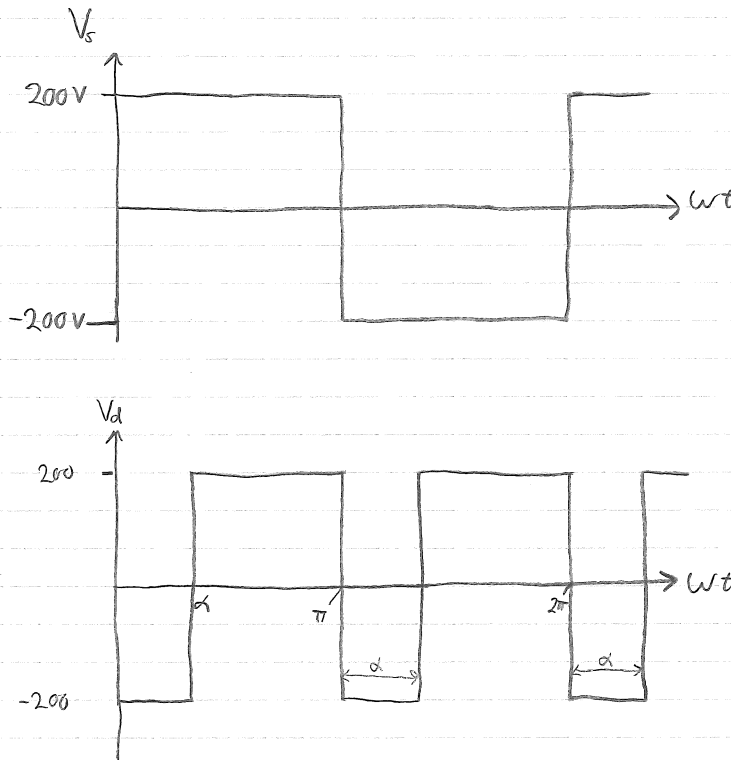
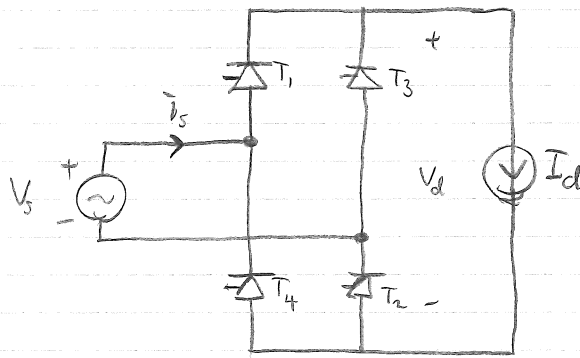


6-3



We start at a time smaller than zero. Then thyristor T3 and T4 conducts. When the time passes zero thyristors T3 and T4 still conduct the current due to that we have not turn thyristor T1 and T2 on yet. At $\omega t = \alpha$ we turn T1 and T2 on and the current is commutated directly over to these thyristors (due to no inductance) and thyristors T3 and T4 are turned off automatically.

$$V_d = \frac{1}{\pi} \int_0^{\pi} V_d(\omega t) d(\omega t) = \frac{1}{\pi} \int_0^{\alpha} -\hat{V}_s d(\omega t) + \frac{1}{\pi} \int_{\alpha}^{\pi} \hat{V}_s d(\omega t) = \frac{1}{\pi} (-\hat{V}_s \alpha + \hat{V}_s \pi - \hat{V}_s \alpha)$$

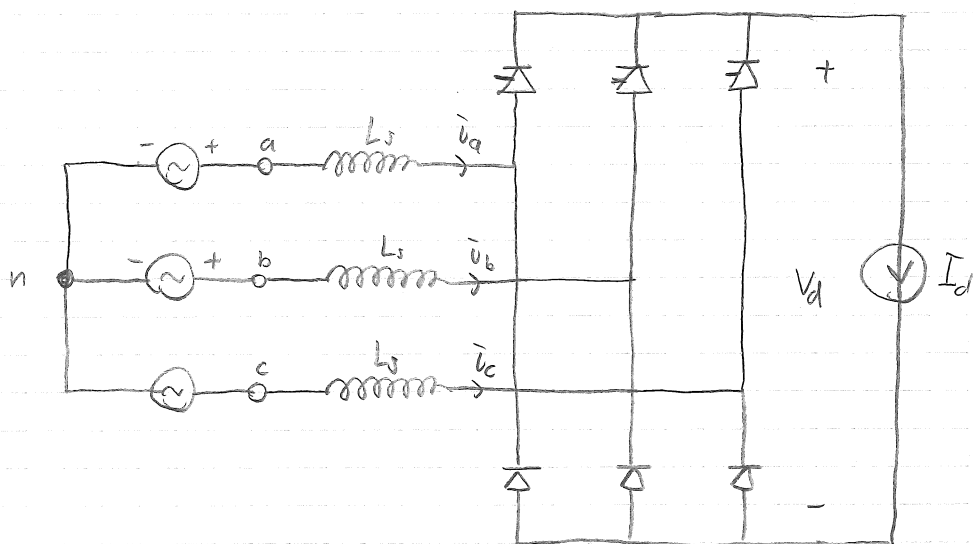
$$\underline{V_d = \hat{V}_s \left(1 - \frac{2\alpha}{\pi}\right)}$$

$$V_d(45^\circ) = \hat{V}_s \left(1 - \frac{2\alpha}{180^\circ}\right) = 200 \left(1 - \frac{90}{180}\right) = \underline{100V}$$

$$V_d(135^\circ) = \hat{V}_s \left(1 - \frac{2 \cdot 135^\circ}{180^\circ}\right) = 200 \left(1 - \frac{270}{180}\right) = \underline{-100V}$$

6-11

1-(2)



Derive the expression for the displacement power factor given in equation 6-64. $\Rightarrow$

$DPF \approx \frac{1}{2} [\cos(\alpha) + \cos(\alpha + \mu)]$ From the text above the equation 6-64 we get the following hint:

- by calculate ac-side and dc-side powers

$$P_d = V_d I_d = \left(\frac{3\sqrt{2}}{\pi} V_{LL} \cos(\alpha) - \frac{3\omega L_s}{\pi} I_d \right) I_d \quad (1)$$

$$P_{Ac} = 3 \cdot P_\phi = 3 V_s I_{s,1} \cos(\phi_1) = \sqrt{3} V_{LL} I_{s,1} \cos(\phi_1) \quad (2)$$

We also know that:

$$\cos(\alpha + \mu) = \cos(\alpha) - \frac{2\omega L_s}{\sqrt{2} V_{LL}} I_d \Rightarrow$$

$$\omega L_s I_d = \frac{\sqrt{2} V_{LL}}{2} (\cos(\alpha) - \cos(\alpha + \mu)) \quad \text{This inserted in (1)} \Rightarrow$$

$$\begin{aligned} P_d &= I_d \left(\frac{3\sqrt{2}}{\pi} V_{LL} \cos(\alpha) - \frac{3\sqrt{2}}{2\pi} V_{LL} (\cos(\alpha) - \cos(\alpha + \mu)) \right) = \\ &= I_d \frac{3\sqrt{2}}{2\pi} V_{LL} (\cos(\alpha) + \cos(\alpha + \mu)) \end{aligned}$$

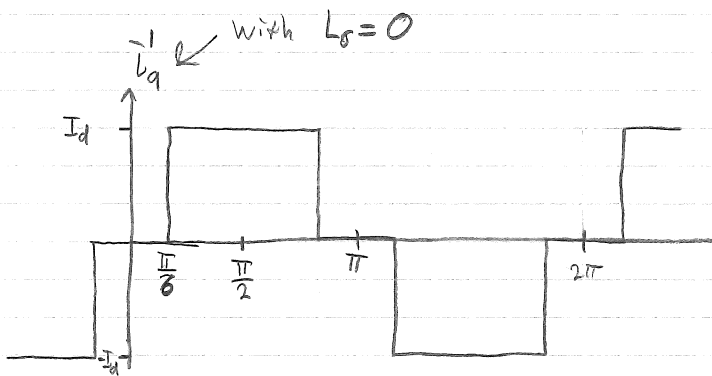
$$P_d = P_{Ac} \Rightarrow$$

$$I_d \frac{3\sqrt{2}}{2\pi} V_{LL} (\cos(\alpha) + \cos(\alpha + u)) = \sqrt{3} V_{LL} I_{s,1} \cos(\phi_1) \Rightarrow$$

$$\text{DPF} = \cos(\phi_1) = \frac{I_d}{I_{s,1}} \sqrt{\frac{3}{2}} \frac{1}{\pi} (\cos(\alpha) + \cos(\alpha + u))$$

We now see that we have to express $I_{s,1}$ in terms of I_d

Here we do a approximation, we approximate $I_{s,1}$ with the fundamental rms current from a converter with no inductance $\Rightarrow$ the phase-current is square-wave shaped $\Rightarrow$



This is a odd half wave function $\Rightarrow$

$$a_h = 0 \text{ for all } h$$

$$b_h = \begin{cases} \frac{4}{\pi} \int_0^{\pi/2} i_a(\omega t) \sin(h\omega t) d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$$

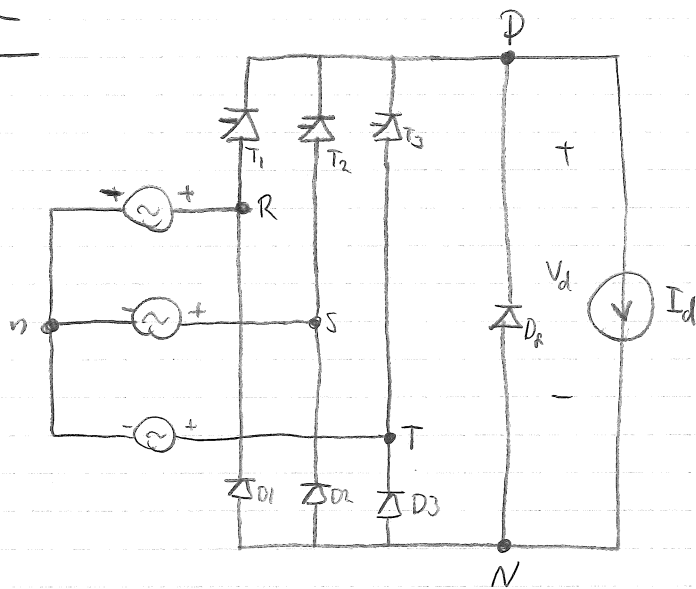
$$b_1 = \frac{4}{\pi} \int_{\pi/6}^{\pi/2} I_d \sin(\omega t) d(\omega t) = \frac{4}{\pi} I_d (\cos(\pi/6) - \cos(\pi/2)) = \frac{4}{\pi} I_d \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{\pi} I_d$$

We now make the approximation that $I_{s,1} \approx \frac{b_1}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} I_d \Rightarrow$

$$\underline{\text{DPF}} = \cos(\phi_1) \approx \frac{I_d}{\frac{\sqrt{6}}{\pi} I_d} \sqrt{\frac{3}{2}} \frac{1}{\pi} (\cos(\alpha) + \cos(\alpha + u)) = \underline{\underline{\frac{1}{2} (\cos(\alpha) + \cos(\alpha + u))}}$$

6-15

1 - (6)



We start by drawing the voltage waveforms for this converter.

See the dot papers

As we notice from the dot-papers the diode D_p will not conduct if $\alpha \leq 60^\circ$, if $\alpha > 60^\circ$ then the diode D_p will conduct during the interval $60^\circ \leq \omega t \leq \alpha$ and during this interval $V_d(t)$ will be zero $\Rightarrow$

$$V_d = \begin{cases} \frac{3}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6} + \alpha} \sqrt{2} V_{LL} \cos(\omega t) d(\omega t) & \alpha \leq 60^\circ \\ \frac{3}{2\pi} \int_{\alpha}^{\frac{\pi}{6}} \sqrt{2} V_{LL} \sin(\omega t) d(\omega t) & \alpha > 60^\circ \end{cases}$$

$\alpha \leq 60^\circ$

$$\begin{aligned} \underline{V_d} &= \frac{3}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6} + \alpha} \sqrt{2} V_{LL} \cos(\omega t) d(\omega t) = \frac{3\sqrt{2} V_{LL}}{2\pi} \left(\sin\left(\frac{\pi}{6} + \alpha\right) + \sin\left(\frac{\pi}{6}\right) \right) = \\ &= \frac{3\sqrt{2} V_{LL}}{2\pi} \left(\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right) \cos(\alpha) + \cos\left(\frac{\pi}{6}\right) \sin(\alpha) \right) = \\ &= \underline{\underline{\frac{3\sqrt{2} V_{LL}}{2\pi} \left(\frac{1}{2} + \frac{1}{2} \cos(\alpha) + \frac{\sqrt{3}}{2} \sin(\alpha) \right)}} \end{aligned}$$

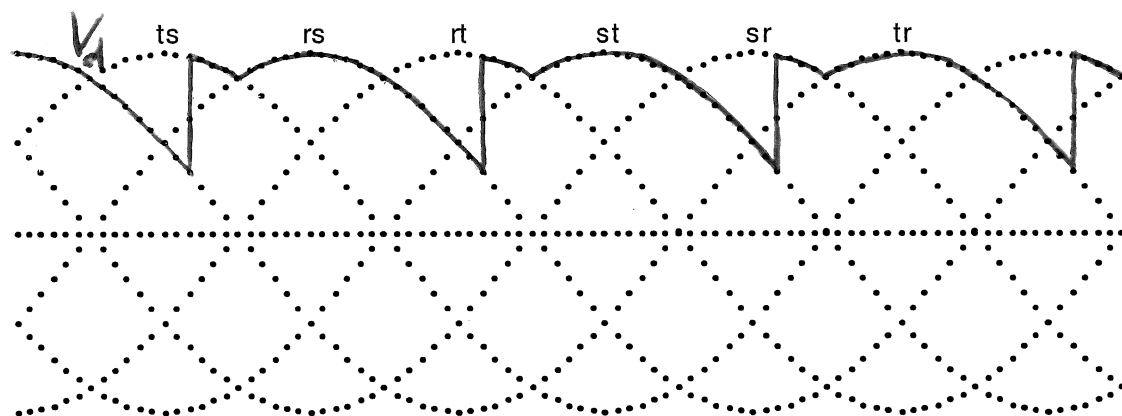
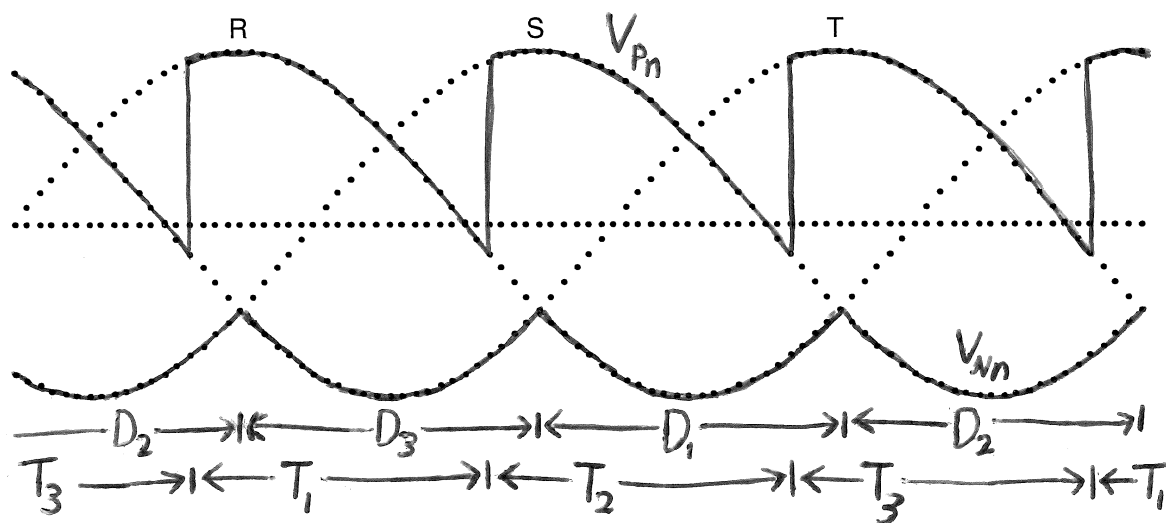
$\alpha > 60^\circ$

$$\underline{V_d} = \frac{3}{2\pi} \int_{\alpha}^{\frac{\pi}{6}} \sqrt{2} V_{LL} \sin(\omega t) d(\omega t) = \underline{\underline{\frac{3\sqrt{2} V_{LL}}{2\pi} (1 + \cos(\alpha))}}$$

6-15

2-(6)

$\alpha = 40^\circ$



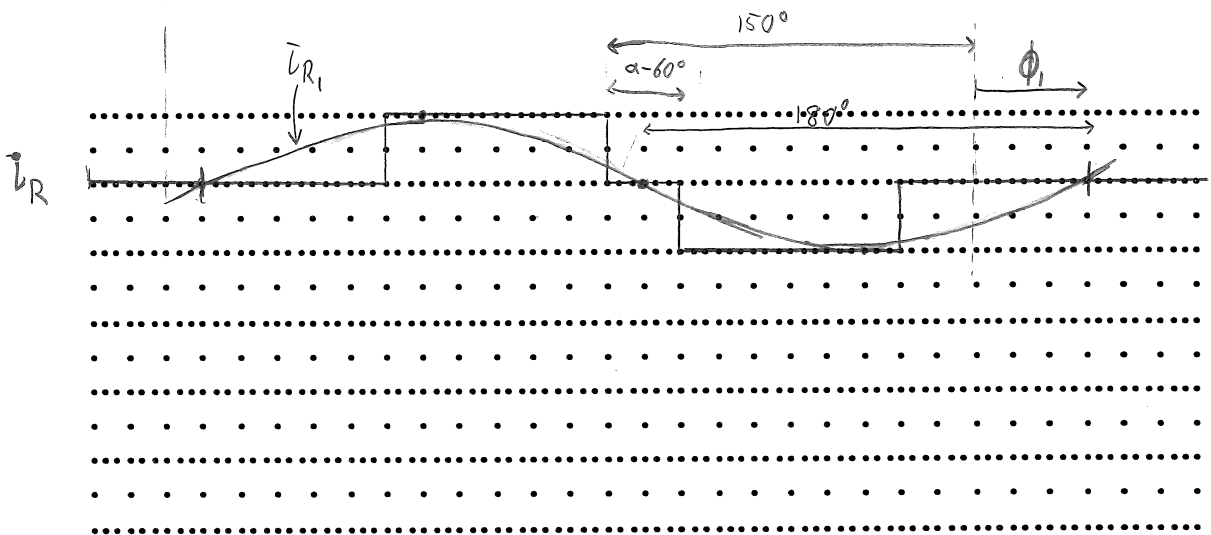
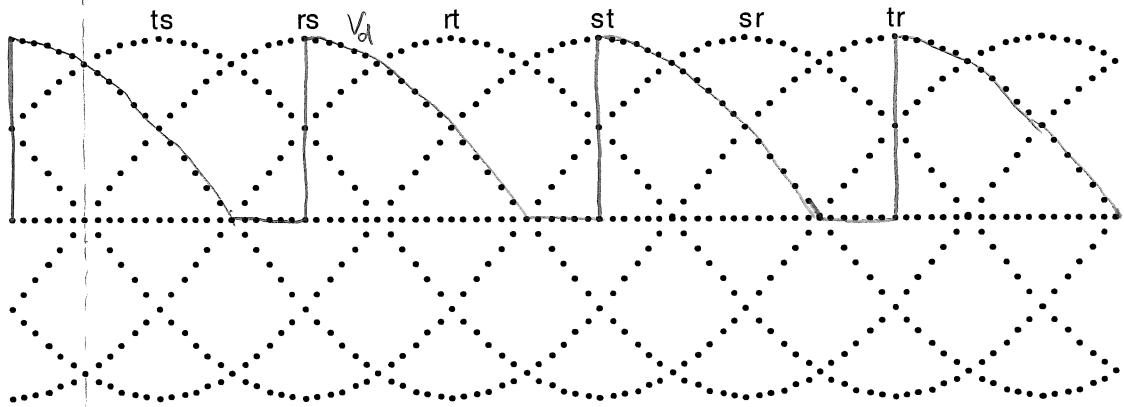
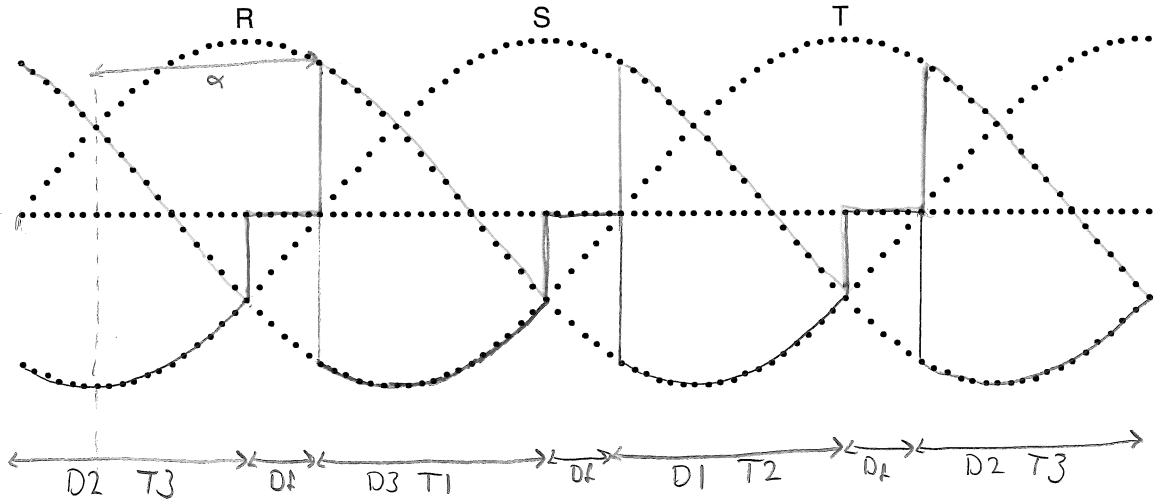
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6-15

$$\alpha > 60^\circ$$

$\alpha = 90^\circ$

3-(6)



6-15 We know that V_{d0} is when we run the converter as a diode rectifier $\Rightarrow$

$$V_{d0} = \frac{3}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{2} V_{LL} \cos(\omega t) d(\omega t) = \frac{3\sqrt{2} V_{LL}}{\pi}$$

We should calculate α for when $V_d = 0,5 V_{d0} = \frac{3\sqrt{2}}{2\pi} V_{LL}$

We assume that $\alpha > 60^\circ \Rightarrow$

$$\frac{3\sqrt{2}}{2\pi} V_{LL} = \frac{3\sqrt{2} V_{LL}}{2\pi} (1 + \cos(\alpha)) \Rightarrow \cos(\alpha) = 0 \Rightarrow \underline{\alpha = 90^\circ}$$

From the dot paper:

$$150^\circ + \phi_1 = 180 + \frac{\alpha - 60}{2} \Rightarrow$$

$$\phi_1 = \frac{\alpha}{2} = \frac{90^\circ}{2} = \underline{45^\circ}$$

$$\Rightarrow \text{DPF} = \cos(\phi_1) = 0,707$$

$$\underline{\text{PF}} = \frac{P}{S} = \frac{V_d I_d}{V_s I_s} = \frac{\frac{3\sqrt{2} V_{LL}}{2\pi} I_d (1 + \cos(\alpha))}{3 \cdot V_s \frac{1}{\sqrt{2}} I_d} = \frac{3\sqrt{3}}{3\pi} (1 + \cos(\alpha)) = \underline{0,551}$$

$$I_s = \sqrt{\frac{1}{180} \int_0^{180} i_R^2(\omega t) d(\omega t)} = \sqrt{\frac{1}{180} \int_{75}^{165} I_d^2 d(\omega t)} = \sqrt{\frac{90}{180}} I_d = \frac{I_d}{\sqrt{2}}$$

$$I_{s,1} = \frac{2}{\sqrt{2}\pi} \int_0^\pi f(\omega t) \sin(\omega t) d(\omega t) = \frac{2}{\sqrt{2}\pi} \int_{\frac{5}{12}\pi}^{\frac{11}{12}\pi} I_d \sin(\omega t) d(\omega t) = \frac{2I_d}{\sqrt{2}\pi} \left(\cos\left(\frac{5}{12}\pi\right) - \cos\left(\frac{11}{12}\pi\right) \right) = \underline{0,551 I_d}$$

$$\underline{\text{PF}} = \frac{P}{S} = \frac{V_s I_{s,1} \cos(\phi_1)}{V_s I_s} = \frac{I_{s,1}}{I_s} \cos(\phi_1) = \frac{0,551 I_d}{0,707 I_d} \cos(45^\circ) = \underline{0,551}$$

$$\underline{\% \text{THD}} = \frac{I_{d,rs}}{I_{s,1}} = \frac{\sqrt{I_s^2 - I_{s,1}^2}}{I_{s,1}} = \frac{\sqrt{\left(\frac{1}{\sqrt{2}} I_d\right)^2 - 0,551^2 I_d^2}}{0,551 I_d} = \underline{0,804}$$

6-15

For a full-bridge converter operating at $V_d = 0.5 V_{d0}$

We know that

$$V_d = \frac{3\sqrt{2}}{\pi} V_{LL} \cos(\alpha) = \frac{3\sqrt{2}}{2\pi} V_{LL} \Rightarrow \cos(\alpha) = 0.5 \Rightarrow \underline{\alpha = 60^\circ}$$

We plot this in the dot-paper

From this we see that

$$\phi_i + 30^\circ = 30^\circ + \alpha \Rightarrow$$

$$\phi_i = \alpha = 60^\circ$$

$$\underline{\text{DPF}} = \cos(\phi_i) = \underline{0.5}$$

$$I_s = \sqrt{\frac{2}{3}} I_d$$

$$I_{s,1} = \frac{\sqrt{6}}{\pi} I_d$$

$$\underline{\text{PF}} = \frac{V_s I_{s,1} \cos(\phi_i)}{V_s I_s} = \frac{\frac{\sqrt{6}}{\pi}}{\sqrt{\frac{2}{3}}} \cos(\alpha) = \frac{\frac{\sqrt{6} \sqrt{3}}{\sqrt{2} \pi} \cos(\alpha)}{\frac{3}{\pi}} = \underline{0.478}$$

$$\underline{\% \text{THD}} = \frac{I_{d,1}}{I_{s,1}} = \frac{\sqrt{I_s^2 - I_{s,1}^2}}{I_{s,1}} = \frac{\sqrt{\frac{2}{3} - \frac{6}{\pi^2}}}{\frac{\sqrt{6}}{\pi}} = \underline{0.311}$$

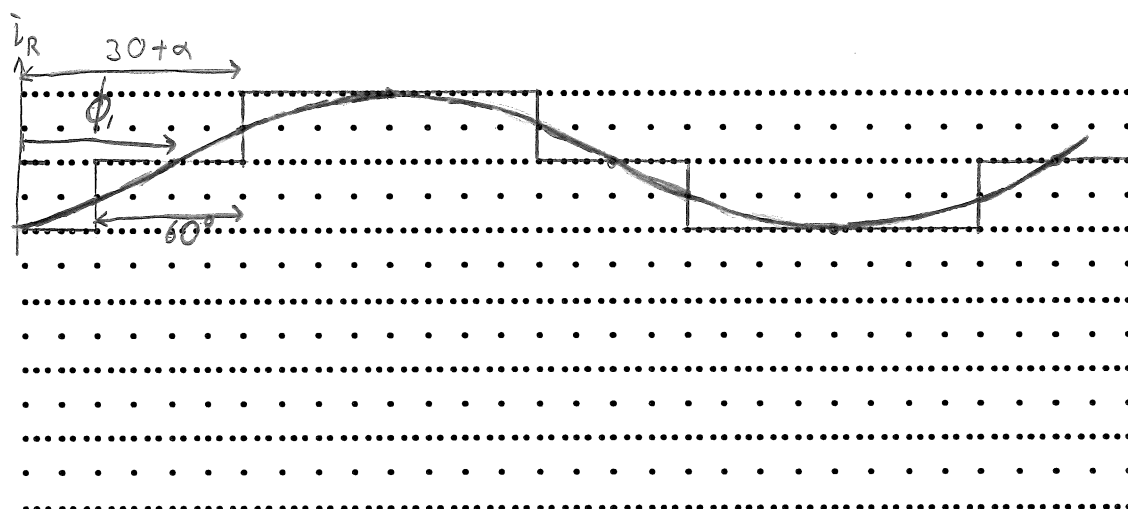
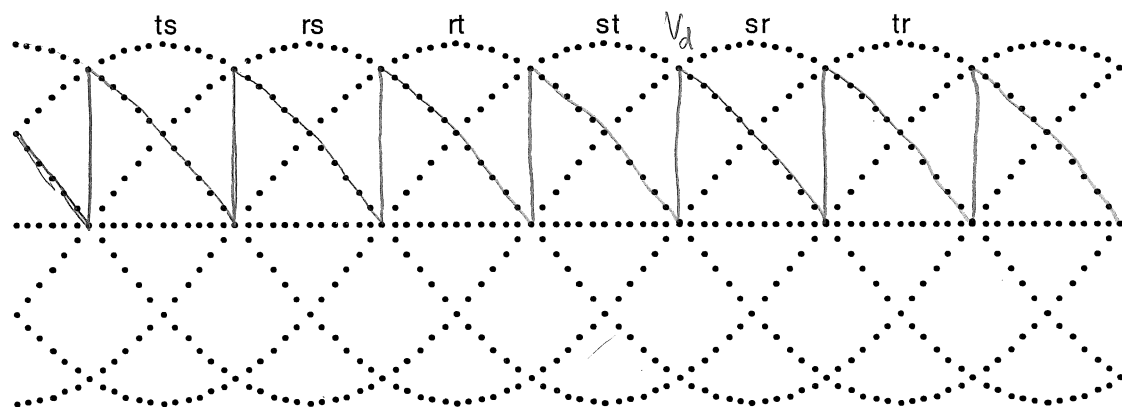
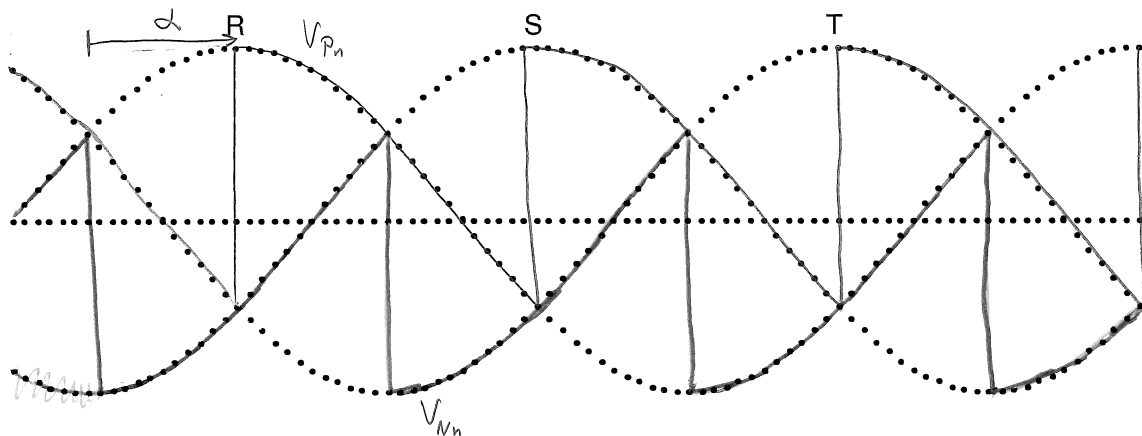
We see that for the half-bridge the power factor is better but the %THD is worse for the half-bridge compare with the full-bridge.

6-15

Full bridge

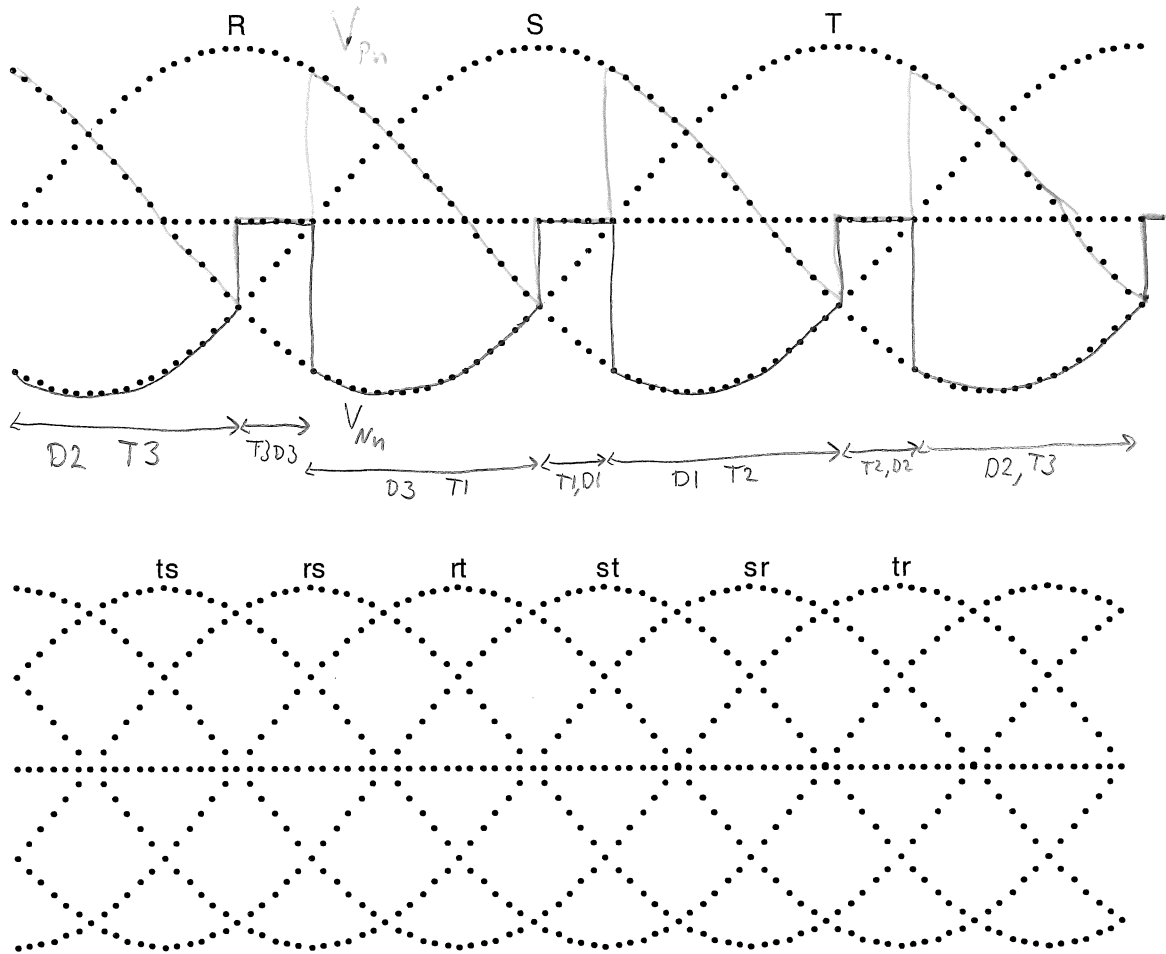
$\alpha = 60^\circ$

6-16)



6-16

For $\alpha \leq 60^\circ$ this converter operates exactly in the same way as the converter in problem 6-15. For $\alpha \geq 60^\circ$ the voltages is as follow. As we can see the voltages and currents has the same wave-forms as in problem 6-15 and therefore



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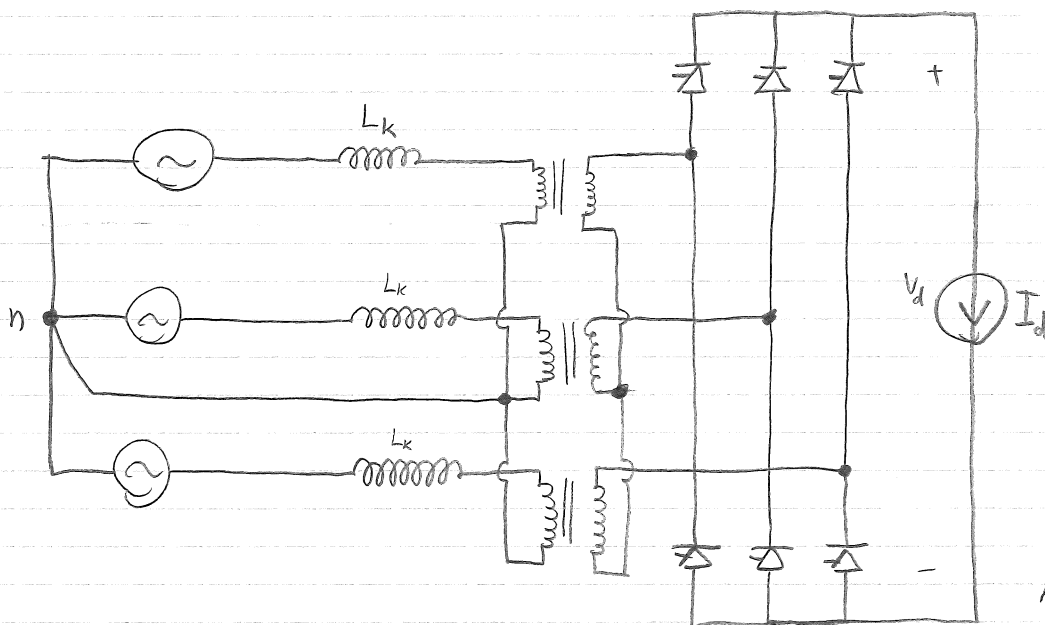
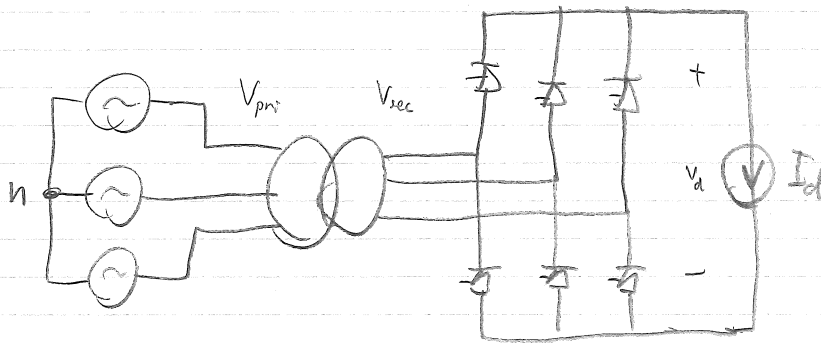
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the results are the same as in problem 6-15. The only difference is the components that conducts.

6-17

$P_{load} = 12 \text{ kW}$, Y-Y 5 kVA $V_{pri} = 120 \text{ V}$ at 60 Hz $L_k = 8\%$
 $V_{dL} = 208 \pm 5\%$ $I_d = I_d$ $V_d = 300 \text{ V}$



$$N = \frac{V_{pri}}{V_{sec}} \quad N = \frac{I_{sec}}{I_{pri}}$$

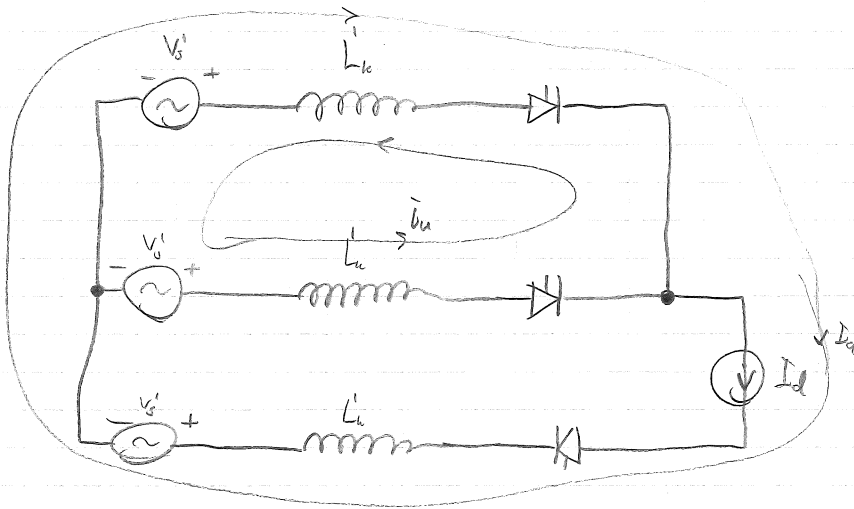
$$Z_{base} = \frac{V_{pri}}{I_{pri}} = \frac{V_{pri}^2}{S} = \frac{120^2}{5 \cdot 10^3} = 2.88 \Omega \Rightarrow$$

$$L'_k = \frac{L_k}{N^2}$$

$$\underline{L_k} = \frac{0.08 \cdot Z_{base}}{\omega} = \frac{0.08 \cdot 2.88}{2 \cdot \pi \cdot 60} = \underline{0.611 \text{ mH}}$$

$$V_s' = \frac{V_s}{N}$$

6-17 During the conduction interval we can draw the converter as
 $\alpha \leq \omega t \leq \alpha + \mu$



$$V_L = V_{L'} = 2L_k' \frac{di_u}{dt} \Rightarrow$$

$$\omega L_k' \int_0^{I_d} di_u = \omega L_k' I_d = \frac{1}{2} \int_{\alpha}^{\alpha+\mu} \sqrt{2} V_{LL}' \sin(\omega t) d(\omega t) = A_u \Rightarrow$$

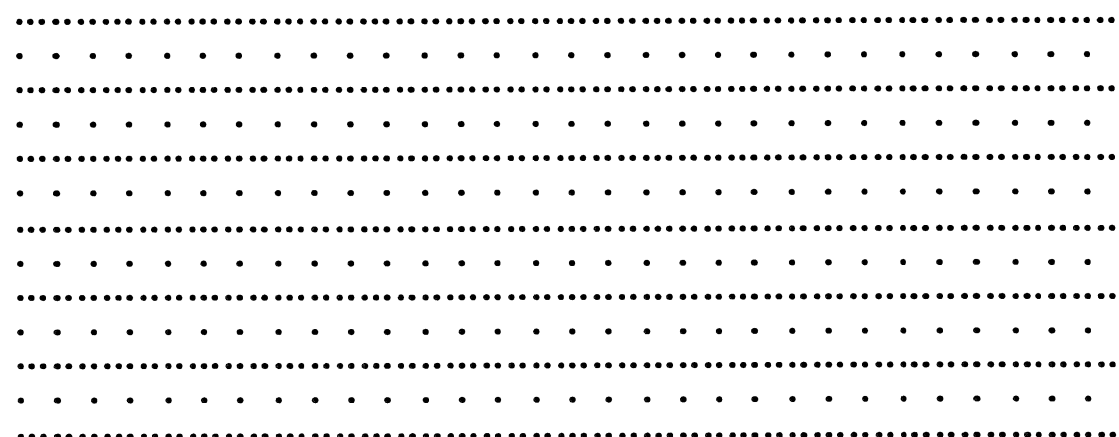
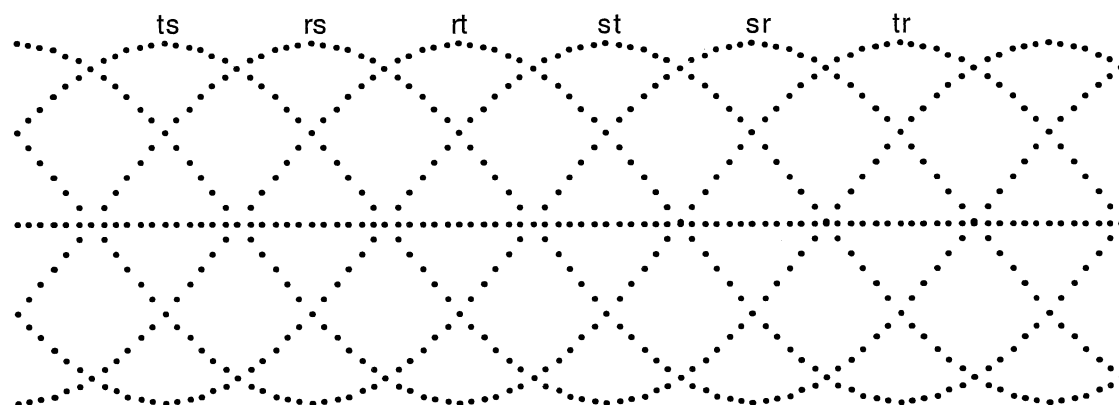
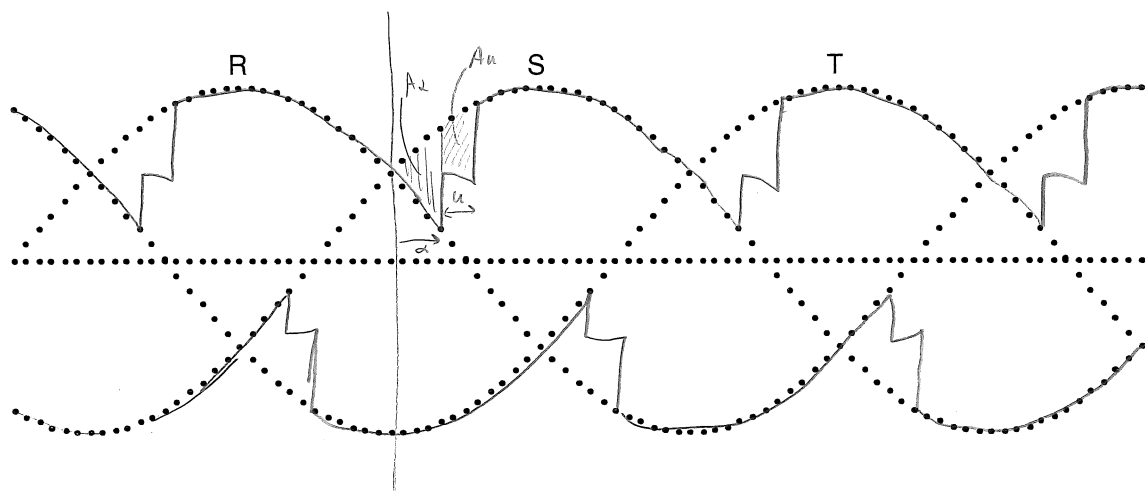
$$A_u = \omega L_k' I_d = \frac{\sqrt{2} V_{LL}'}{2} (\cos(\alpha) + \cos(\alpha + \mu))$$

$$V_d = \frac{3}{\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{2} V_{LL}' \cos(\omega t) d(\omega t) - A_\alpha - A_u \right) = \frac{3}{\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{2} V_{LL}' \cos(\omega t) d(\omega t) - \int_0^\alpha \sqrt{2} V_{LL}' \sin(\omega t) d(\omega t) - \omega L_k' I_d \right) \Rightarrow$$

$$V_d = \frac{3}{\pi} \left(\sqrt{2} V_{LL}' - \sqrt{2} V_{LL}' (1 - \cos(\alpha)) - \omega L_k' I_d \right) = \frac{3\sqrt{2}}{\pi} V_{LL}' \cos(\alpha) - \frac{3\omega L_k' I_d}{\pi}$$

$$V_d = \frac{3\sqrt{2}}{\pi N} V_{LL} \cos(\alpha) - \frac{3\omega L_k I_d}{\pi N^2}$$

We now see that if V_{LL} is at its minimum and I_d is at its maximum $\cos(\alpha)$ must be at its maximum $\Rightarrow$



6-17

4-(4)

$$V_d = \frac{3\sqrt{2}}{\pi N} V_{L, \min} - \frac{3\omega L_k P_d}{\pi N^2 V_d} \Rightarrow$$

$$N^2 - \frac{3\sqrt{2}}{\pi} \frac{V_{L, \min}}{V_d} N = - \frac{3\omega L_k P_d}{\pi V_d^2} \Rightarrow$$

$$N = \frac{3}{\sqrt{2}\pi} \cdot \frac{V_{L, \min}}{V_d} \pm \sqrt{\left(\frac{3}{\sqrt{2}\pi} \frac{V_{L, \min}}{V_d}\right)^2 - \frac{3\omega L_k P_d}{\pi V_d^2}} \Rightarrow$$

$$N = \frac{3}{\sqrt{2}\pi} \frac{208,9}{300} \pm \sqrt{\left(\frac{3}{\sqrt{2}\pi} \frac{208,9}{300}\right)^2 - \frac{3 \cdot 2\pi \cdot 60 \cdot 0,611 \cdot 10^{-3} \cdot 12 \cdot 10^3}{\pi \cdot 300^2}}$$

$$\underline{N} = 0,421 \pm \sqrt{0,178 - 0,0293} = 0,421 \pm 0,386 = \underline{\underline{0,807}}$$

$$V_{LL} = 208 + 10\% \Rightarrow$$

$$\cos(\alpha) = \frac{1}{\sqrt{2} V_{LL}} \left(\frac{\pi N}{3} V_d + \frac{\omega L_k P_d}{N V_d} \right) = \frac{1}{\sqrt{2} \cdot 208 \cdot 1,05} \left(\frac{\pi \cdot 0,807}{3} 300 + \frac{2\pi \cdot 60 \cdot 0,611 \cdot 10^{-3} \cdot 12 \cdot 10^3}{0,807 \cdot 300} \right)$$

$$\cos(\alpha) = 0,858 \Rightarrow \underline{\underline{\alpha = 30,9^\circ}}$$