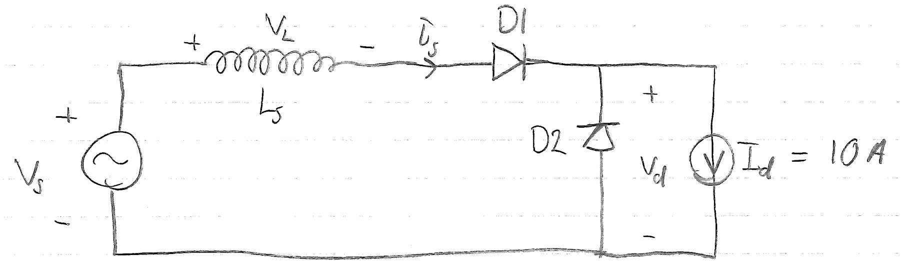
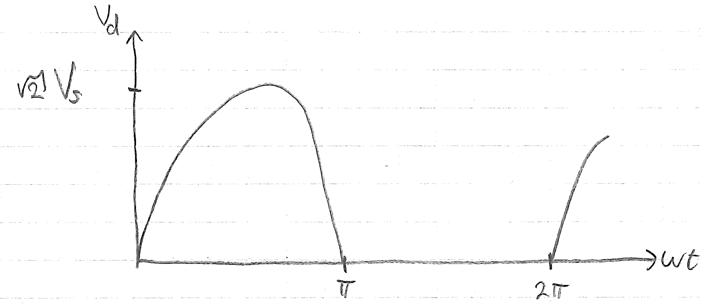
5-5



a) $V_s = 120V$ at 60 Hz $L_s = 0$

When $\begin{cases} V_s > 0 & \text{D1 will conduct and D2 will block} \\ V_s < 0 & \text{D1 will block and D2 will conduct} \end{cases}$ $\Rightarrow$

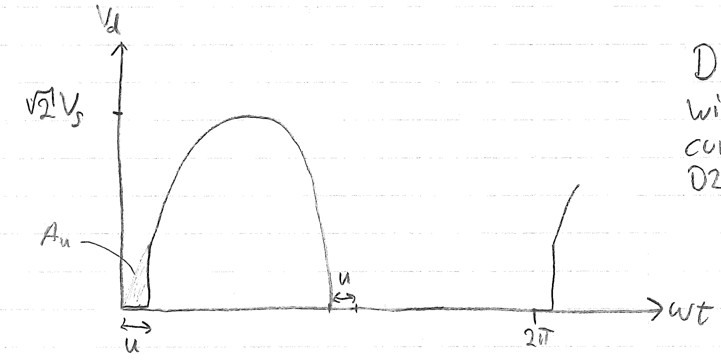


$$V_d = \frac{1}{2\pi}\int_0^{2\pi} V_d(wt)\,dwt = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V_s \sin(wt)\,d(wt) = \frac{\sqrt{2}V_s}{2\pi}\left[-\cos(wt)\right]_0^{\pi} =$$

$$= \frac{\sqrt{2}\cancel{2}V_s}{\cancel{2}\pi} = \frac{\sqrt{2}}{\pi}V_s = \frac{\sqrt{2}}{\pi}120 = \underline{\underline{54V}}$$

$$P_d = V_d I_d = 52 \cdot 10 = \underline{\underline{540W}}$$

b) $V_s = 120V$ at 60 Hz $L_s = 5mH$

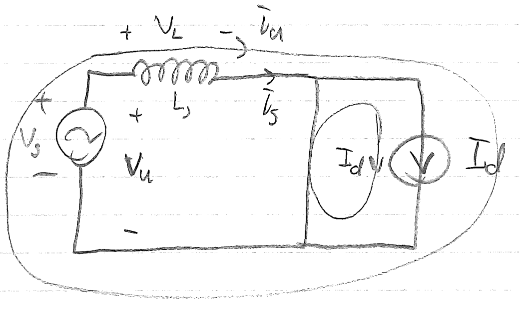


Due to the inductance L_s it will take some time to commutate the current from D1 to D2 and from D2 to D1.

As we can notice from the figure: when we should commutate the current from D2 to D1 ($wt = 0$) both D1 and D2 will conduct the current for a while $\Rightarrow$

During the commutation intervall we can draw the circuit as:

$0 \leq \omega t \leq u$



We see that when $i_s = i_u$ is equal to I_d the commutation is finished

$$\left.\begin{aligned} i_u &= i_s \\ V_u &= V_s = V_L \end{aligned}\right\} \Rightarrow$$

$V_L = L_s \frac{d i_u}{dt}$ multiply both sides with $d(\omega t)$ and integrate over the commutation intervall $\Rightarrow$

$$\omega L_s \int_0^{I_d} d i_u = \int_0^u V_L(\omega t)\, d(\omega t) = \int_0^u \sqrt{2} V_s \sin(\omega t)\, d(\omega t) = A_u \Rightarrow$$

$$A_u = \omega L_s I_d = \sqrt{2} V_s \left[-\cos(\omega t)\right]_0^u = \sqrt{2} V_s (1 - \cos(u)) \Rightarrow$$

$$\cos(u) = 1 - \frac{\omega L_s I_d}{\sqrt{2} V_s} = 1 - \frac{2\pi 60 \cdot 5 \cdot 10^{-3} \cdot 10}{\sqrt{2} \cdot 120} = 0{,}889 \Rightarrow$$

$$\underline{\underline{u = 27{,}26^\circ}}$$

As we see the commutation from D1 to D2 is similar to the commutation from D2 to D1 and therefore the commutation angle is the same.

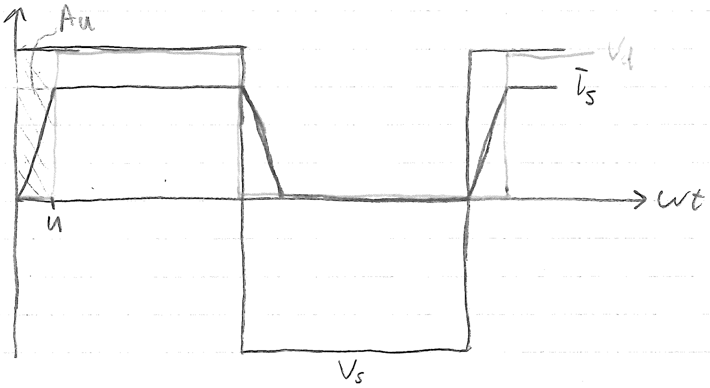
For the average output voltage V_d we calculate it in the same way as for problem a) but we substract the area A_u from the integral to compensate for what we missed $\Rightarrow$

$$V_d = \frac{1}{2\pi}\left(\int_0^{\pi} \sqrt{2} V_s \sin(\omega t)\, d(\omega t) - A_u\right) = \frac{1}{2\pi}\left(\sqrt{2}\, 2 V_s - \omega L_s I_d\right) =$$

$$= \frac{\sqrt{2} V_s - 0{,}5 \omega L_s I_d}{\pi} = \frac{\sqrt{2}\, 120 - \pi 60 \cdot 5 \cdot 10^{-3} \cdot 10}{\pi} = \underline{51\,V}$$

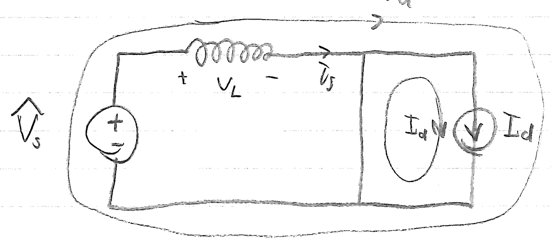
$$P_d = V_d I_d = 51 \cdot 10 = 510\,W$$

c) V_s is a 60 Hz square wave form with 200V amplitude



During the commutation from D2 to D1 we can draw the circuit as

$0 \leq \omega t \leq u$



$\Rightarrow \left.\begin{array}{l} i_u = i_s \\ V_L = \hat{V}_s \end{array}\right\} \Rightarrow$

$V_L = \hat{V}_s = L_s \frac{di_s}{dt} = L_s \frac{di_u}{dt}$ multiply with $d(\omega t)$ and integrate both sides over the commutation intervall $\Rightarrow$

$$\omega L \int_0^{I_d} di_u = \int_0^u V_L(\omega t)\, d(\omega t) = \int_0^u \hat{V}_s\, d(\omega t) = A_u \Rightarrow$$

$$A_u = \omega L I_d = \hat{V}_s u \Rightarrow$$

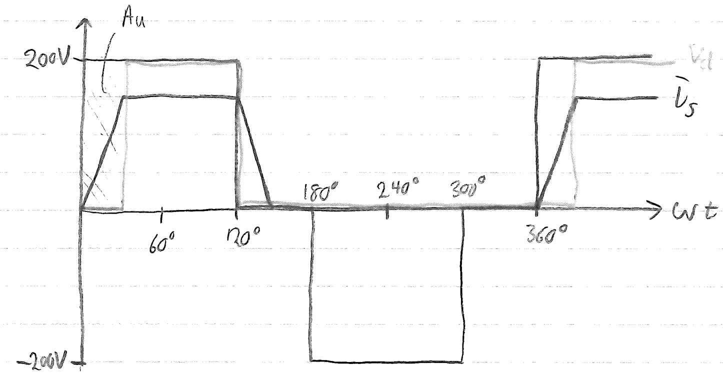
$$u = \frac{\omega L I_d}{\hat{V}_s} = \frac{2\pi 60 \cdot 5 \cdot 10^{-3} \cdot 10}{200} = 0{,}0942 \text{ rad} = \underline{\underline{5{,}4^\circ}}$$

In the same way as before

$$V_d = \frac{1}{2\pi} \int_u^{\pi} \hat{V}_s\, d(\omega t) = \frac{\pi - u}{2\pi} \hat{V}_s = \frac{\pi - 0{,}0942}{2\pi} 200 = 97V$$

$$P_d = V_d I_d = 97 \cdot 10 = 970 W$$

d)



Since we have the same inductance $L_s = 5\,mH$ and the same peak value of the pulse waveform as for the square-waveform in problem c) the commutation angle u will be the same ⇒

$$\underline{\underline{u}} = \frac{\omega L I_d}{\hat{V}_s} = \underline{\underline{5{,}4°}}$$

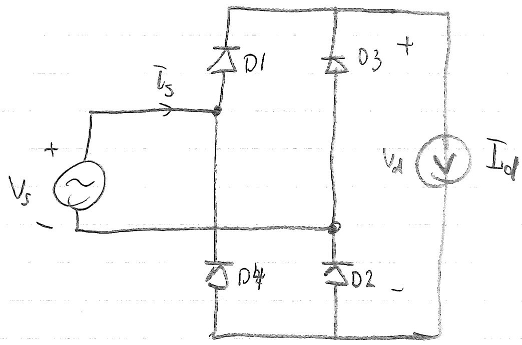
The area A_u will also be the same ⇒ $A_u = \omega L I_d$ ⇒

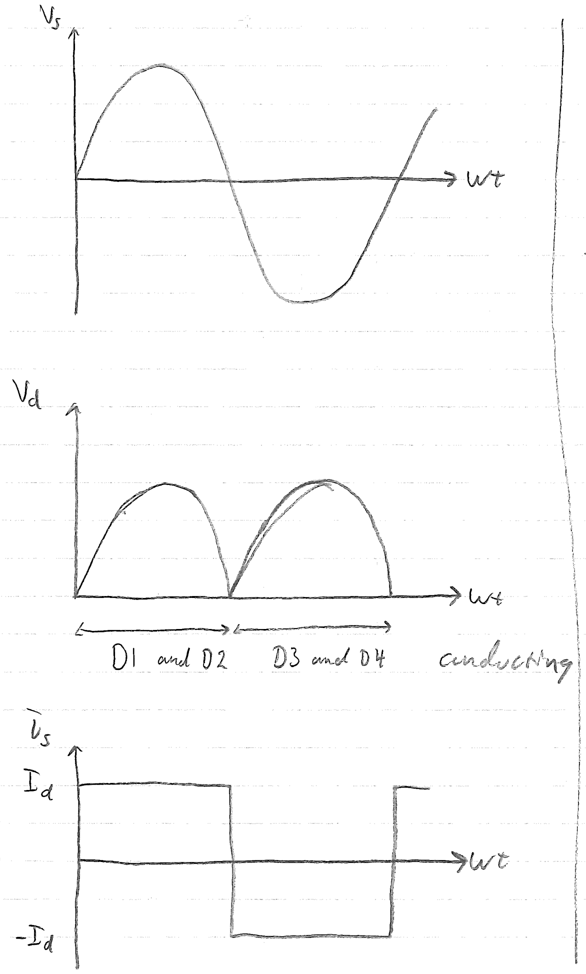
$$\underline{V_d} = \frac{1}{2\pi}\int_u^{\frac{2}{3}\pi} \hat{V}_s\, d(\omega t) = \frac{\frac{2}{3}\pi - u}{2\pi}\hat{V}_s = \frac{V_s}{3} - \frac{\omega L I_d}{2\pi} = \frac{200}{3} - \frac{2\pi \cdot 60 \cdot 5 \cdot 10^{-3} \cdot 10}{2\pi} =$$

$$= \underline{63{,}7\,V}$$

$\underline{P_d} = V_d I_d = 63{,}7 \cdot 10 = \underline{637\,W}$

5-6





from the i_s plot we see that the average current through each diode can be calculated by

$$\underline{I_{diode,AVG}} = \frac{1}{2\pi}\int_0^{\pi} i_s(wt)\,d(wt) =$$

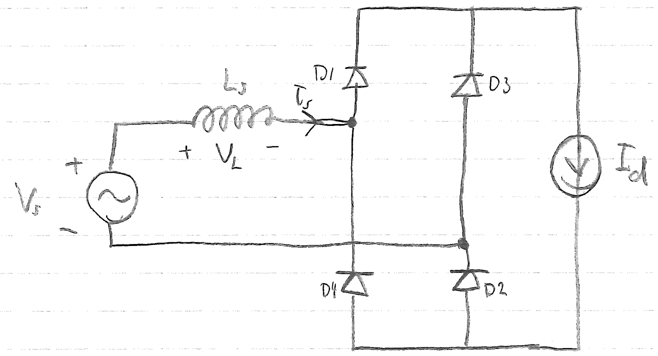
$$= \frac{\pi - 0}{2\pi} I_d = \underline{\frac{1}{2} I_d}$$

and the rms

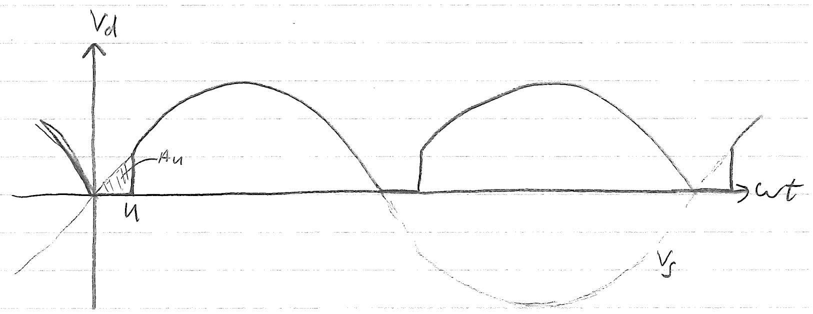
$$\underline{I_{diode,RMS}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_s^2(wt)\,d(wt)} =$$

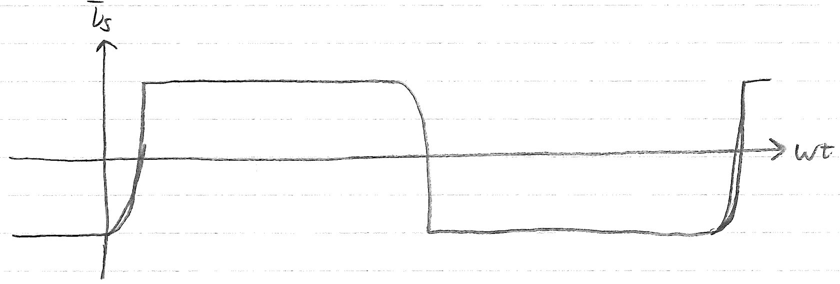
$$= \sqrt{\frac{\pi - 0}{2\pi} I_d^2} = \underline{\frac{1}{\sqrt{2}} I_d}$$

5-8 $V_s = 120V$ at $60 Hz$ $L_s = 1 mH$ and $I_d = 10 A$



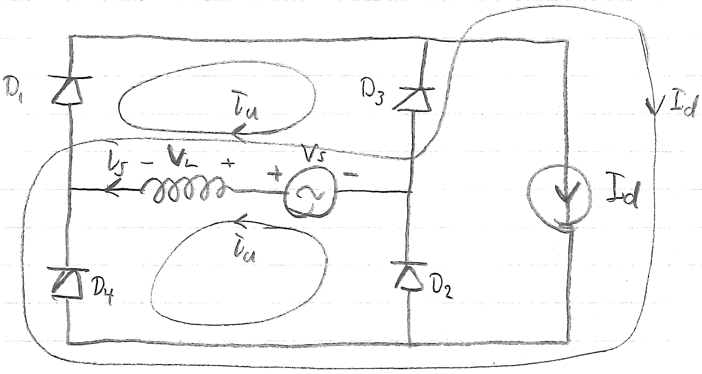
We start at a time somewhat smaller then zero. At this time we have a negative voltage V_s and a negative curren $I_s = -I_d$ and Diode D3 and D4 are conducting.
After we have passed $wt = 0$ then we have a positive voltage V_s which means that diode D1 and D2 will conduct but due to the inductance L_s we will still have a negative current I_s and this gives that diode D3 and D4 are still conducting $\Rightarrow$ All four diodes are conducting.





During the commutation we can draw the circuit as

$0 \leq \omega t \leq u$



$$\Rightarrow \left.\begin{array}{l} i_{D1} = i_{D2} = i_u \\ i_{D3} = i_{D4} = I_d - i_u \\ i_s = -I_d + 2 i_u \\ v_L = v_s \end{array}\right\} \quad \text{when } i_u = I_d \text{ then the commutation is finished}$$

$$v_L = v_s = L_s \frac{d i_s}{dt} \quad \text{multiply with } d(\omega t) \text{ and integrate} \Rightarrow$$

$$\omega L_s \int_{-I_d}^{I_d} d i_s = \int_0^u v_L(\omega t)\, d(\omega t) = \int_0^u \sqrt{2} V_s \sin(\omega t)\, d(\omega t) = A_u \Rightarrow$$

$$A_u = 2 \omega L_s I_d = \sqrt{2} V_s (1 - \cos(u)) \Rightarrow$$

$$\cos(u) = 1 - \frac{\sqrt{2} \omega L_s I_d}{V_s} = 1 - \frac{\sqrt{2}\, 2\pi 60 \cdot 1 \cdot 10^{-3} \cdot 10}{120} = 0{,}9556 \Rightarrow$$

$$u = 17{,}14^\circ$$

$$\underline{\underline{V_d}} = \frac{1}{\pi}\left(\int_0^\pi \sqrt{2} V_s \sin(\omega t)\, d(\omega t) - A_u \right) = \frac{1}{\pi}\left(2\sqrt{2} V_s - 2 \omega L_s I_d\right) \Rightarrow$$

$$V_d = \frac{2\sqrt{2} V_s - \omega L_s I_d}{\pi} = \frac{2\sqrt{2}\, 120 - 2 \cdot 2\pi 60 \cdot 1 \cdot 10^{-3} \cdot 10}{\pi} = \underline{\underline{105{,}6\,V}}$$

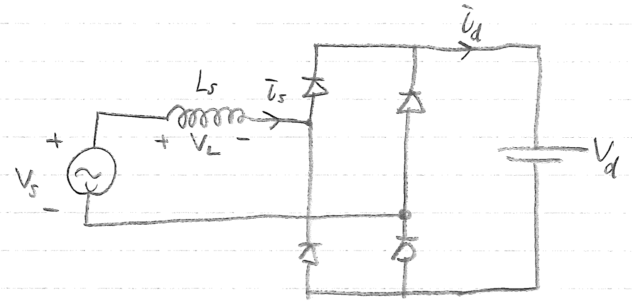
$$\underline{\underline{P_d}} = V_d I_d = 105{,}6 \cdot 10 = \underline{\underline{1056\,W}}$$

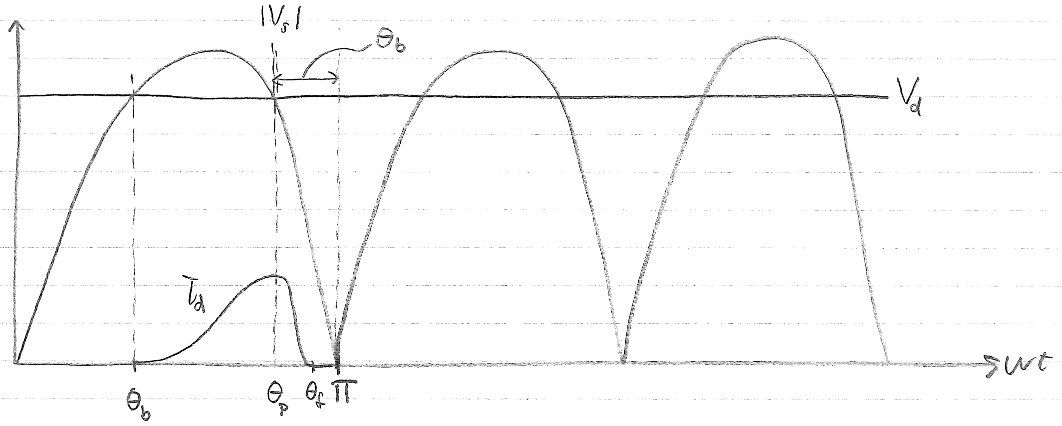
$$\underline{\underline{\Delta V_d}} = 1 - \frac{V_d}{V_{d\,L_s=0}} = 1 - \frac{105{,}6}{\frac{2\sqrt{2}}{\pi} V_s} = 1 - \frac{105{,}6}{0{,}9 \cdot 120} = \underline{\underline{2{,}22\,\%}}$$

5-11

$V_s = 120\,V$ at $60\,Hz$ $\quad L_s = 1\,mH \quad V_d = 150\,V$

Calculate i_d and indicate θ_b θ_f and $I_{d,peack}$ $I_{d,AVG}$





From the figure we see that:

$$V_d = \sqrt{2}\,V_s \sin(\theta_b) \Rightarrow \quad \sin(\theta_b) = \frac{V_d}{\sqrt{2}\,V_s} = \frac{150}{\sqrt{2}\,120} = 0{,}884 \Rightarrow$$

$$\theta_b = \underline{62{,}11^\circ} = \underline{1{,}084\,rad}$$

We also see that during the intervall $\theta_b \le \omega t \le \theta_f$

$$V_L = V_s - V_d = L_s \frac{di_s}{dt} = L_s \frac{di_d}{dt}$$ multiplying with $d(\omega t)$ and integrating gives

$$\omega L_s \int_0^{i_d(\theta)} di_d = \int_{\theta_b}^{\theta} (\sqrt{2}\,V_s \sin(\omega t) - V_d)\, d(\omega t) \Rightarrow$$

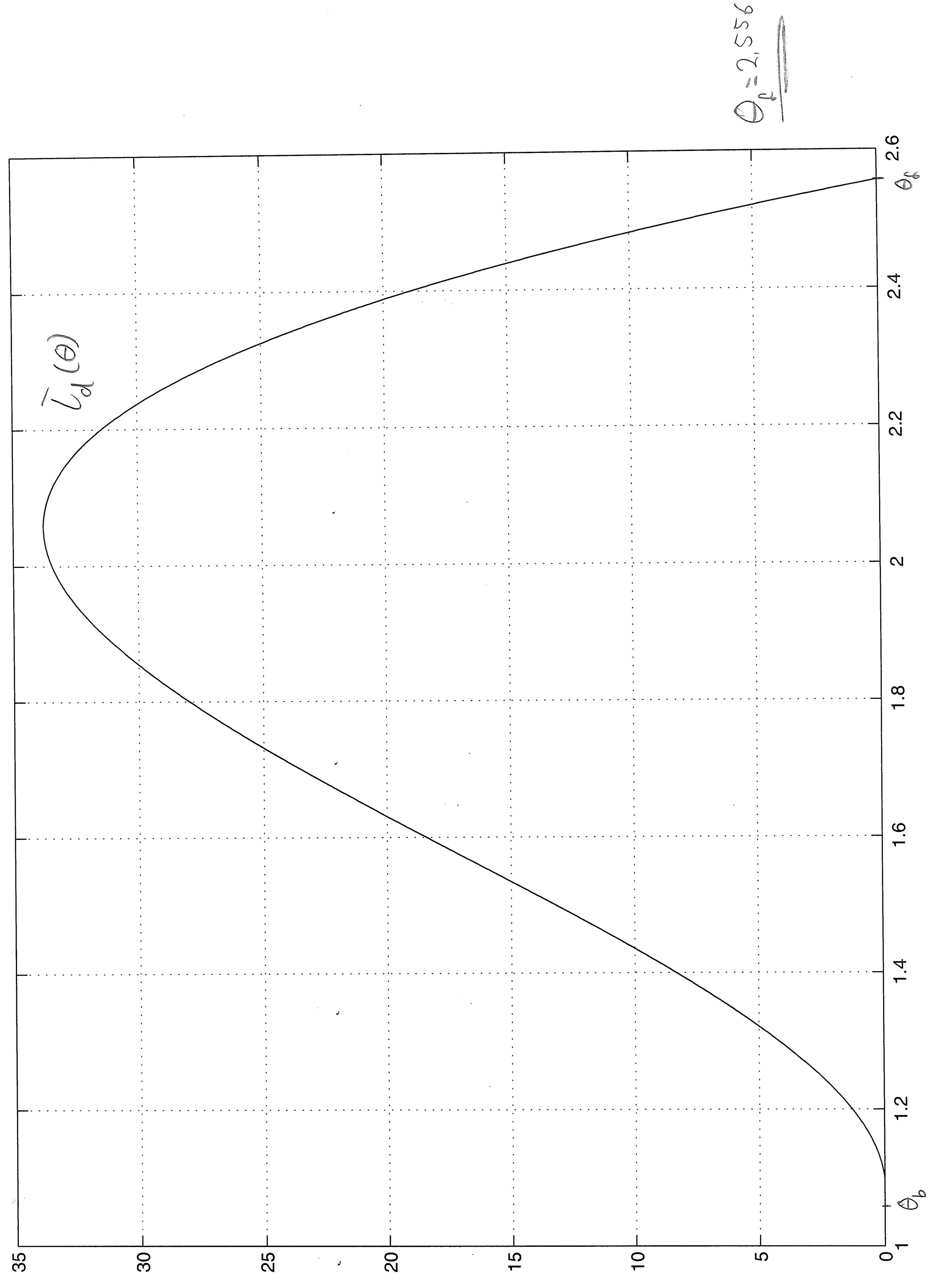
$$i_d(\theta) = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} (\sqrt{2}\,V_s \sin(\omega t) - V_d)\, d(\omega t) = \frac{1}{\omega L_s} \left[-\sqrt{2}\,V_s \cos(\omega t) - V_d\,\omega t\right]_{\theta_b}^{\theta} \Rightarrow$$

$$i_d(\theta) = \frac{1}{\omega L_s}\left(\sqrt{2}\,V_s \cos(\theta_b) - \sqrt{2}\,V_s \cos(\theta) - V_d(\theta - \theta_b)\right) =$$

$$= 641{,}9 - 450{,}2\cos(\theta) - 397{,}9\,\theta$$

5-11

2-(3)



$\theta_f = 2.556$
$\bar{l}_d(\theta)$
θ_b
θ_f
0
5
10
15
20
25
30
35
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

$\underline{i_{d,peak}} = i_d(\pi-\theta_b) = i_d(2{,}058) = 641{,}9 - 450{,}2\cos(2{,}058) - 397{,}9\cdot 2{,}058 =$

$= \underline{\underline{33{,}8\ A}}$

From the figure we see that

$i_d(\theta_f) = 0 = 641{,}9 - 450{,}2\cos(\theta_f) - 397{,}9\cdot\theta_f \Rightarrow$

$\cos(\theta_f) + 0{,}884\,\theta_f = 1{,}426$

The solution to this equation we must find numerically

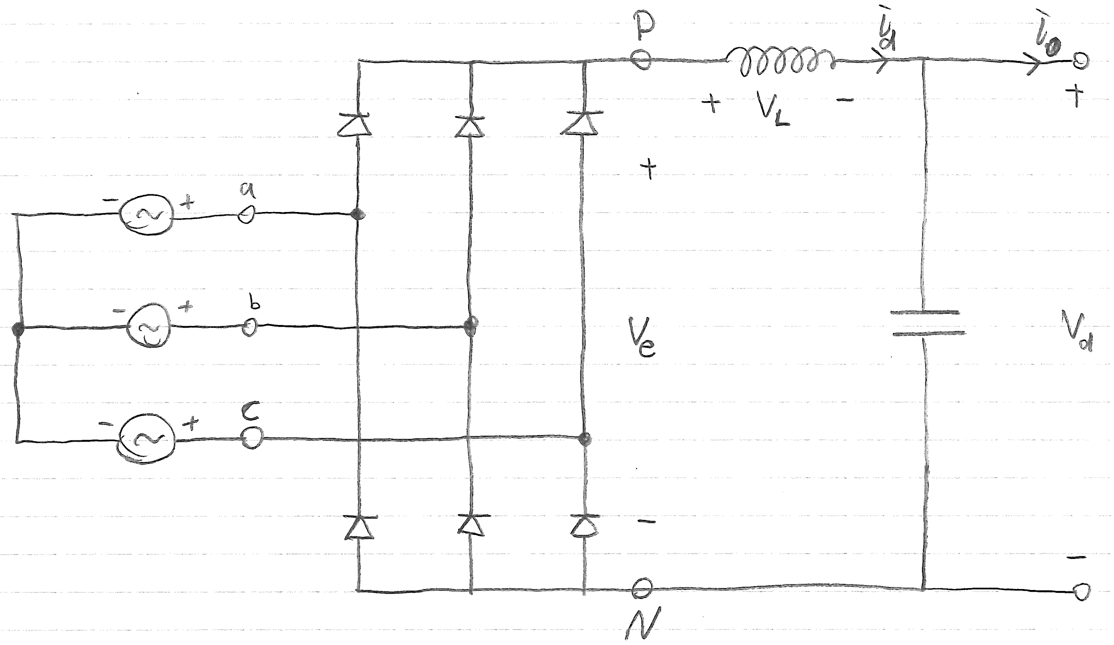
$\underline{\underline{\theta_f = 2{,}556 \text{ rad}}}$

$\underline{\underline{I_{d,AVG}}} = \frac{1}{\pi}\int_{\theta_b}^{\theta_f} 641{,}9 - 450{,}2\cos(\theta) - 397{,}9\theta\; d\theta =$

$= \frac{1}{\pi}\left[641{,}9\theta - 450{,}2\sin(\theta) - \frac{397{,}9}{2}\theta^2\right]_{1{,}084}^{2{,}556} =$

$= \frac{92{,}106 - 64{,}139}{\pi} = \underline{\underline{8{,}9\ A}}$

5-26 Calculate the minimum L_d in terms of V_{LL}, ω, I_d that will result in a continuous i_d, assum $v_d = V_d$



We know that the average of the inductor voltage moust be zero in steady-state $\Rightarrow$ $V_d = V_{e,AVG}$

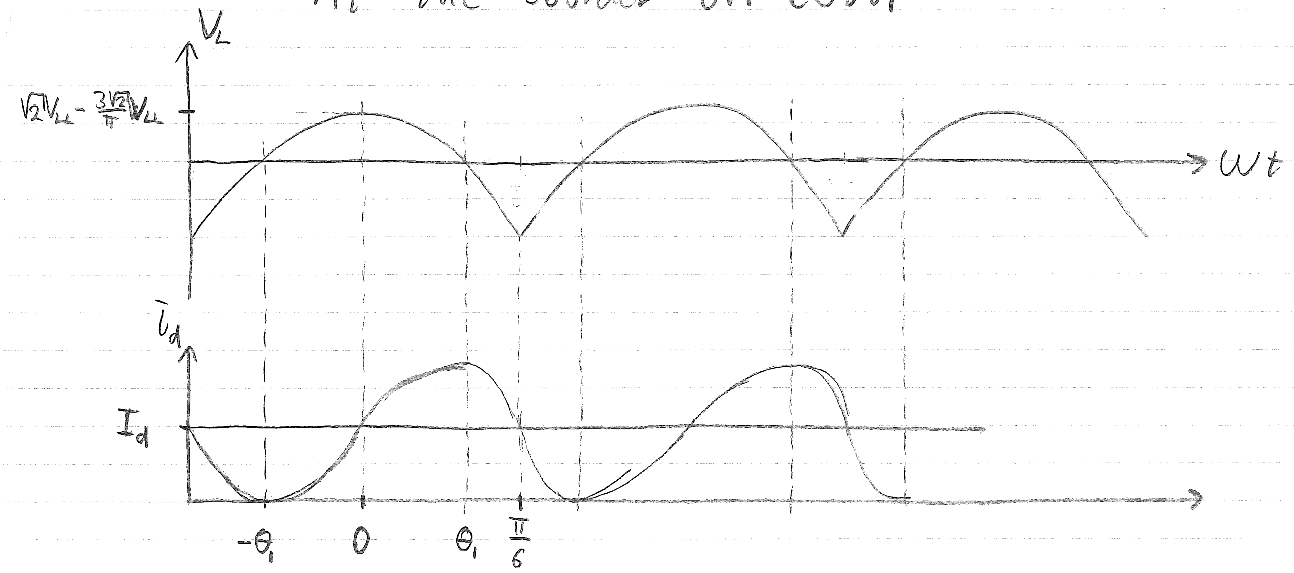
From the figure on the next page we see that

$$V_{e,AVG} = 2\frac{3}{\pi}\int_0^{\frac{\pi}{6}} \sqrt{2}V_{LL}\cos(\omega t)\,d\omega t = 2\frac{3}{\pi}\sqrt{2}V_{LL}\left[\sin(\omega t)\right]_0^{\frac{\pi}{6}} = \frac{\cancel{2}\cdot 3\sqrt{2}}{\pi}\frac{1}{\cancel{2}}V_{LL} =$$

$$= \frac{3\sqrt{2}}{\pi}V_{LL} = \underline{1{,}35\,V_{LL}} \Rightarrow$$

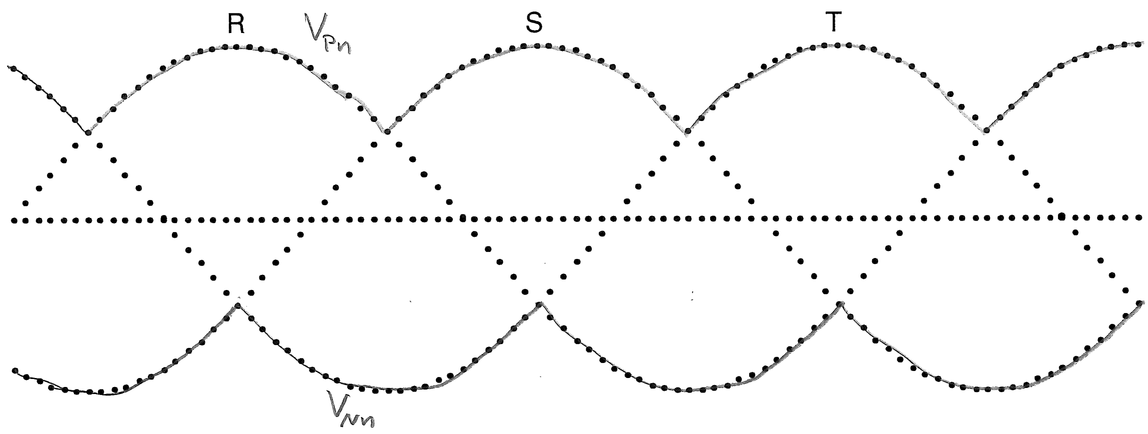
$$\underline{V_d = \frac{3\sqrt{2}}{\pi}V_{LL}} \Rightarrow V_L(\omega t) = V_e(\omega t) - \frac{3\sqrt{2}}{\pi}V_{LL}$$

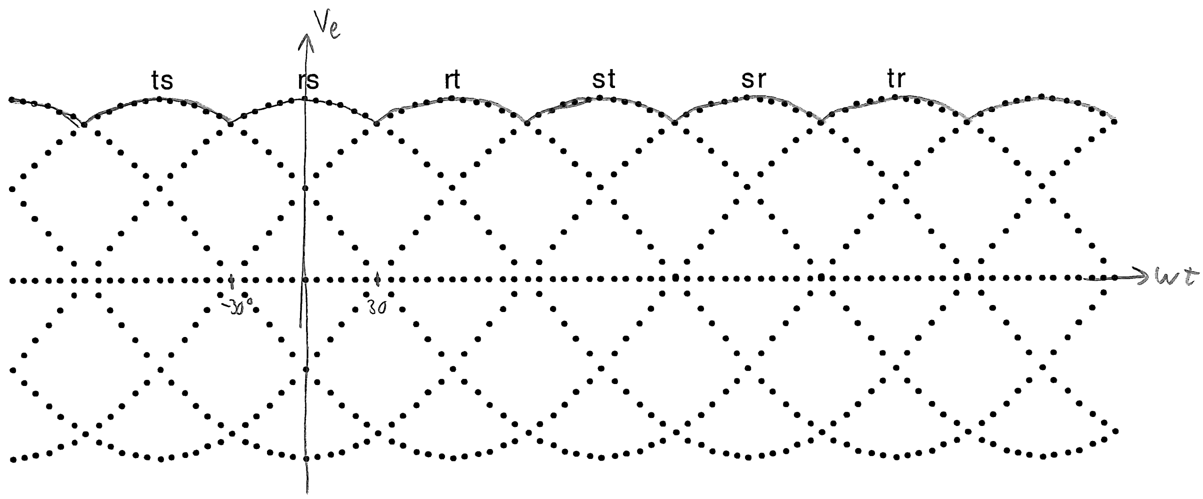
At the bourder off CCM



5-26

2-(3)





5-26

$$V_L(\omega t) = \sqrt{2} V_{LL} \cos(\omega t) - \frac{3\sqrt{2}}{\pi} V_{LL} = L_d \frac{d i_d}{dt} \qquad -\frac{\pi}{6} \le \omega t \le \frac{\pi}{6}$$

multiply with $d(\omega t)$ and integrate from $-\frac{\pi}{6}$ to θ $\Rightarrow$

$$\omega L_d \int_{i_d(-\frac{\pi}{6})}^{i_d(\theta)} d i_d = \omega L_d \left(i_d(\theta) - i_d\left(-\frac{\pi}{6}\right)\right) = \int_{-\frac{\pi}{6}}^{\theta} \sqrt{2} V_{LL} \cos(\omega t) - \frac{3\sqrt{2}}{\pi} V_{LL} \, d(\omega t) \Rightarrow$$

$$i_d(\theta) = i_d\left(-\frac{\pi}{6}\right) + \frac{1}{\omega L_d}\left(\sqrt{2} V_{LL} \sin(\theta) - \frac{3\sqrt{2}}{\pi} V_{LL}\theta - \sqrt{2} V_{LL} \frac{1}{2} - \frac{3\sqrt{2}}{\pi} V_{LL} \frac{\pi}{6}\right)$$

$$i_d(\theta) = \frac{\sqrt{2} V_{LL}}{\omega L_d}\left(\sin(\theta) - \frac{3}{\pi}\theta + k\right)$$ where k is a constant

from the figure we notice that

$i_d(-\theta_1) = 0$ and that $\cancel{\sqrt{2} V_{LL}} \cos(\theta_1) = \frac{3\cancel{\sqrt{2}}}{\pi} \cancel{V_{LL}}$ $\Rightarrow$

$$\cos(\theta_1) = \frac{3}{\pi} \Rightarrow \theta_1 = 0{,}301 \text{ rad} \Rightarrow$$

$$i_d(-0{,}301) = 0 = \frac{\sqrt{2} V_{LL}}{\omega L_d}\left(\sin(-0{,}301) + \frac{3}{\pi} 0{,}301 + k\right) \Rightarrow$$

$$k = -\sin(-0{,}301) - \frac{3}{\pi} 0{,}301 = 0{,}00904 \Rightarrow$$

$$i_d(\theta) = \frac{\sqrt{2} V_{LL}}{\omega L_d}\left(\sin(\theta) + \frac{3}{\pi}\theta + 0{,}00904\right) \qquad -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$$

$$I_d = \frac{3}{\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} i_d(\theta)\, d\theta = \frac{3}{\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sqrt{2} V_{LL}}{\omega L_d}\left(\sin(\theta) + \frac{3}{\pi}\theta + 0.00904\right) d\theta =$$

$$= \frac{3}{\pi}\frac{\sqrt{2} V_{LL}}{\omega L_d}\left[-\cos(\theta) + \frac{3}{2\pi}\theta^2 + 0{,}00904\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} =$$

$$= \frac{\cancel{3}}{\cancel{\pi}}\frac{\sqrt{2} V_{LL}}{\omega L_d} 0{,}00904 \cdot \frac{2\cancel{\pi}}{\cancel{6}} = 0{,}00904 \frac{\sqrt{2} V_{LL}}{\omega L_d} = \frac{0{,}013 V_{LL}}{\omega L_d} \Rightarrow$$

By looking in the i_d plot we see that if we have a load current $I_o = I_d$ then if we decrease the inductance we will have a larger ripple $\Rightarrow$ discontinuous current and therefore the equation gives the minimum inductance value that guarantees CCM operation $\Rightarrow$

$$L_{min} = \frac{0{,}013 V_{LL}}{\omega I_d}$$