3-1 RMS voltage and current phasor

$$\mathbb{V} = Ve^{j0} \qquad \mathbb{I} = Ie^{-j\phi^\circ}$$

$$v(t) = \sqrt{2}\,V\cos(\omega t) \qquad i(t) = \sqrt{2}\,I\cos(\omega t - \phi)$$

$$p(t) = v(t)\,i(t) = 2VI\cos(\omega t)\cos(\omega t - \phi)$$

$$\left.\begin{array}{l}\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right) \\ \cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\end{array}\right\} \Rightarrow$$

$$p(t) = VI\cos(\phi) + VI\cos(2\omega t - \phi) =$$

$$= VI\cos(\phi) + VI\cos(\phi)\cos(2\omega t) + VI\sin(\phi)\sin(2\omega t) =$$

$$= P + P\cos(2\omega t) + Q\sin(2\omega t)$$

$$P = VI\cos(\phi)$$

$$Q = VI\sin(\phi)$$

3-2

$$V = 120\ V \qquad \mathbb{I} = e^{-j30^\circ}$$

$$v(t) = \sqrt{2}\,120\cos(\omega t)$$

$$i(t) = \sqrt{2}\cos(\omega t - 30^\circ)$$

Figure on the next page !!

$$p(t) = v(t)\,i(t)$$

$$i_p(t) = \sqrt{2}\,I_p\cos(\omega t) = \sqrt{2}\,I\cos(\phi)\cos(\omega t)$$

$$i_q(t) = \sqrt{2}\,I_q\sin(\omega t) = \sqrt{2}\,I\sin(\phi)\sin(\omega t)$$

$$P = VI\cos(\phi) = 120 \cdot 1 \cdot \cos(30^\circ) = 103{,}9\ W$$

$$Q = VI\sin(\phi) = 120 \cdot 1 \cdot \sin(30^\circ) = 60\ VA_r$$

$$p_2 = V i_q = \sqrt{2}\,120\cos(\omega t)\,\sqrt{2}\sin(30^\circ)\sin(\omega t) = 120\sin(30^\circ)\sin(2\omega t) \Rightarrow$$

$$\hat{p}_2 = 120\sin(30^\circ) = Q = 60$$

$$PF = \frac{P}{S} = \frac{P}{VI} = \frac{103{,}9}{120} = 0{,}87$$ ~~l~~ inductive current is lagging the voltage

$\Rightarrow$ the load draws positive vars !!

2-2

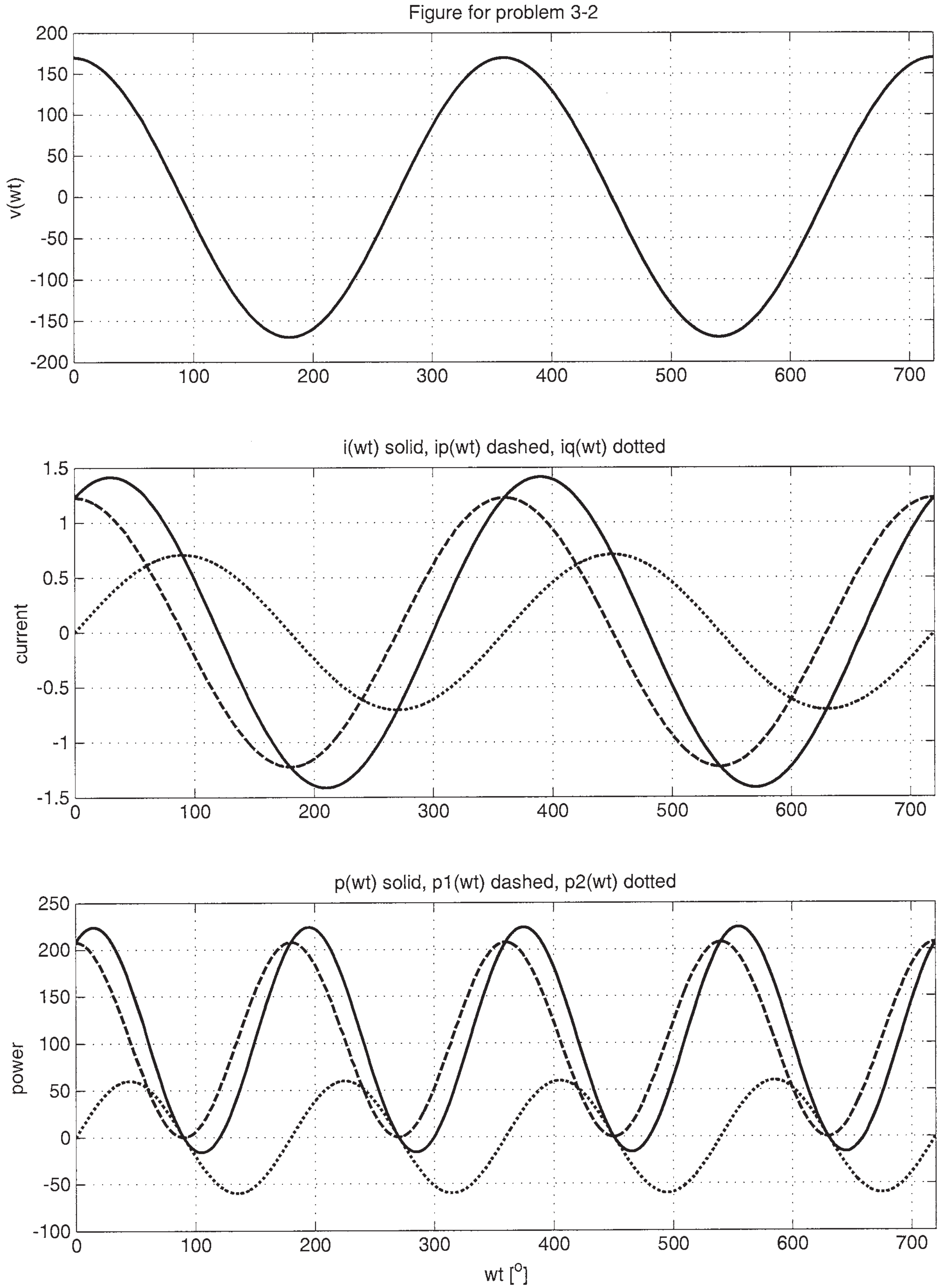
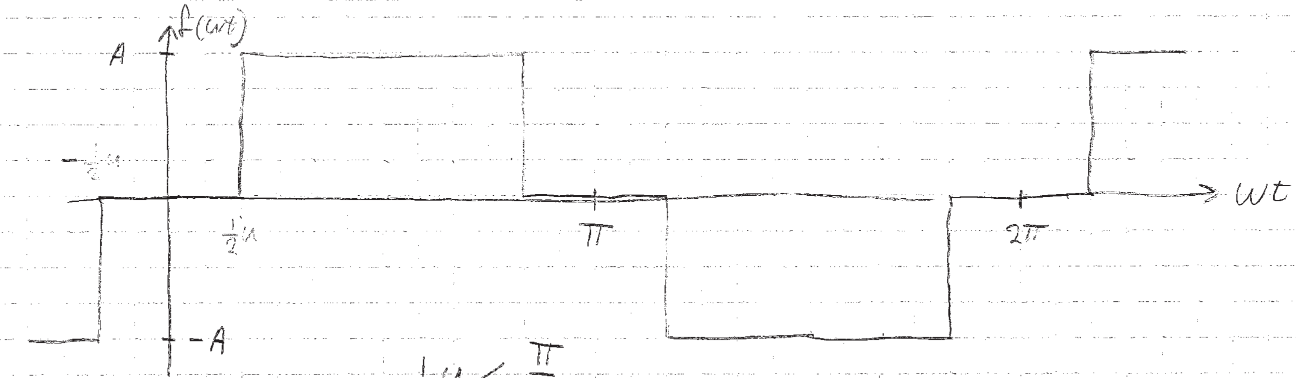


Figure for problem 3-2
v(wt)
i(wt) solid, ip(wt) dashed, iq(wt) dotted
current
p(wt) solid, p1(wt) dashed, p2(wt) dotted
power
wt [^{o}]

3-3 to 3-5 fig b



assume $\frac{1}{2}u < \frac{\pi}{2}$

Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

$$a_n = \frac{2}{T}\int_a^{a+T} f(t)\cos(n\omega t)\,dt \qquad n \geq 0$$

$$b_n = \frac{2}{T}\int_a^{a+T} f(t)\sin(n\omega t)\,dt \qquad n \geq 1$$

if $f(t) = g(\omega t)$ $\quad \omega t = \theta \Rightarrow \quad \omega = \frac{2\pi}{T}$

$$g(\theta) = g(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\theta) + b_n \sin(n\theta)\right]$$

$$a_n = \frac{1}{\pi}\int_b^{b+2\pi} g(\theta)\cos(n\theta)\,d\theta$$

$$b_n = \frac{1}{\pi}\int_b^{b+2\pi} g(\theta)\sin(n\theta)\,d\theta$$

From the figure we see that

$f(t)$ is odd $\quad f(t) = -f(-t)$ and

$f(t)$ is Half-wave $\quad f(t) = -f(t+\frac{T}{2})$ this gives

$a_n = 0$ for all n

$$b_n = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} g(\theta)\sin(n\theta)\,d\theta & \text{for odd } n \\ 0 & \text{for even } n \end{cases}$$

$a_n = 0 \Rightarrow \underline{F_{av} = 0 = \frac{1}{2}a_0}$

for odd n

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} g(\theta)\sin(n\theta)\,d\theta = \frac{4}{\pi}A\int_{\frac{1}{2}u}^{\pi/2}\sin(n\theta)\,d\theta =$$

$$= -\frac{4}{\pi}\frac{A}{n}\Big[\cos(n\theta)\Big]_{\frac{1}{2}u}^{\pi/2} = -\frac{4}{\pi}\frac{A}{n}\Big(\underbrace{\cos(n\tfrac{\pi}{2})}_{=0 \text{ for odd } n} - \cos(\tfrac{nu}{2})\Big) \Rightarrow$$

$$b_n = \frac{4}{\pi}\frac{A}{n}\cos\left(\frac{nu}{2}\right) \leftarrow \text{peak amplitude}$$

The rms value of the fundamental and the harmonics are

$$\underline{\underline{b_{n,rms} = \frac{2\sqrt{2}}{\pi}\frac{A}{n}\cos\left(\frac{nu}{2}\right)}}$$

3-4

a) $F_{rms} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2}$ $F_n = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}$, $F_0 = \frac{1}{2}a_0 \Rightarrow$

$$F_{rms} = \sqrt{\frac{4 \cdot 2 \cdot A^2}{\pi^2 \, 2}\sum_{n=1}^{\infty}\frac{\cos^2(\frac{nu}{2})}{n^2}} = \ldots \text{ skip this!!!}$$

b) $$\underline{\underline{F_{rms}}} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} g^2(\theta)\,d\theta} = \sqrt{\frac{A^2}{2\pi}(2\pi - 2u)} =$$

$$= A\sqrt{1 - \frac{u}{\pi}} = 10\sqrt{1 - \frac{20}{180}} = \underline{\underline{9{,}43}}$$

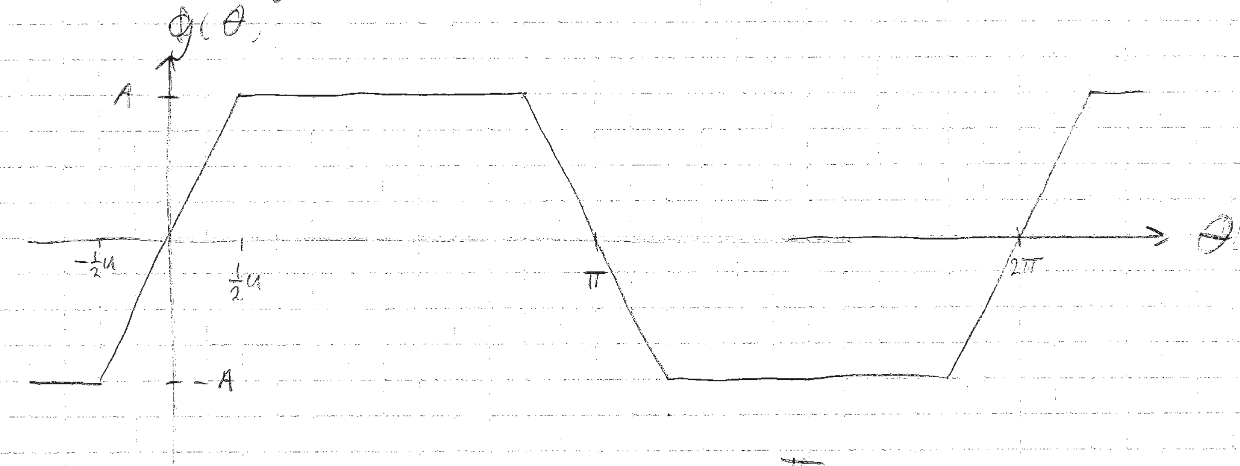
3-5

$$\frac{b_{n,rms}}{F_{rms}} = \frac{\frac{2\sqrt{2}}{\pi}\frac{10}{1}\cos(10^\circ)}{9{,}43} = \frac{8{,}87}{9{,}43} = \underline{\underline{0{,}94}}$$

$$\frac{F_{dis}}{F_{rms}} = \frac{\sqrt{F_{rms}^2 - b_{1,rms}^2}}{F_{rms}} = \frac{\sqrt{9{,}43^2 - \left(\frac{2\sqrt{2}}{\pi}\frac{10}{1}\cos(10^\circ)\right)^2}}{9{,}43} = \frac{\sqrt{9{,}43^2 - 8{,}87^2}}{9{,}43}$$

$$= \underline{\underline{0{,}34}}$$

3-3 to 3-5 fig c



We assume that $\frac{1}{2}u < \frac{\pi}{2}$

From the figure we see that:

$g(\theta)$ is odd $g(\theta) = -g(-\theta)$ and

$g(\theta)$ is Half-wave $g(\theta) = -g(\theta+\pi)$ this gives

$\underline{a_n} = \underline{0}$ for all n

$$b_n = \begin{cases} \frac{4}{\pi}\int_0^{\frac{\pi}{2}} g(\theta)\sin(n\theta)\,d\theta & \text{for odd } n \\ 0 & \text{for even } n \end{cases}$$

for odd n

$$\underline{b_n} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} g(\theta)\sin(n\theta)\,d\theta = \frac{4}{\pi}\int_0^{\frac{1}{2}u}\frac{2}{u}A\,\theta\sin(n\theta)\,d\theta + \frac{4}{\pi}\int_{\frac{1}{2}u}^{\frac{\pi}{2}} A\sin(n\theta)\,d\theta$$

$$= \frac{4}{\pi}\frac{2A}{u}\frac{1}{n^2}\Big[\sin(n\theta) - n\theta\cos(n\theta)\Big]_0^{\frac{1}{2}u} - \frac{4}{\pi}\frac{A}{n}\Big[\cos(n\theta)\Big]_{\frac{1}{2}u}^{\frac{\pi}{2}} =$$

$$= \frac{4}{\pi}\frac{2A}{u}\frac{1}{n^2}\left(\sin\left(\frac{nu}{2}\right) - \frac{nu}{2}\cos\left(\frac{nu}{2}\right)\right) - \frac{4}{\pi}\frac{A}{n}\left(\underbrace{\cos\left(n\frac{\pi}{2}\right)}_{=0 \text{ for odd } n} - \cos\left(\frac{nu}{2}\right)\right) =$$

$$= \frac{8}{\pi}\frac{A}{un^2}\sin\left(\frac{nu}{2}\right) \cancel{- \frac{4}{\pi}\frac{2A}{u}\frac{1}{n^2}\frac{nu}{2}\cos\left(\frac{nu}{2}\right)} \cancel{+ \frac{4}{\pi}\frac{A}{n}\cos\left(\frac{nu}{2}\right)} =$$

$$= \underline{\frac{8}{\pi}\frac{A}{un^2}\sin\left(\frac{nu}{2}\right)}$$

$G_{av} = 0$ becouse $a_0 = 0$

$b_{n,rms} = \frac{4\sqrt{2}}{\pi} \frac{A}{u n^2} \sin\left(\frac{nu}{2}\right)$

3-4 a) Skip this

b) $$G_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} g^2(\theta)\,d\theta} = \sqrt{\frac{2}{\pi}\int_0^{\frac{\pi}{2}} g^2(\theta)\,d\theta} =$$

$$= \sqrt{\frac{2}{\pi}\int_0^{\frac{1}{2}u}\left(\frac{2}{u}A\theta\right)^2 d\theta + \frac{2}{\pi}\int_{\frac{1}{2}u}^{\frac{\pi}{2}} A^2\,d\theta} =$$

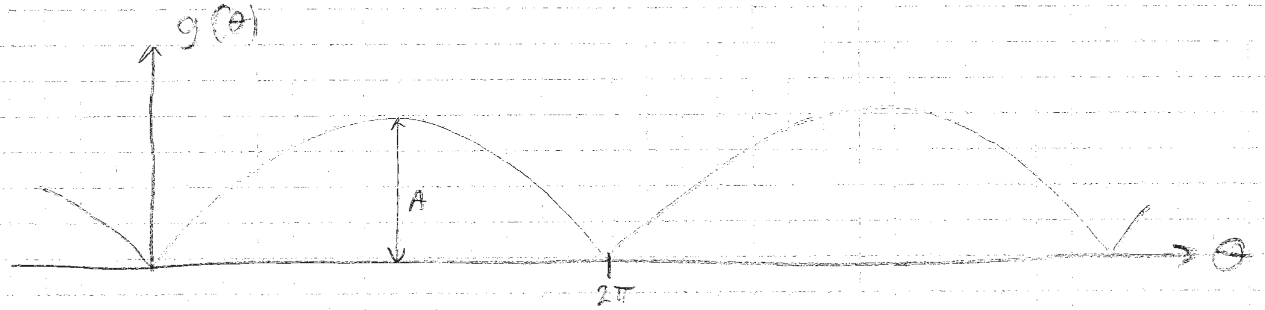
$$= \sqrt{\frac{8A^2}{\pi u^2 3}\left[\theta^3\right]_0^{\frac{1}{2}u} + \frac{2}{\pi}A^2\left(\frac{\pi}{2} - \frac{1}{2}u\right)} = \sqrt{\frac{A^2 u}{\pi 3} + A^2 - \frac{u}{\pi}A^2} =$$

$$= A\sqrt{1 - \frac{2u}{3\pi}} = 10\sqrt{1 - \frac{2}{3}\frac{20}{180}} = \underline{\underline{9{,}62}}$$

3-5 a) $$\frac{b_{1,rms}}{G_{rms}} = \frac{\frac{4\sqrt{2}}{\pi}\frac{10}{\frac{20\pi}{180}}\sin(10^\circ)}{9{,}62} = \frac{8{,}96}{9{,}62} = \underline{\underline{0{,}93}}$$

$$\frac{G_{dis}}{G_{rms}} = \frac{\sqrt{G_{rms}^2 - b_{1,rms}^2}}{G_{rms}} = \frac{\sqrt{9{,}62^2 - 8{,}96^2}}{9{,}62} = \underline{\underline{0{,}36}}$$

3-3 to 3-5 for fig f



$$g(\theta) = A\left|\sin\left(\frac{\theta}{2}\right)\right|$$

From the figure we see that

$g(\theta)$ is even $g(\theta) = g(-\theta) \Rightarrow$

$b_n = 0$ for all n

$$\underline{a_n} = \frac{2}{\pi}\int_0^{\pi} g(\theta)\cos(n\theta)\,d\theta = \frac{2}{\pi}\int_0^{\pi} A\sin\left(\frac{\theta}{2}\right)\cos(n\theta)\,d\theta =$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin\left(\left(\tfrac{1}{2}-n\right)\theta\right) + \sin\left(\left(\tfrac{1}{2}+n\right)\theta\right)d\theta =$$

$$= -\frac{A}{\pi}\left[\frac{1}{\frac{1}{2}-n}\cos\left(\left(\tfrac{1}{2}-n\right)\theta\right) + \frac{1}{\frac{1}{2}+n}\cos\left(\left(\tfrac{1}{2}+n\right)\theta\right)\right]_0^{\pi} =$$

$$= \frac{A}{\pi}\left(\frac{1}{\frac{1}{2}-n} + \frac{1}{\frac{1}{2}+n} - \frac{1}{\frac{1}{2}-n}\underbrace{\cos\left(\frac{\pi}{2}-\pi n\right)}_{=0 \text{ for all } n} - \frac{1}{\frac{1}{2}+n}\underbrace{\cos\left(\frac{\pi}{2}+\pi n\right)}_{=0 \text{ for all } n}\right) =$$

$$= \frac{A}{\pi}\left(\frac{\frac{1}{2}+n+\frac{1}{2}-n}{\frac{1}{4}-n^2}\right) = \underline{\underline{\frac{4A}{\pi(1-4n^2)}}}$$

$$\underline{\underline{G_{av}}} = \frac{1}{2}a_0 = \underline{\underline{\frac{2A}{\pi}}}$$

$$\underline{\underline{a_{n,rms}}} = \underline{\underline{\frac{2\sqrt{2}\,A}{\pi(1-4n^2)}}}$$

3-4 b

$$G_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} g^2(\theta)\,d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} A^2 \sin^2\left(\frac{\theta}{2}\right)d\theta} =$$

$$= \sqrt{\frac{A^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin(\theta)}{2}\right]_0^{\pi}} =$$

$$= \sqrt{\frac{A^2}{\pi}\,\frac{\pi}{2}} = \frac{A}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7{,}07$$

3-5 b

$$\frac{G_{av}}{G_{rms}} = \frac{\frac{2A}{\pi}}{\frac{A}{\sqrt{2}}} = \frac{2\sqrt{2}}{\pi} = 0{,}9$$