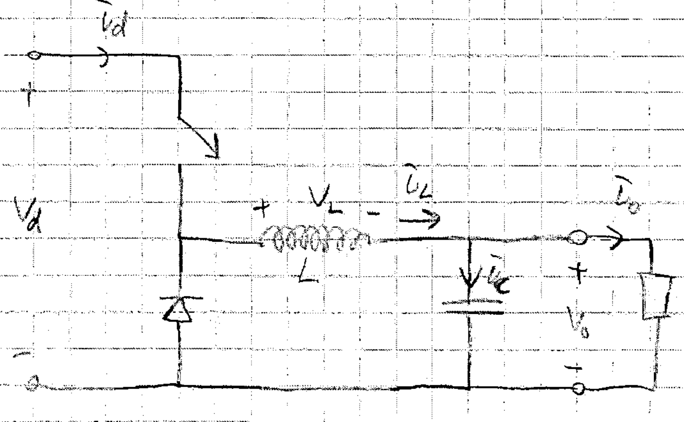
7-1 Step-down



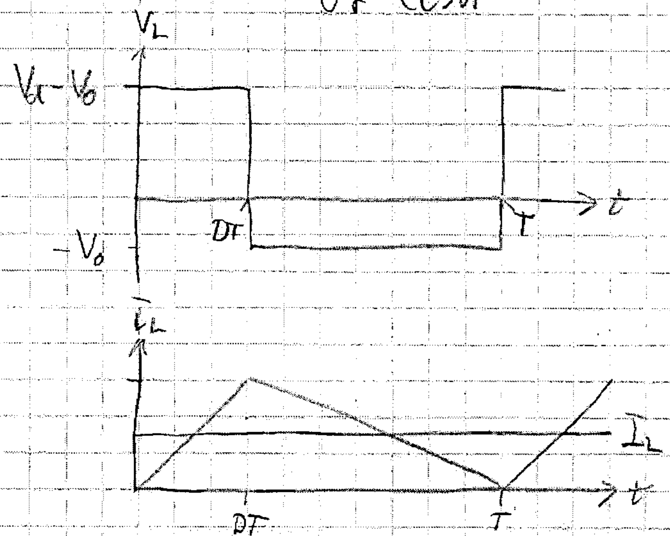
Let $v_o \simeq V_o$ be held constant 5V

V_d is 10 - 40 V $P_o \geq 5W$ $f_s = 50kHz$

calculate L required to keep the converter in CCM

I assume steady state operation and CCM

The waveforms for the converter at the border of CCM



In steady state we know that $\bar{V}_L = 0$

$0 = \bar{V}_L = \frac{1}{T}\left((V_d - V_o)DT - V_o(1-D)T\right) \Rightarrow$

$V_o = V_d D$

$I_c = 0 \Rightarrow$

$i_L = i_c + i_o$ and $i_o = I_o$

due to that $v_o \simeq V_o \Rightarrow$

$$0 = I_c = \frac{1}{T}\int_0^T i_L - I_o \, dt = \frac{1}{T}\int_0^T i_L \, dt - I_o = I_L - I_o \Rightarrow I_L = I_o$$

for CCM we know that

$$\Delta i_L \leq 2 I_L = 2 I_o = 2 \frac{P_o}{V_o}$$

$V_L = L\frac{di_L}{dt}$, during the time the switch is open $V_L = -V_o$ = const $\Rightarrow$

$$V_L = L\frac{\Delta i_L}{\Delta t} \Rightarrow \Delta i_L = \frac{V_L \Delta t}{L} = \frac{V_o(1-D)T}{L} \Rightarrow$$

$$\frac{V_o(1-D)}{f_s L} \leq 2\frac{P_o}{V_o} \Rightarrow$$

$$L \geq \frac{V_o^2(1-D)}{2 f_s P_o}$$

We must now find D and P_o within the operation interval that gives the highest value of $\frac{V_o^2(1-D)}{2 f_s P_o}$

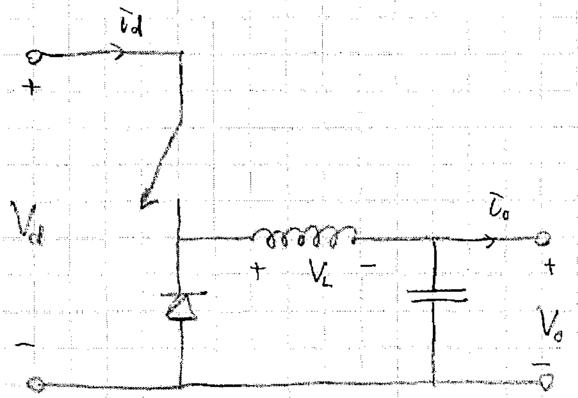
This due to the fact that this is the lowest value of L that guarantee CCM operation within the interval.

We directly see that the lowest D and lowest P_o gives the highest value $\Rightarrow$

$$V_o = V_d D \Rightarrow \quad D_{min} = \frac{V_o}{V_{d,max}} = \frac{5}{40} = \frac{1}{8}$$

$$L_{min} = \frac{5^2}{2 \cdot 50 \cdot 10^3 \cdot 5}\left(1-\frac{1}{8}\right) = 43,75\,\mu H$$

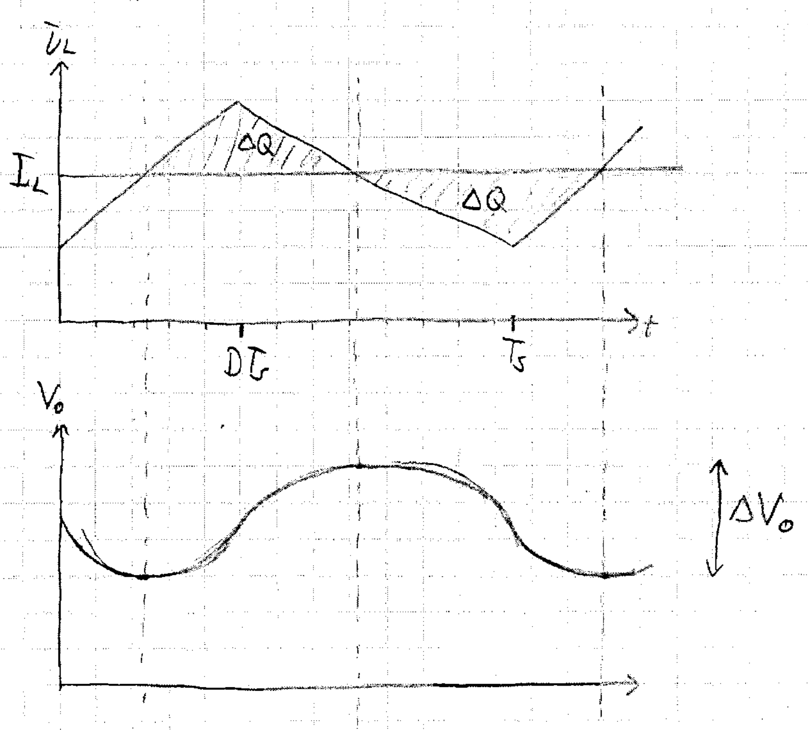
7.2 Step-Down converter, consider all components to be ideal. Assume: $V_o = 5V$, $f_s = 20kHz$, $L = 1mH$, $C = 470\mu F$
calculate ΔV_o (peak-peak) if $V_d = 12{,}6V$ and $I_o = 200mA$



As before, we starts by assuming CCM. ⇒

$$D = \frac{V_o}{V_d} = \frac{5}{12{,}6} \simeq 0{,}40$$

we plot the inductor current:



We check if the converter is operating in CCM

$$\Delta I_L = \frac{V_L}{L}\Delta t = \frac{-V_o}{L}(1-D)T_s = -\frac{5}{1\cdot 10^{-3}}(1-0{,}4)\frac{1}{20\cdot 10^{3}} \simeq 151\,mA$$

We see that $\Delta I_L < 2I_L = 2I_o = \underline{400mA}$ ⇒ $\underline{\underline{CCM}}$

We assumed right!!!

7,2 forts:

We now assume that all current ripple through the inductance is taken care of by the capacitance ⇒

$i_o = I_o = I_L$ and $i_c = i_{L,ripple}$ ⇒

When $i_L > I_L$ the capacitor is charged and
when $i_L < I_L$ — " — discharged.

We know that $V_c(t) = C \frac{d\,i_c(t)}{dt}$ ⇒

$$V_c(t) = V_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(\varepsilon)\,d\varepsilon \Rightarrow$$

$$\underbrace{V_c(t) - V_c(t_0)}_{\text{change in voltage}} = \frac{1}{C}\underbrace{\int_{t_0}^{t} i_c(\varepsilon)\,d\varepsilon}_{\text{change in charge}} \Rightarrow$$

$$\Delta V_c = \frac{1}{C}\Delta Q$$

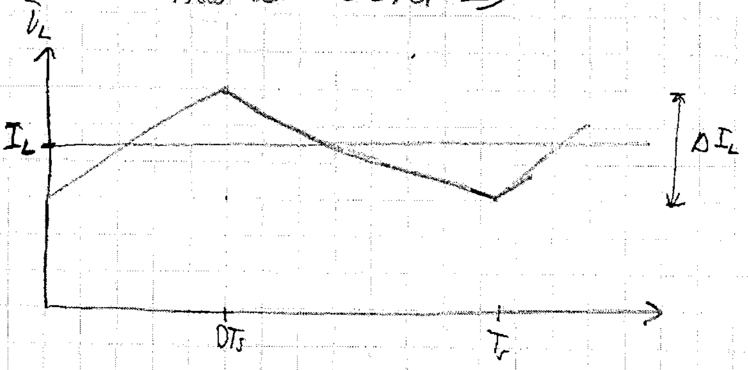
Shade ΔQ in the current plott.

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{1}{C}\left(\frac{\Delta I_L}{2}\cdot\frac{T_s}{2}\frac{1}{2}\right) = \frac{T_s}{8C}\,\frac{V_o}{L}(1-D)T_s = \frac{V_o}{8f_s^2CL}\left(1-\frac{V_o}{V_d}\right) =$$

$$= \frac{5}{8(20\cdot10^3)^2\cdot470\cdot10^{-6}\cdot1\cdot10^{-3}}\left(1-\frac{5}{12,6}\right) = \underline{2,01\,mV}$$

- Note that the ripple is independent of the output power when the converter operates in CCM

7.3 Assume CCM $\Rightarrow$

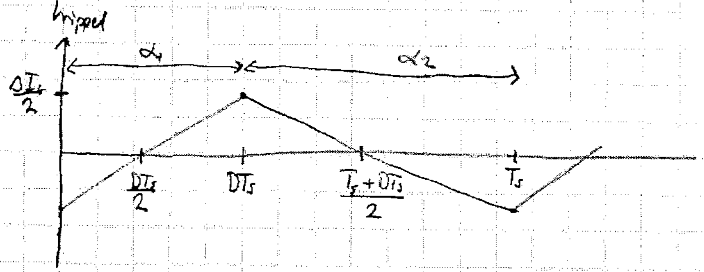


$$D = \frac{5}{12{,}6}$$

$$\Delta I_L = \frac{V_o}{L}(1-D)T_s = \frac{5}{1\cdot 10^{-3}}\left(1-\frac{5}{12{,}6}\right)\frac{1}{20\cdot 10^{3}} \simeq 151\,mA$$

$$I_L = I_o = 200\,mA > \frac{\Delta I_L}{2} \Rightarrow \underline{CCM}$$

Calculate the rms value off the ripple current.
The ripple current can be drawn as



$$\frac{T_s - DT_s}{2} + DT_s = \frac{T_s + DT_s}{2}$$

$$I_{ripple,RMS} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i_{ripple}^2(t)\,dt} = \sqrt{\frac{DT_s}{T_s}\frac{1}{DT_s}\int_0^{DT_s} i_{ripple}^2(t)\,dt + \frac{T_s-DT_s}{T_s}\frac{1}{T_s-DT_s}\int_{DT_s}^{T_s} i_{ripple}^2(t)\,dt}$$

$$= \sqrt{\frac{DT_s}{T_s} I^2_{ripple,RMS,\alpha_1} + \frac{T_s-DT_s}{T_s} I^2_{ripple,RMS,\alpha_2}}$$

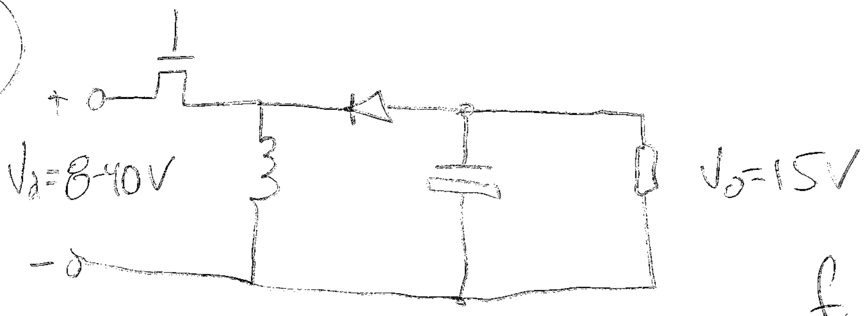
$$I^2_{ripple,RMS,\alpha_1} = \frac{1}{DT_s}\int_0^{DT_s} i_{ripple}^2(t)\,dt \overset{\text{Simpson's}}{=} \frac{1}{6}\left(\frac{\Delta I_L}{2}\right)^2 + \frac{1}{6}\left(\frac{\Delta I_L}{2}\right)^2 = \frac{1}{12}\Delta I_L^2$$

$$I^2_{ripple,RMS,\alpha_2} = \frac{1}{T_s-DT_s}\int_{DT_s}^{T_s} i_{ripple}^2(t)\,dt = \frac{1}{12}\Delta I_L^2$$

$$I_{ripple,RMS} = \sqrt{\frac{DT_s}{T_s}\frac{1}{12}\Delta I_L^2 + \frac{T_s-DT_s}{T_s}\frac{1}{12}\Delta I_L^2} = \frac{1}{\sqrt{12}}\Delta I_L = \frac{151\cdot 10^{-3}}{\sqrt{12}} =$$

$$= 44\,mA$$

7.12

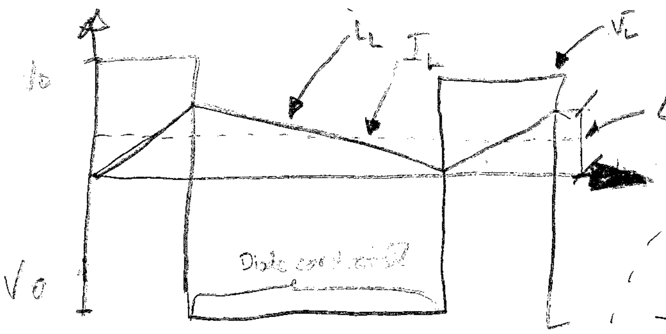


$f_{sw} = 20kHz$

$C = 470\mu F$

$P_o \geq 2.0W$

Calculate L_{min} required to operate in CCM?



$$\Delta i_L = \frac{1}{L}\int_0^{DT} V_D \, dt = \frac{D \cdot T \cdot V_D}{L}$$

$$\Delta i_L = \frac{V_o(1-D)}{L \cdot f_{sw}}$$

We have to express Δi_L in terms of I_o or I_{in}.

$$I_o = I_D = \frac{1}{T}\int_0^T i_D \, dt = \frac{1}{T}\int_{DT}^T i_d \, dt$$

But i_d equals to i_L within this interval?

$$I_D = \frac{1}{T}\int_{DT}^T i_L \, dt = \frac{T-DT}{T} \underbrace{\frac{1}{T-DT}\int_{DT}^T i_L \, dt}_{I_L} \Rightarrow I_D = \frac{T-DT}{T} \cdot I_L \Rightarrow I_o = (1-D) \cdot I_L$$

ideal converter:

$P_d = P_o \Rightarrow V_d I_d = V_o \cdot I_o$

Steady state: $0 = \int_0^T v_L \, dt =$

$= \int_0^{DT} V_d \, dt + \int_{DT}^T -V_o \, dt = V_d DT - T V_o + DT V_o = 0$

$\Rightarrow V_d \cdot D = V_o(1-D) \Rightarrow V_o = V_d \cdot \frac{D}{1-D}$

Steady state 2: $I_C = 0 \Rightarrow$

$0 = \frac{1}{T}\int_0^T i_d - i_{out} \, dt = \frac{1}{T}\int_0^T i_d \, dt - \frac{1}{T}\int_0^T I_o \, dt =$

$= I_d - I_o = 0 \Rightarrow I_d = I_o$

ideal converter: $V_D \cdot I_D = V_o I_o$

$\Rightarrow V_d \cdot I_d = V_d \frac{D}{1-D} I_o \Rightarrow I_d = I_o \frac{D}{1-D}$

This gives: $\underbrace{\frac{I_o}{1-D}}_{\Delta i_L = 2 I_L} = \frac{V_o(1-D)}{2 \cdot L \cdot f_{sw}} \Rightarrow L = \frac{V_o(1-D)^2}{2 \cdot I_o \cdot f_{sw}} = \frac{V_o^2(1-D)^2}{2P_o \cdot f_{sw}}$

$$L = \frac{V_o^2}{2 \cdot P_o f_{sw}} \cdot (1-D)^2 \Rightarrow \begin{cases} \text{A small } D \text{ gives the highest value of } L \\ \text{A low } P_o \text{ gives the highest value of } L \end{cases}$$

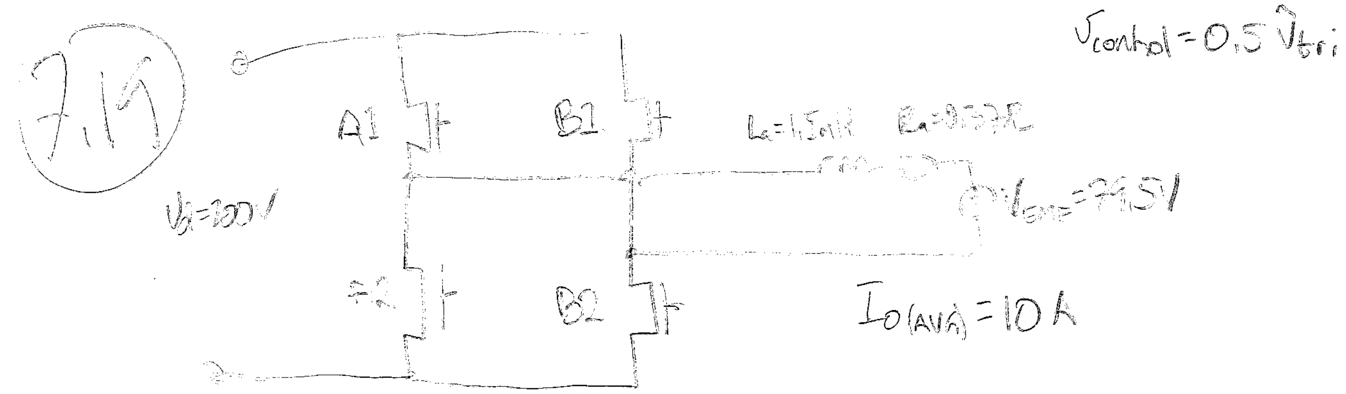
$$D_{min}: \quad V_o = V_d \frac{D}{1-D} \Rightarrow D = \frac{V_o}{V_d} - \frac{V_o}{V_d} \cdot D = D \Rightarrow$$

$$\Rightarrow \frac{V_o}{V_d} = D\left(1 + \frac{V_o}{V_d}\right)$$

$$\Rightarrow D = \frac{V_o}{V_d + V_o} = \frac{15}{15+40} = 0,2727$$

$$L_{min} = \frac{15^2}{2 \cdot 20k \cdot 2} \cdot (1-0.2727)^2 = \underline{1,49 \text{ mH}}$$

7.19



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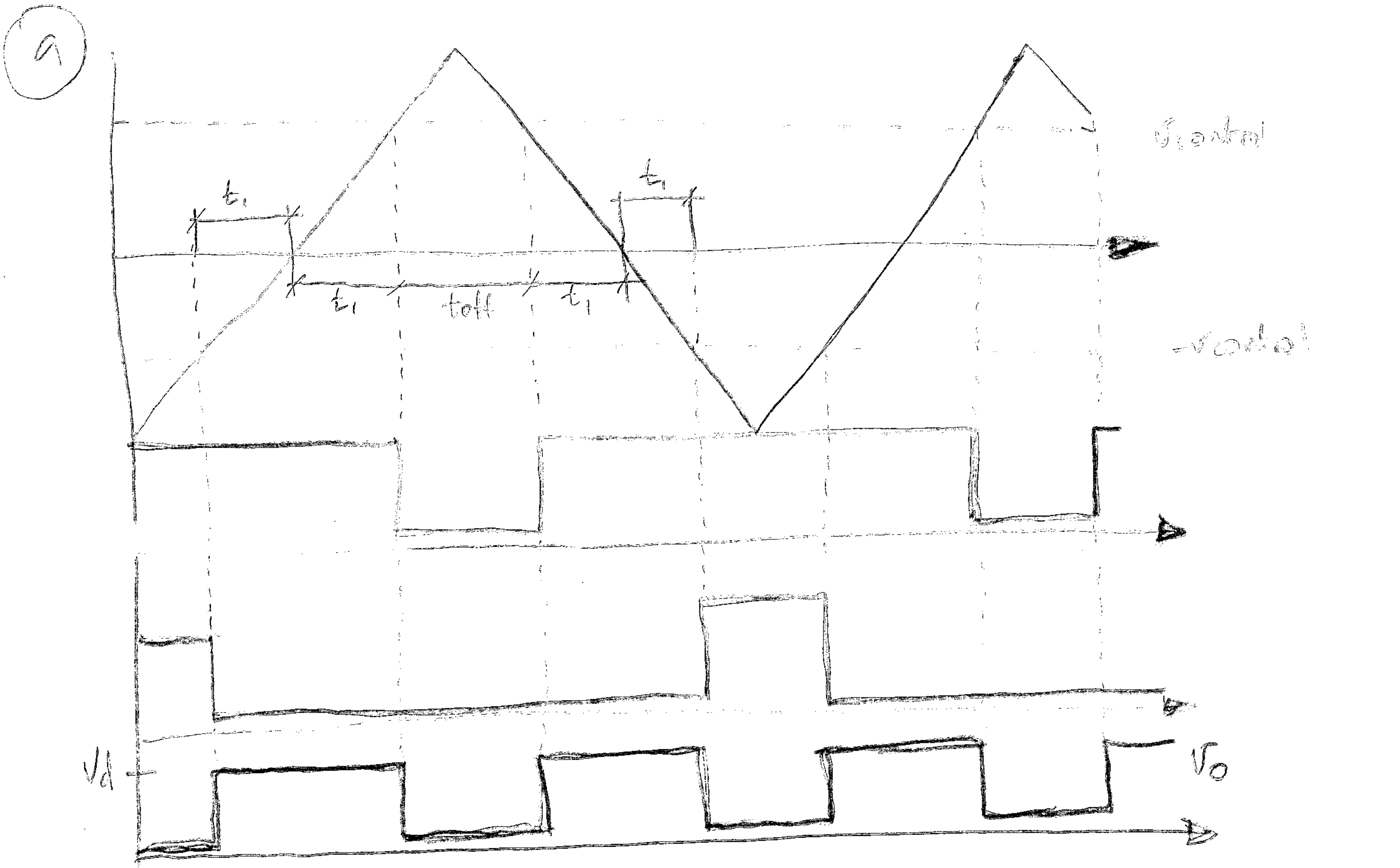


$$V_{o(AVG)} = \frac{1}{T}\int_0^T v_o\,dt = \frac{1}{T}\left(0\cdot\frac{t_{off}}{2} + 2t_1V_d + 0\cdot t_{off} + 2\cdot t_1V_d + 0\,\frac{t_{off}}{2}\right) = \frac{4t_1V_d}{T}$$

So, we have to express t_1 in known terms, see 7.13:

$$v_c = \frac{4}{T}t_1\cdot\hat{V}_{tri} \Rightarrow t_1 = \frac{v_{control}\cdot T}{4\cdot\hat{V}_{tri}}$$

$$V_{o(AVG)} = \frac{4V_d}{T}\cdot\frac{v_{control}T}{4\hat{V}_{tri}} = V_d\cdot\frac{v_{control}}{\hat{V}_{tri}} = \underline{\underline{0.5\,V_d}}$$

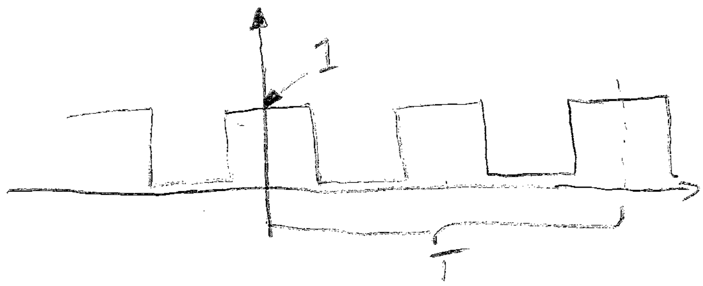
Same for the current:

$$I_{d(AVG)} = \frac{1}{T}\int i_d\,dt = \frac{4t_1I_o}{T} \Rightarrow I_{d(AVG)} = I_o\cdot\frac{v_{control}}{\hat{V}_{tri}} = \underline{\underline{0.5\,I_o}}$$

(b) Fourier Analysis

Calculate the fourier coefficients for v_o and i_d

⇒ normalize the axis, calculate the fourier series for $\frac{v_o}{V_d}$ and $\frac{i_d}{I_o}$, gives unity axis...



Even function: $f(t) = f(-t)$ gives $b_n = 0$

Periodic function gives: $a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega t)\,dt$

$$a_n = \frac{4}{T}\int_0^{t_1}\cos(n\omega t) + \frac{4}{T}\int_{\frac{T}{2}-t_1}^{T/2}\cos(n\omega t) = \frac{4}{T}\left(\left[\frac{\sin(n\omega t)}{n\omega}\right]_0^{t_1} + \left[\frac{\sin(n\omega t)}{n\omega}\right]_{\frac{T}{2}-t_1}^{T/2}\right) =$$

$$= \frac{4}{n\omega T}\left(\sin(n\omega t_1) + \sin\left(n\omega \frac{T}{2}\right) - \sin\left(n\omega\left(\frac{T}{2}-t_1\right)\right)\right) = \left[\begin{array}{l}\omega = \frac{2\pi}{T} \\ t_1 = \frac{V_{control}\cdot T}{4\cdot \hat{V}_{tri}}\end{array}\right] =$$

$$= \frac{2}{n\pi}\left(\sin\left(n\frac{2\pi}{T}\frac{V_{control}\cdot T}{4\cdot\hat{V}_{tri}}\right) + \underbrace{\sin(n\pi)}_{=0} - \sin n\frac{2\pi}{T}\left(\frac{T}{2}-\frac{V_{control}\cdot T}{4\cdot \hat{V}_{tri}}\right)\right) =$$

$$= \frac{2}{n\pi}\left(\sin\left(n\pi\cdot\frac{V_{control}}{2\hat{V}_{tri}}\right) - \sin\left(n\pi - n\pi\frac{V_{control}T}{2\hat{V}_{tri}}\right)\right) =$$

$$= \frac{2}{n\pi}\left(\sin(0{,}25n\pi) - \underbrace{\sin(n\pi)\cdot\cos\left(n\pi\frac{V_{control}T}{2\hat{V}_{tri}}\right)}_{=0} + \cos(n\pi)\sin\left(n\pi\frac{V_{control}T}{2\hat{V}_{tri}}\right)\right)$$

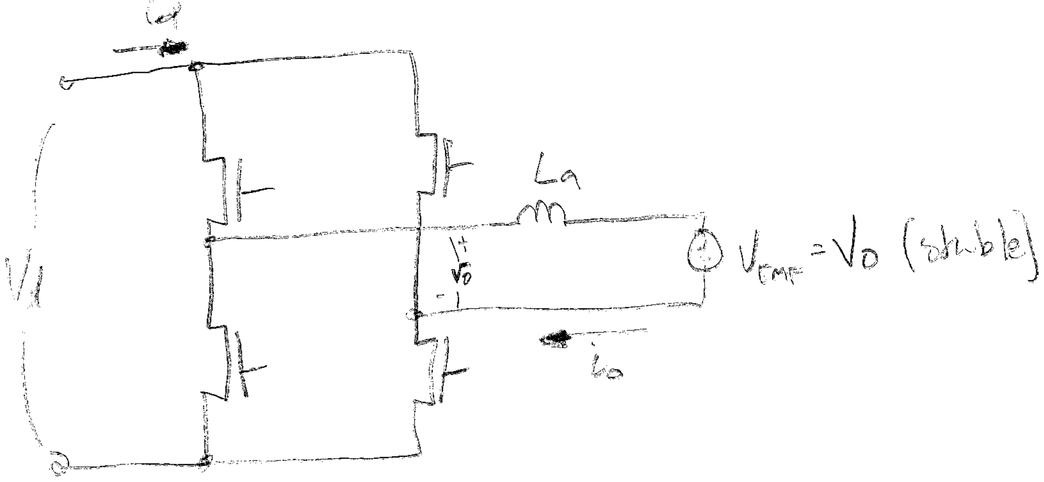
$$= \frac{2}{n\pi}\left(\sin(0{,}25n\pi) + (-1)^n\sin(0{,}25n\pi)\right)$$

⟹

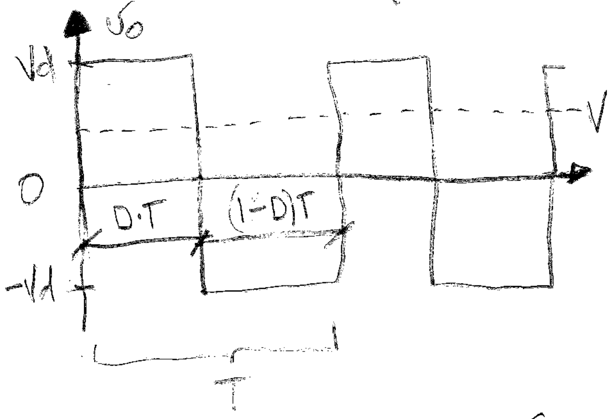
The fourier coefficients can then be calculated as:

n		
1	$\frac{2}{1\pi}\left(\sin(0{,}25\pi) + (-1)^1 \sin(0{,}25\pi)\right)$	0
2	$\frac{2}{2\pi}\left(\sin(0{,}5\pi) + (-1)^2 \sin(0{,}5\pi)\right)$	0,6366
3	$\frac{2}{3\pi}\left(\sin(0{,}75\pi) + (-1)^3 \sin(0{,}75\pi)\right)$	0
4	$\frac{2}{4\pi}\left(\sin(\pi) + (-1)^4 \sin(\pi)\right)$	0
5	$\frac{2}{5\pi}\left(\sin(1{,}25\pi) + (-1)^5 \sin(1{,}25\pi)\right)$	0
6	$\frac{2}{6\pi}\left(\sin(1{,}5\pi) + (-1)^6 \sin(1{,}5\pi)\right)$	-0,212

7.22



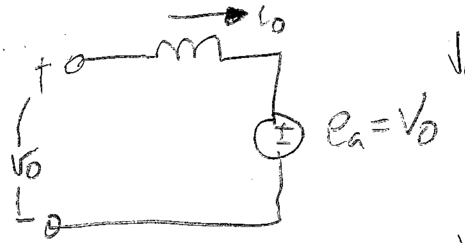
Analytically obtain the value of $\frac{V_o}{V_d}$ which results in the maximum peak-to-peak ripple in the output current?

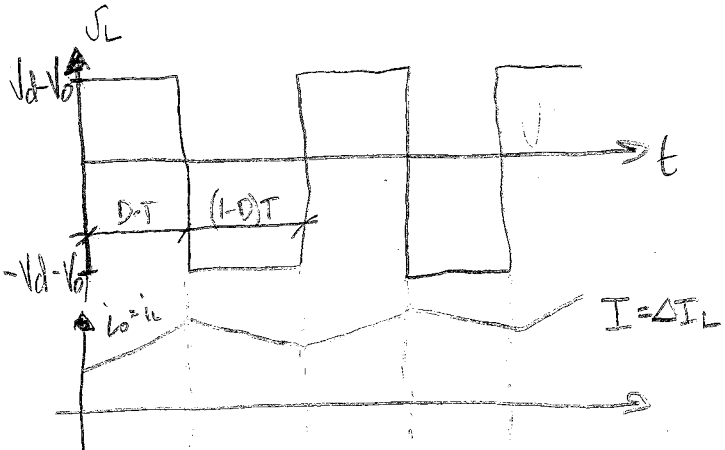


Bipolar switching scheme

Notes:

V_o is positive and lower than V_d





$$\Delta I_L = \frac{1}{L_a}\int^{t} v_L(t)\,dt =$$

$$= \frac{1}{L_a}\int_0^{DT = t_1} (V_d - V_o)\,dt = \frac{V_d - V_o}{L_a}\cdot DT = \Delta I_L$$ So, we have to find another expression for DT

$$\Rightarrow D: \; 0 = \frac{1}{L_a}\int_0^T v_L(t)\,dt = \frac{1}{L_a}\int_0^{DT} V_d - V_o\,dt + \int_{DT}^{T} -V_d - V_o\,dt = \frac{1}{L_a}\left(DT\cdot(V_d - V_o) + T(-V_d - V_o) - DT(-V_d - V_o)\right)$$

$$= \frac{1}{L_a}\left(V_d DT - DTV_o - TV_d - TV_o + V_d DT + DTV_o\right) = \frac{1}{L_a T}\left(2V_d D - V_d - V_o\right) = 0$$

$$\Rightarrow 0 = (2 \cdot V_d \cdot D - V_d - V_o) \Rightarrow D = \frac{V_d + V_o}{2V_d} = \underline{\underline{\frac{1}{2}\left(1 + \frac{V_o}{V_d}\right)}}$$

lengthen with $\frac{V_d}{V_d}$

$$\Rightarrow \Delta I_L = \frac{V_d - V_o}{L_a} DT = \frac{V_d - V_o}{L_a} \frac{1}{2}\left(1 + \frac{V_o}{V_d}\right) \cdot T = \frac{T}{2L_a}\left(1 + \frac{V_o}{V_d}\right)\left(\frac{V_d \cdot V_d}{V_d} - \frac{V_o \cdot V_d}{V_d}\right) =$$

$$= \frac{T}{2L_a}\left(1 + \frac{V_o}{V_d}\right)\left(\frac{V_d \cdot V_d - V_o V_d}{V_d}\right) = \frac{TV_d}{2L_a}\left(1 + \frac{V_o}{V_d}\right)\left(\frac{V_d - V_o}{V_d}\right) =$$

$$= \frac{TV_d}{2L_a}\left(1 + \frac{V_o}{V_d}\right)\left(1 - \frac{V_o}{V_d}\right)$$

When you want to search for a maximum, find the first derivative and set it to 0, then find the correct $\frac{V_o}{V_d}$

$$\Delta I_L = \frac{TV_d}{2L_a}\left(1 - \left(\frac{V_o}{V_d}\right)^2\right) \Rightarrow \frac{\partial \Delta I_L}{\partial \left(\frac{V_o}{V_d}\right)} = \frac{-TV_d}{2L_a} \cdot 2\left(\frac{V_o}{V_d}\right) = 0 \Rightarrow$$

$$\Rightarrow \frac{V_o}{V_d} = 0$$ to get a stationary point?

(A negative second derivative gives a maximum point)

Use $\frac{V_o}{V_d} = 0$ in the equation for ΔI_L gives

$$\boxed{\Delta I_{L(max)} = \frac{TV_d}{2L_a} = \frac{V_d}{2L_a f_{sw}}}$$