the practical labs to have a firsthand experience of how real DC/DC converters operate.

## Control of DC/DC Converters

- The buck converter from Lecture 4 with controller circuit

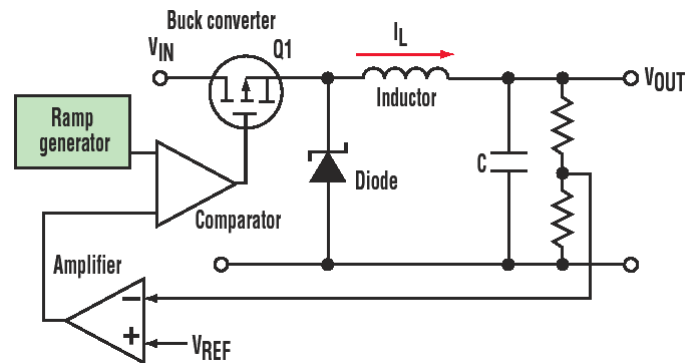


- The PWM pattern to the switch is obtained by comparing a control voltage with a saw tooth carrier

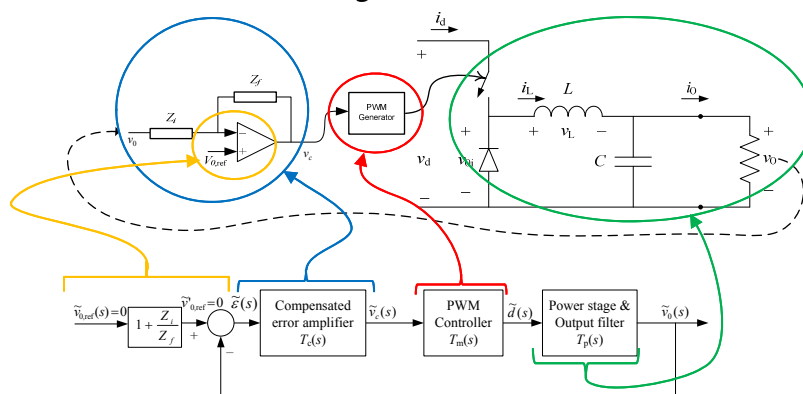


## Control of DC/DC Converters

- Voltage mode control – the output voltage is measured and compared to a reference value

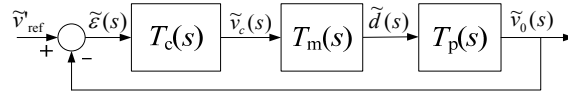


## The Dynamic Model



- The behavior of each block is analyzed with its transfer function and corresponding bode plot

## The Dynamic Model

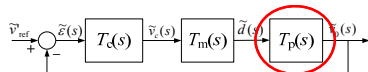


- Once each block is known, the total open-loop transfer function is used to determine the gain and phase margins
- The transfer function of each block depends on e.g. circuit parameters and accounts for the small signal variation around a specific operating point

### Small signal perturbation model

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}, \quad T_m(s) = \frac{\tilde{d}(s)}{\tilde{v}_c(s)}, \quad T_c(s) = \frac{\tilde{v}_c(s)}{\tilde{e}(s)} \Rightarrow T_{OL}(s) = T_p(s)T_m(s)T_c(s)$$

## The Power Stage State-Space Modeling

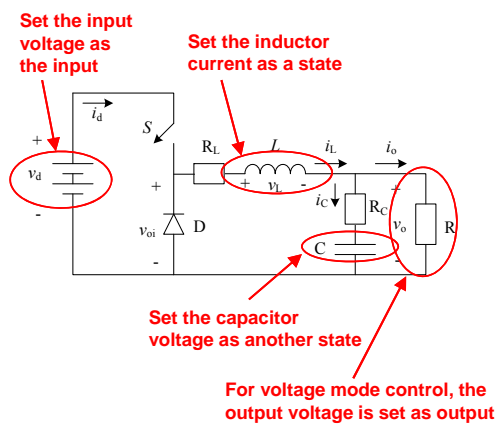


- State-space description of the power stage in a buck converter

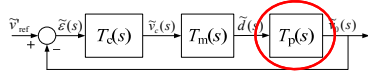
$$\dot{x} = Ax(t) + Bu(t)$$

$$y = Cx(t) + Du(t)$$

- The capacitor voltage and the inductor current are set as states ( $x(t)$ ) since they can be expressed with their time derivatives
- The system input ( $u(t)$ ) is set to  $V_d$  and the output ( $y$ ) is the quantity that we want to control ( $v_o$ )



## The Power Stage State-Space Modeling



1. State variable description of each circuit state

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ y = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{cases} \quad \left\{ \begin{array}{ll} \dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1v_d & v_o = \mathbf{C}_1\mathbf{x} \quad \text{During } d \cdot T_{sw} \\ \dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2v_d & v_o = \mathbf{C}_2\mathbf{x} \quad \text{During } (1-d) \cdot T_{sw} \end{array} \right.$$

2. Average the state-variables with  $d$

$$\begin{aligned} \dot{\mathbf{x}} &= [\mathbf{A}_1d + \mathbf{A}_2(1-d)]\mathbf{x} + [\mathbf{B}_1d + \mathbf{B}_2(1-d)]v_d \\ v_o &= [\mathbf{C}_1d + \mathbf{C}_2(1-d)]\mathbf{x} \end{aligned}$$

then linearize the model around an operating point, i.e. describe,

$$d = D + \tilde{d}, \quad \mathbf{x} = \mathbf{X} + \tilde{\mathbf{x}}, \quad v_d = V_d + \tilde{v}_d = V_d, \quad v_o = V_o + \tilde{v}_o$$

## The Power Stage State-Space Modeling

Define the different matrices using  $D$  as

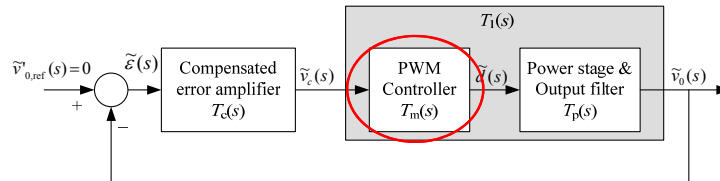
$$\mathbf{A} = \mathbf{A}_1D + \mathbf{A}_2(1-D) \quad \mathbf{B} = \mathbf{B}_1D + \mathbf{B}_2(1-D) \quad \mathbf{C} = \mathbf{C}_1D + \mathbf{C}_2(1-D)$$

$$\Rightarrow \begin{aligned} \dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + [(\mathbf{A}_1 - \mathbf{A}_2)\mathbf{X} + (\mathbf{B}_1 - \mathbf{B}_2)V_d]\tilde{d} & \text{where } \dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}V_d = 0 \\ \tilde{v}_o &= \mathbf{C}\tilde{\mathbf{x}} + [(\mathbf{C}_1 - \mathbf{C}_2)\mathbf{X}]\tilde{d} & \text{where } V_o &= \mathbf{C}\mathbf{X} = -\mathbf{C}\mathbf{A}^{-1}\mathbf{B}V_d \end{aligned}$$

3. Transform to the Laplace domain and determine the final transfer function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)} = \mathbf{C}[s\mathbf{I} - \mathbf{A}]^{-1}[(\mathbf{A}_1 - \mathbf{A}_2)\mathbf{X} + (\mathbf{B}_1 - \mathbf{B}_2)V_d] + (\mathbf{C}_1 - \mathbf{C}_2)\mathbf{X}$$

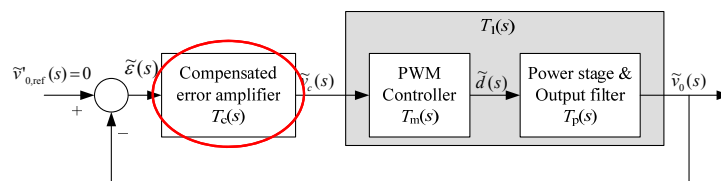
## The Modulator dynamic model



$$T_m(s) = \frac{\tilde{d}(s)}{\tilde{v}_c(s)} = \frac{1}{\hat{V}_r} \Rightarrow T_1(s) = \frac{\tilde{v}_o(s)}{\tilde{v}_c(s)} = \frac{\tilde{v}_o(s)}{\tilde{d}(s)} \frac{\tilde{d}(s)}{\tilde{v}_c(s)} = T_p(s) T_m(s)$$

Undeland, Power Electronics  
Figure 10-19, page 323

## The Compensated Error Amplifier



- Once the transfer functions of the PWM controller and the Power stage are known, the error amplifier shall be dimensioned

$$T_c(s) = \frac{\tilde{v}_c(s)}{\tilde{e}(s)}$$

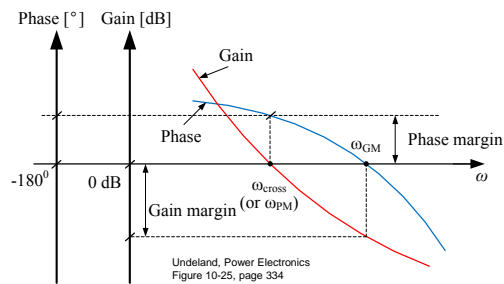
- The total open-loop transfer function ( $T_{OL}(s)$ ) shall get a suitable behavior.

$$T_{OL}(s) = T_c(s) \cdot T_1(s)$$

- High gain at low frequencies to minimize steady state errors
- High cross-over frequency ( $f_{cross} = 0.1 f_{sw}$ ) for fast response and high filtering
- A phase margin in the range of  $45^\circ - 60^\circ$  for good transient response

## Controller Design Gain and Phase Margin

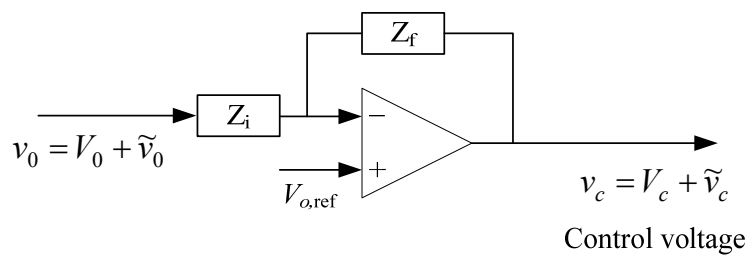
- The gain margin and the phase margin (and associated frequencies  $\omega_{GM}$  and  $\omega_{PM}$ ) indicates the relative stability of the closed-loop system formed by applying unity negative feedback
- The gain margin is the amount of gain increase or decrease required to make the loop gain unity at the frequency  $\omega_{GM}$  where the phase angle is  $-180$  degrees
- The phase margin is the difference between the phase of the response and  $-180$  degrees when the loop gain is 1. The frequency  $\omega_{PM}$  at which the magnitude is 1.0 is called the gain crossover frequency.
- Recommendation: gain margins of three or more combined with a phase margin between 30 and 60 degrees for good transient performance and stability.



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## Controller Design Circuit Realization



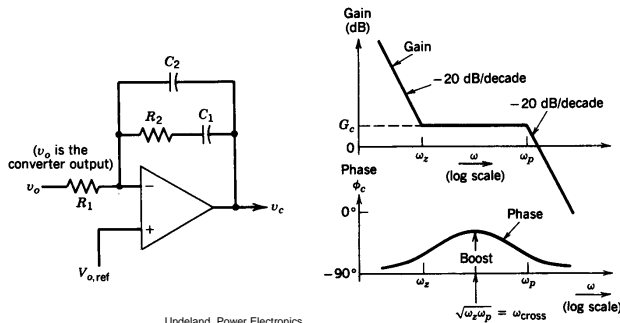
- For small signals: 
$$\frac{\tilde{v}_c(s)}{\tilde{v}_0(s)} = -\frac{Z_f(s)}{Z_i(s)} = -T_c(s)$$

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## Controller Design Example Integrated-compensation Network

- A zero-pole pair to give a phase boost
- Maximum phase boost of 90 deg can be provided ( $\omega_z$  and  $\omega_p$  design parameters)
- Integrator at origin gives high DC gain for good load regulation



Undeland, Power Electronics  
Figure 10-27, page 335

$$T_c(s) = \frac{1}{R_1 C_2 s} \frac{s + \omega_z}{s + \omega_p}$$

$$\omega_z = \frac{1}{R_2 C_1}$$

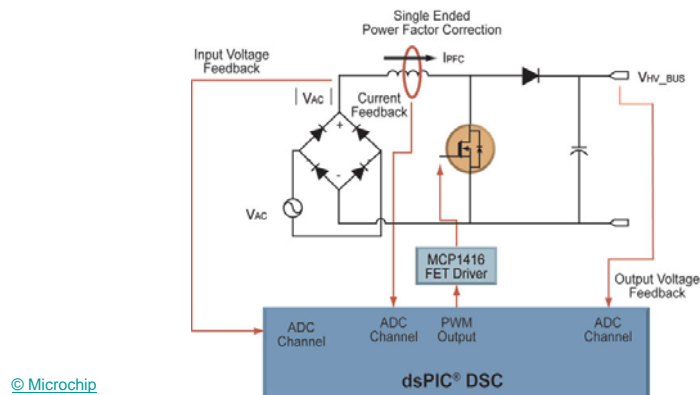
$$\omega_p = \frac{C_1 + C_2}{R_2 C_1 C_2}$$

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## Digital Control

- Can serve as an additional monitoring and control functions to an existing analog controller or as a complete digital control
- Boost converter: Microcontrollers, ADC/DAC, digital input-analog output



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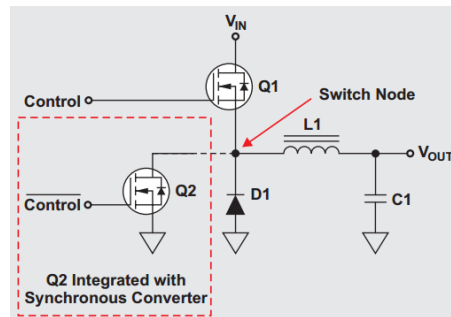
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## Synchronous Step-Down Converters

- The freewheeling diode is replaced by a second (low-side) MOSFET that freewheels the current
- Main advantage is lower voltage drop over the low-side MOSFET than a corresponding diode
- Synchronous rectification can also be applied to other topologies as well



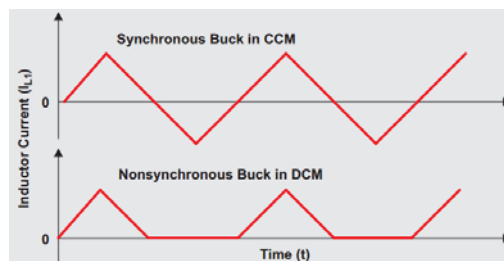
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## Synchronous Step-Down Converters

- In a standard buck converter with a free-wheeling diode, the inductor current can only flow in one direction
- In a synchronous buck, the current through the low-side MOSFET can flow in both directions which will increase the losses



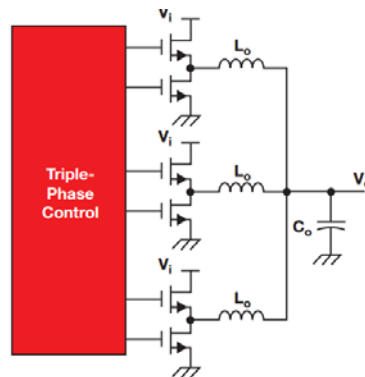
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## Interleaved Step-Down Converters

- By connecting several phase shifted converters in parallel, the resulting current and voltage ripple can be decreased
- Typically used in high current and low voltage applications (e.g. processor power supplies)

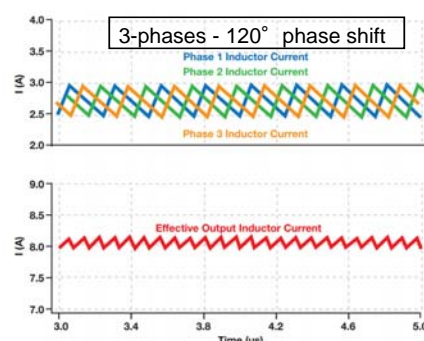
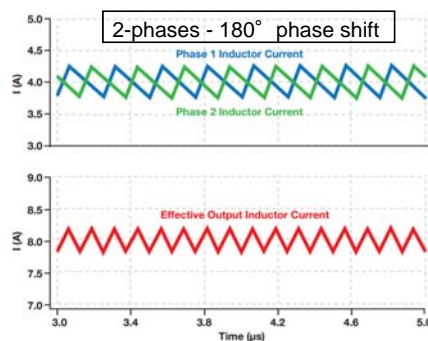


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## Interleaved Step-Down Converters

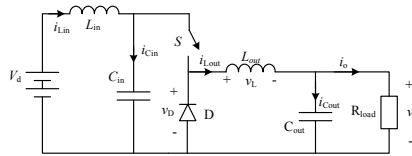
- The inductor currents add together which gives a lower effective ripple
- The current ripple determines the resulting output voltage ripple in combination with the output capacitors
- The inductor in each phase can be realized with lower inductance which gives higher saturation current ratings



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## PSpice 7



Nominal values	
Source voltage ( $V_d$ )	15V
Output Inductance ( $L$ )	2.2μH
Output Capacitance ( $C$ )	150μF
Load Resistance ( $R_{load}$ )	2Ω
Switching frequency ( $f_{sw}$ )	300kHz
Duty ratio steady state ( $D$ )	0.667

- Buck converter with output voltage controller. How does the controller compensate for changes in the system such as  $V_{o,ref}$  and  $R_{load}$  changes, both during an increase and a decrease?

$$T_c(s) = \frac{A}{s} \frac{s + \omega_z}{s + \omega_p}$$

## Summary

- Why a controller is needed and how it works
- The concept of controller design: bode diagram, phase and gain margins
- Digital Control
- Synchronous rectification in the step-down converter
- Interleaved step-down converters
- Learning outcome:
  - ❖ Basics in dynamic modelling and control of power electronic converters