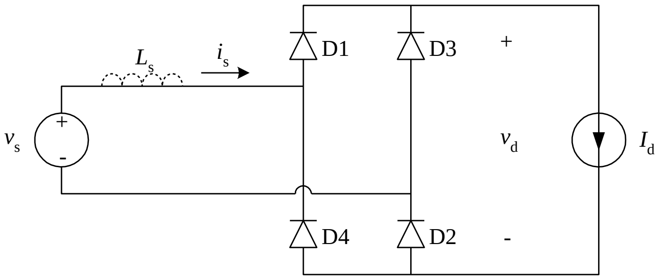
Solution of demonstration 10

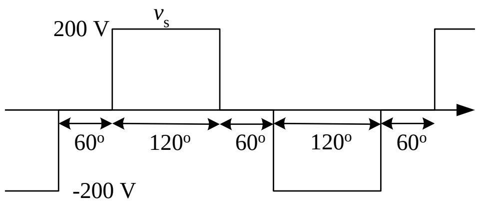
Problem 1 (P5-4 in Undeland book)

A single-phase rectifier is operating without grid inductance (L_s) loaded with a constant dc- current of 10A.

a) Calculate the average power supplied to the load if the supply voltage (v_s) is a sinusoidal voltage with $V_s = 120V$ at 60Hz

b) Calculate the average power delivered to the load if the supply voltage (v_s) has the pulses waveform shown below

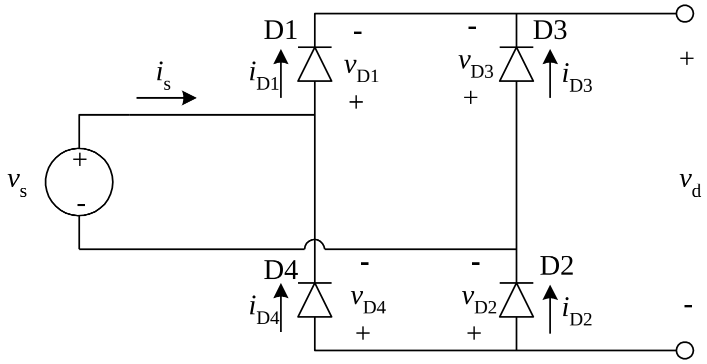


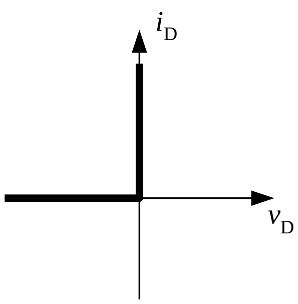


Solution

a) Calculate the average power supplied to the load if the supply voltage (v_s) is a sinusoidal voltage with $V_s = 120V$ at 60Hz

By assuming that ideal rectifiers are used, they have the following *IV*-characteristics:





From the current-voltage diagram for the ideal diode it can be seen that the diode will conduct when the voltage applied over the diode is positive and it will block when the voltage over the diode is negative. To determine the voltages over the diodes, KVL is applied to the circuit:

$$v_s - v_{D1} - v_d - v_{D2} = 0 \quad \rightarrow \quad v_s - v_d = v_{D1} + v_{D2}$$
$$v_s + v_{D4} + v_d + v_{D3} = 0 \quad \rightarrow \quad -(v_s + v_d) = v_{D4} + v_{D3}$$

We assume that we have a equal voltage division over the diodes ($v_{D1} + v_{D2}$ and $v_{D4} + v_{D3}$). This gives that diodes D_1 and D_2 will start to conduct if the supply voltage exceeds the DC-link voltage ($v_s > v_d$), this since the voltage over diodes D_1 and D_2 will become positive. The opposite relation is also valid; diodes D_4 and D_3 will start to conduct if the supply voltage is lower than the negative DC-link voltage ($v_s < -v_d$) since the voltage over diodes D_4 and D_3 will become positive.

If the feeding voltage exceeds the dc voltage, ($|v_s| > v_d$), then two diodes will conduct and the DC-voltage will become equal to the supply voltage ($|v_s| = v_d$).

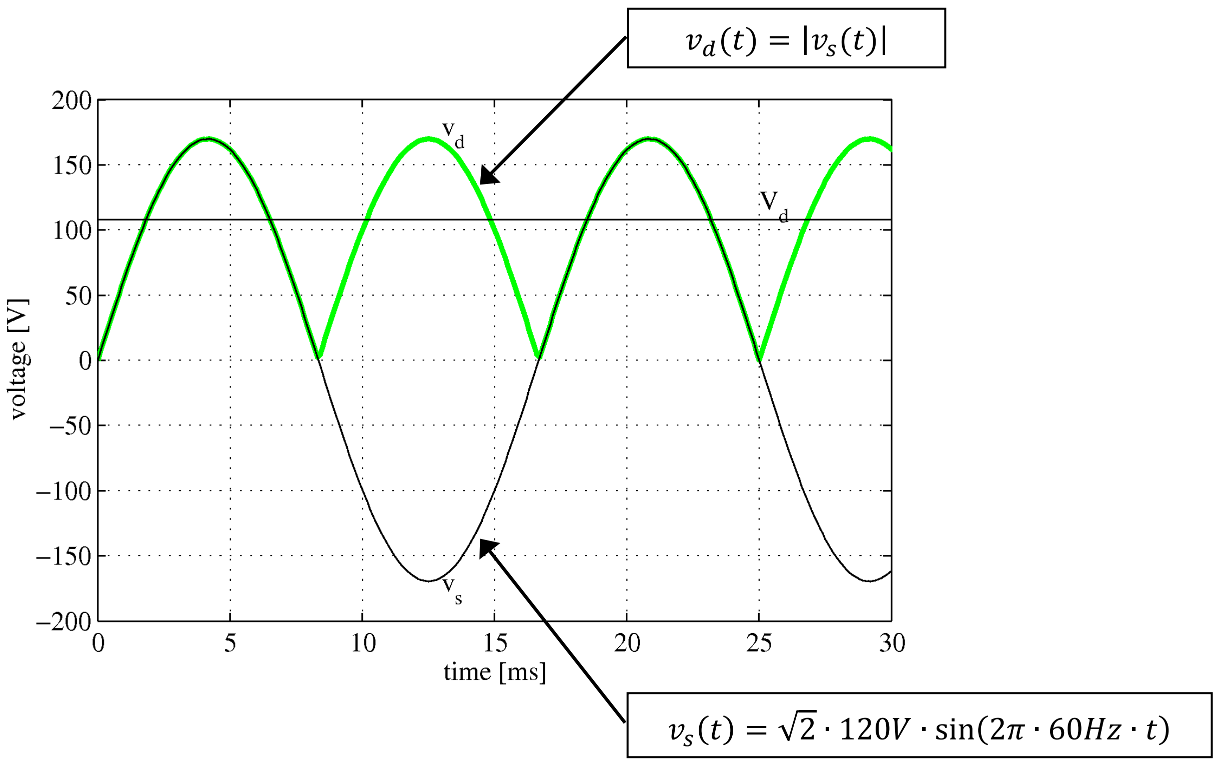
But what does it mean that we have a current source on the dc side? The constant load current can be represented by large inductance, since the current through a large inductor cannot change instantaneously. The DC-side is then said to be current stiff.

For the first task, the supply voltage (v_s) is a sinusoidal voltage with $V_s = 120V$ at 60Hz. The average power supplied to the load can be calculated by:

$$P_o = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v_d(t) \cdot I_d dt = I_d \frac{1}{T}\int_0^T v_d(t)dt = I_d V_d$$

The average power is equal to the DC-current (I_d)multiplied with the average output voltage (V_d). So, we have to calculate the average output voltage.

We know that if the input voltage is positive ($v_s > 0$) then diodes D_1 and D_2 will conduct and the DC-voltage will be equal to the input voltage ($v_d = v_s$), and if the input voltage is negative ($v_s < 0$), then diodes D_4 and D_3 will conduct and the DC-voltage becomes equal to minus the input voltage ($v_d = -v_s$). As always, we start by drawing the waveforms:



The average of v_d can now be calculated as:

$$V_d = \frac{2}{T}\int_0^{T/2} \sqrt{2}V_s \sin(2\pi f t)dt = \frac{2}{T}\frac{\sqrt{2}V_s}{2\pi f}[-\cos(2\pi f t)]_0^{T/2} = \frac{2}{T}\frac{\sqrt{2}V_s}{2\pi f}\left(\cos 0° - \cos\left(2\pi f \frac{T}{2}\right)\right)$$

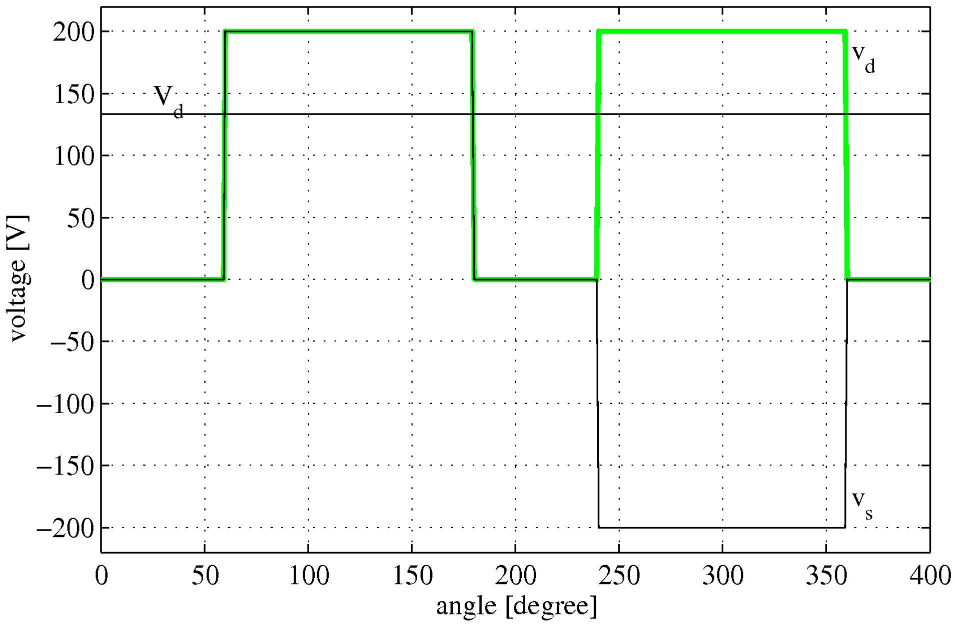
$$V_d = \frac{2}{T}\frac{\sqrt{2}V_s}{2\pi f}2 = \frac{2\sqrt{2}V_s}{\pi} = 0.9V_s = 0.9 \cdot 120V = 108V$$

Which results in an easy multiplication to reach the final answer.

$$P_d = I_d V_d = 10A \cdot 108V = 1080W$$

b) Calculate the average power delivered to the load if the supply voltage (v_s) has the pulsed waveform shown below

The voltages v_d and v_s are drawn:

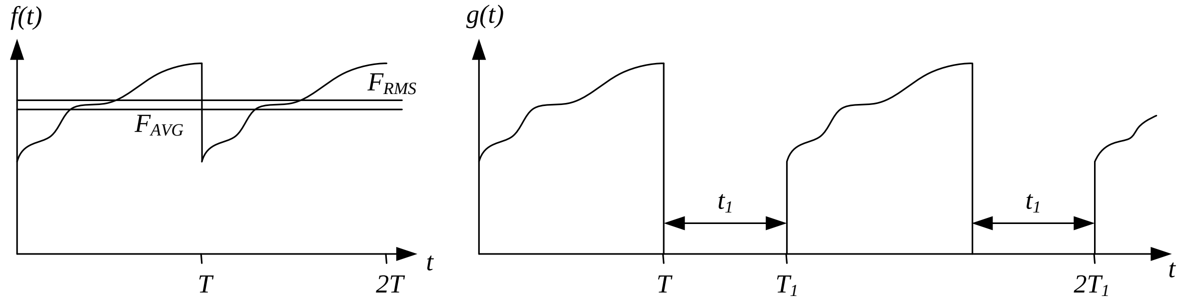


We know from the last problem that we need to calculate the average of the voltage v_d in order to calculate the average output power. To do this, a technique is presented below which facilitates the calculations.

We assume that we have an arbitrary waveform ($f(t)$), which is periodic with a period time(T) and an average value (F_{AVG}) and a RMS-value (F_{RMS}). If we take one period of the arbitrary waveform ($f(t)$) and add a piece which is zero for a specified amount of time (t_1), we get a new waveform ($g(t)$). The new waveform is also periodic with a period time of $T_1 = T + t_1$ and can be written as:

$$g(t) = \begin{cases} 0 \leq t \leq T & \rightarrow \quad f(t) \\ T \leq t \leq T_1 & \rightarrow \quad 0 \end{cases}$$

If illustrated in a figure:



The average and the RMS value of $g(t)$ can now be calculated as:

$$G_{AVG} = \frac{1}{T_1}\int_0^{T_1} g(t)dt = \frac{T}{T_1}\frac{1}{T}\int_0^{T} f(t)dt = \frac{T}{T_1}F_{AVG}$$

$$G_{RMS} = \sqrt{\frac{1}{T_1}\int_0^{T_1} \left(g(t)\right)^2 dt} = \sqrt{\frac{T}{T_1}\frac{1}{T}\int_0^{T} \left(f(t)\right)^2 dt} = \sqrt{\frac{T}{T_1}}F_{RMS}$$

How can we now use this to calculate the average of v_d? From the figure it can be seen that the average between 60° and 180° is equal to the peak value 200V. Hence can the average output voltage be calculated as:

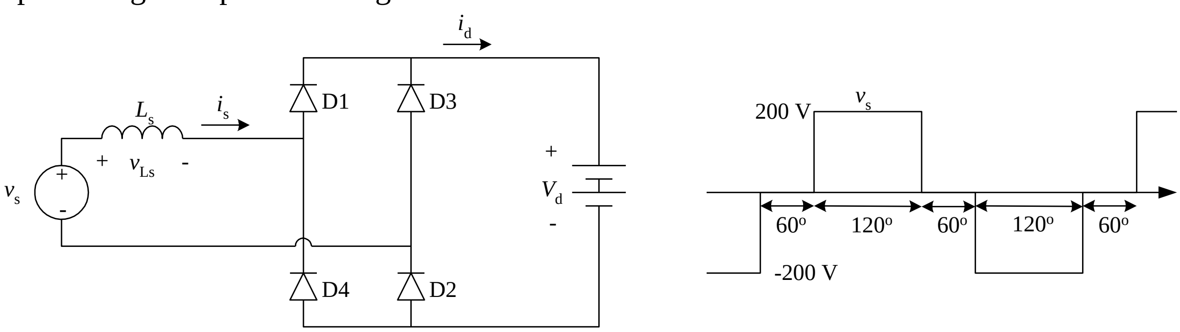
$$V_d = \frac{120°}{180°}200V = 133.3V$$

And the average output power becomes:

$$P_d = I_d V_d = 10A \cdot 133.3V = 1.33kW$$

Problem 2 (P5-10 in Undeland book)

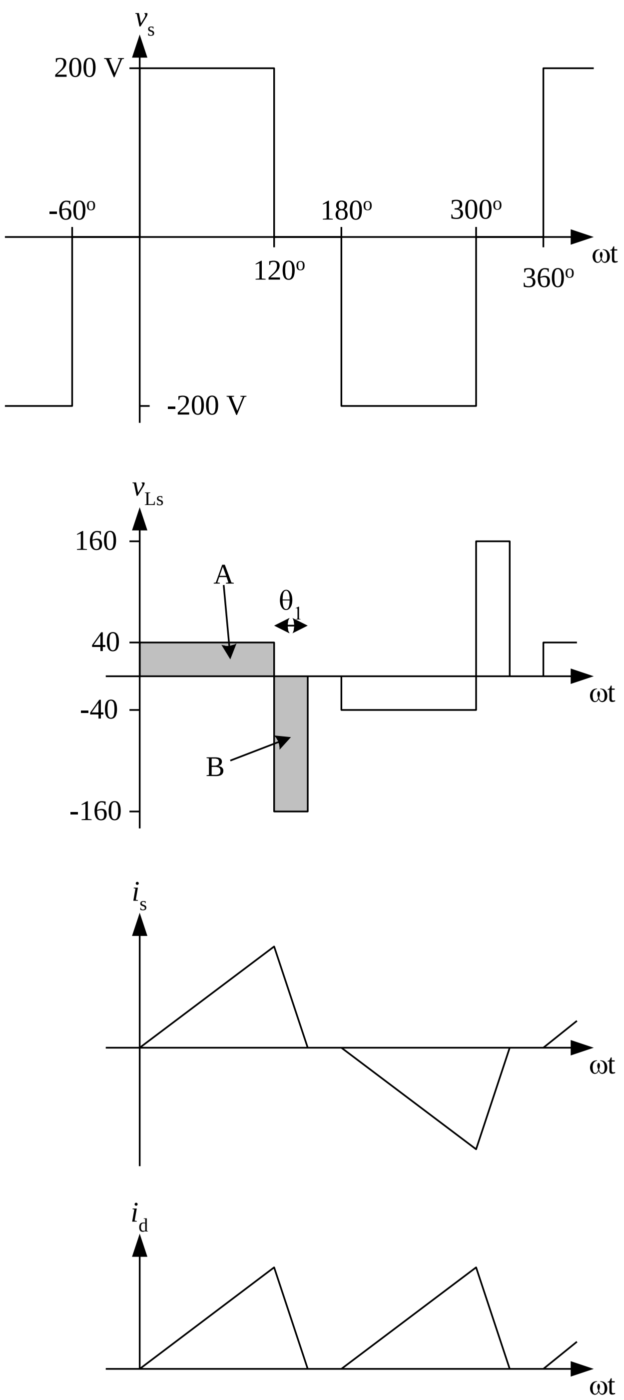
In a single phase rectifier the line inductance is $L_s = 10mH$ and the DC-voltage is $V_d = 160V$. The input voltage is a pulsed voltage shown below



Plot i_d and i_s waveforms. Hint: these currents flow discontinuously.

Solution

Plot i_d and i_s waveforms. Hint: these currents flow discontinuously.



From the hint we know that $i_s(0) = i_d(0) = 0$. We see that just before 0° all diodes will block since the input voltage is less than the dc voltage.

At 0° we see that the input voltage is greater than the dc-voltage and therefore diodes D_1 and D_2 will start to conduct. This gives that the voltage over the inductor is:

$$v_{L_s} = v_s - v_d = 200V - 160V = 40V$$

This also gives that the input current will increase linearly. From the figure we see that when diodes D_1 and D_2 conducts the DC-current is equal to the input current ($i_s = i_d$).

At 120° we see that the input voltage drops to zero. Will the diodes D_1 and D_2 block now? We know that at 120° we have a current through the inductance and this current cannot be changed instantaneously, so therefore will diodes D_1 and D_2 continue to conduct as long as the inductor current is greater than zero. The voltage over the inductor then becomes

$$v_{L_s} = v_s - v_d = 0V - 160V = -160V$$

How long time does it take to reduce the current to zero? We know that the areas A and B must be equal, steady state, this gives:

$$40V \cdot 120° = 160V \cdot \theta_1$$

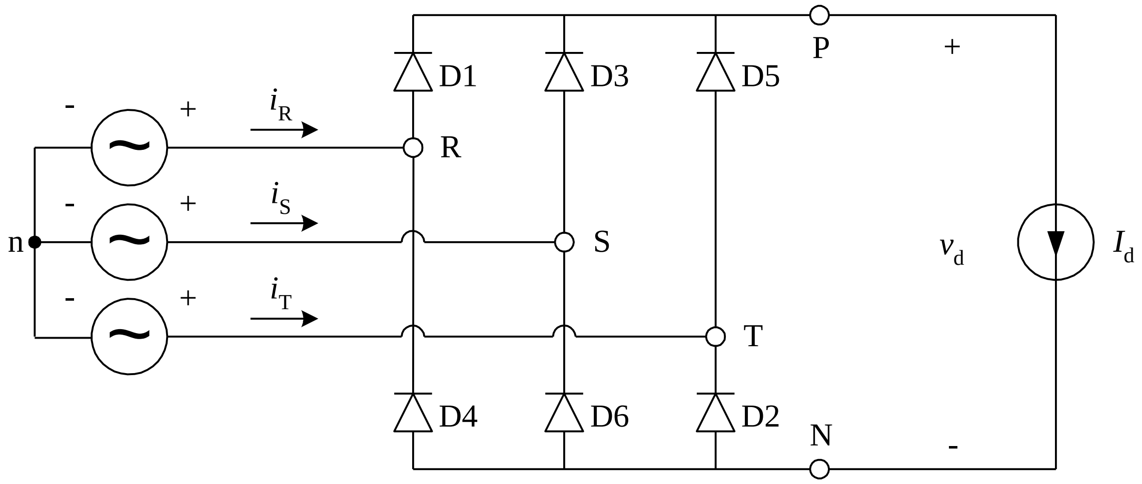
$$\theta_1 = 120° \frac{40V}{160V} = 30°$$

At 180° everything is repeated again with the only difference that the input voltage is negative and diodes D_4 and D_3 conducts instead. This gives that:

$$v_{Ls} = v_s + v_d$$
$$i_d = -i_s$$

Problem 3 (P5-23 in Undeland book)

In the three-phase rectifier depicted below, calculate the average and the RMS-values of the current through each diode as a ratio of the dc-side current (I_d).



Solution

We know that the phase with the highest voltage will make the upper diode to conduct and the phase with the lowest voltage will make the lower diode to conduct. The phase voltages can be written as:

$$v_{Pn} = max(v_{Rn}, v_{Sn}, v_{Tn})$$

$$v_{Nn} = min(v_{Rn}, v_{Sn}, v_{Tn})$$

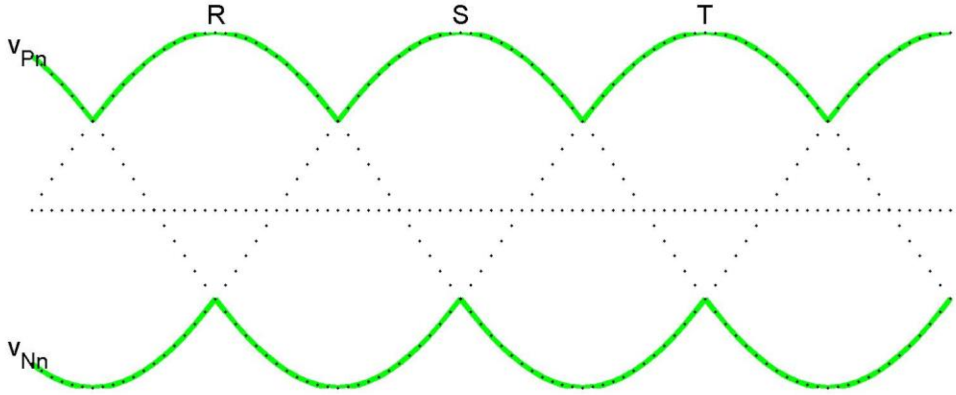
The DC-link voltage is equal to the difference between the positive and negative phase voltage and can be expressed as:

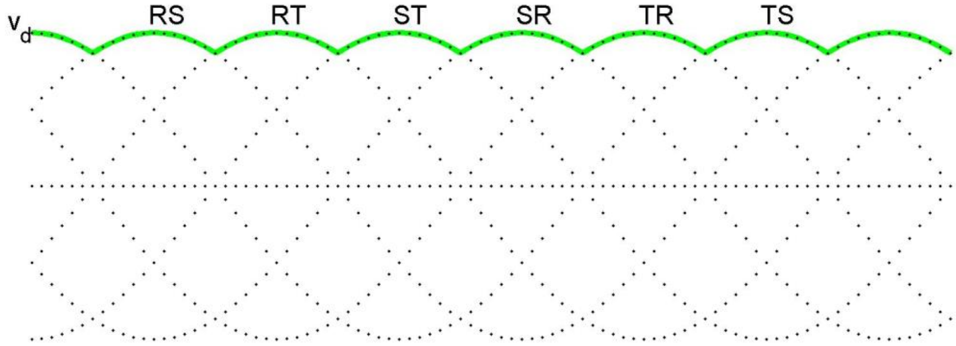
$$v_d = v_{Pn} - v_{Nn}$$

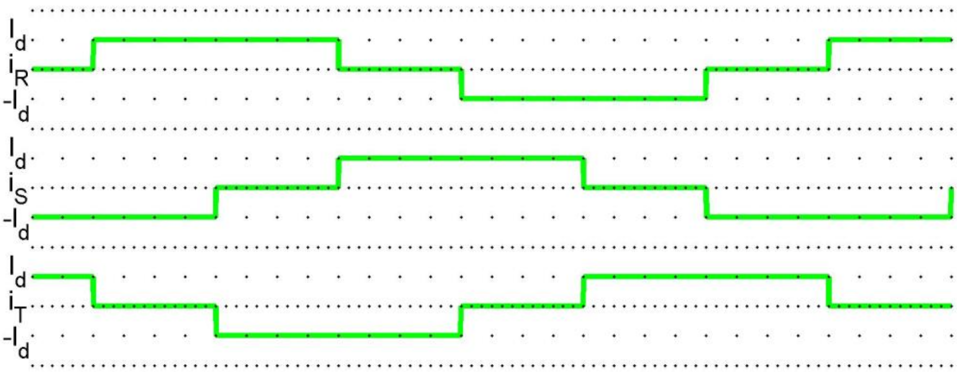
The voltages can be drawn on a dot-paper (see next page). From this we see that the upper diode for the phase with the highest voltage will conduct the dc-current. So the diode current will be equal to the DC-current for one third of the period and zero for the rest, from this and with the result from problem 5-4 (b) we directly can derive the average and rms-value of the diode current

$$I_{D(AVG)} = \frac{120°}{360°} I_d = 0.33 I_d$$

$$I_{D(RMS)} = \sqrt{\frac{120°}{360°}} I_d = 0.58 I_d$$


R
S
T
v_{Pn}
v_{Nn}


RS
RT
ST
SR
TR
TS
v_d


I_d
i_R
$-I_d$
I_d
i_S
$-I_d$
I_d
i_T
$-I_d$

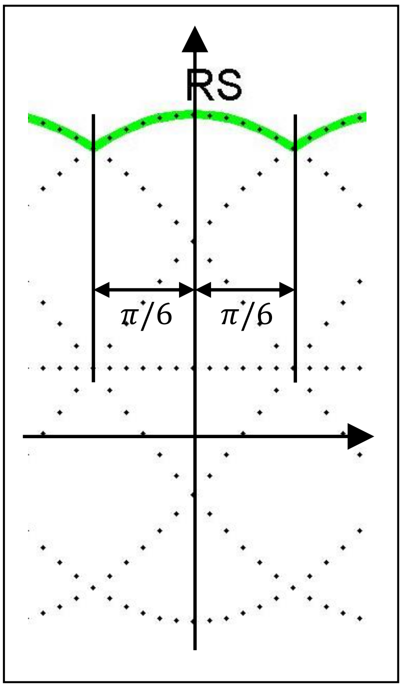
Problem 4

Calculate the average output voltage for the previous task (problem 3) if the input phase voltages are symmetrical and 220V.

Solution

To obtain the average output voltage, it is enough to analyze 60º $(\pi/3)$ since the curve form repeats itself with that period. The function is defined symmetrically according to:

$$v_d = \sqrt{2} \cdot V_{LL} \cdot \cos(\omega t) \quad \text{for} \quad \left\{ {}^{-\pi}/_{6} \leq \omega t \leq {}^{\pi}/_{6} \right\}$$



The average can be calculated as:

$$V_d = \frac{1}{{}^{\pi}/_{6}} \int_0^{{}^{\pi}/_{6}} \sqrt{2} \cdot V_{LL} \cdot \cos(\omega t) d\omega t =$$

$$= \frac{6 \cdot \sqrt{2} \cdot V_{LL}}{\pi} \left[\sin(\omega t) \right]_0^{{}^{\pi}/_{6}} = \frac{6 \cdot \sqrt{2} \cdot V_{LL}}{\pi} \left(\frac{1}{2} - 0 \right) \rightarrow$$

$$V_{d(AVG)} = \frac{3 \cdot \sqrt{2} \cdot V_{LL}}{\pi} = 1.35 \cdot V_{LL}$$

If numerical values are inserted:

$$V_{d(AVG)} = \frac{3 \cdot \sqrt{2}}{\pi} \cdot \sqrt{3} \cdot 220V = 515V$$

And the peak voltage becomes:

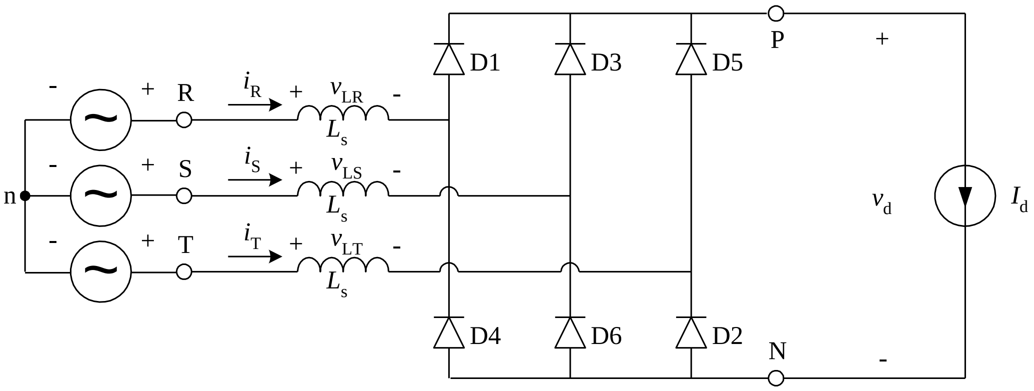
$$V_{d(peak)} = \sqrt{3} \cdot \sqrt{2} \cdot 220V = 539V$$

Problem 5 (P5-24 in Undeland book)

We have the following Three-phase rectifier with finite line impedance and a constant DC-side current. For simplification in the three-phase rectifier circuit, assume that the commutation voltage increases linearly instead of sinusoidal.

a) Obtain the expression for the commutation angle u

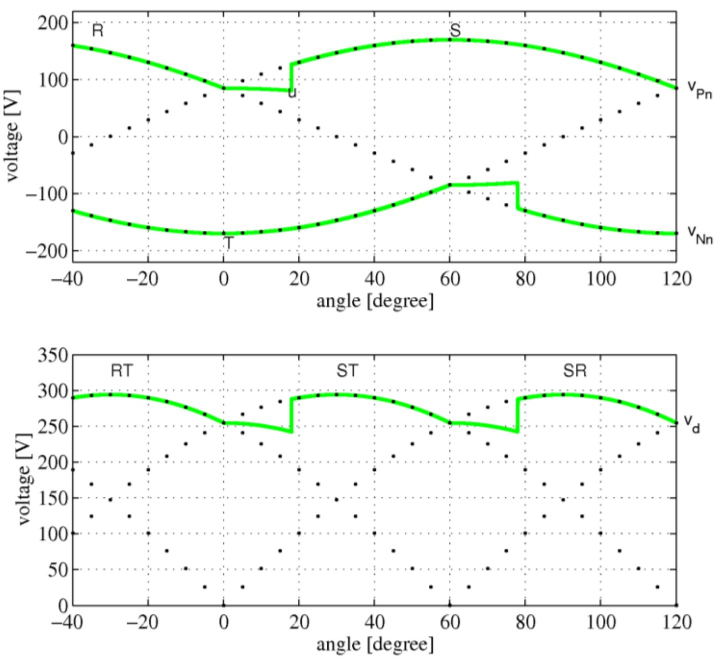
b) For $V_{LL} = 208V@60Hz$, $L_s = 2mH$ and $I_d = 10A$, compare the results in task (a) with the more realistic case where a sinusoidal commutation voltage occurs.



Solution

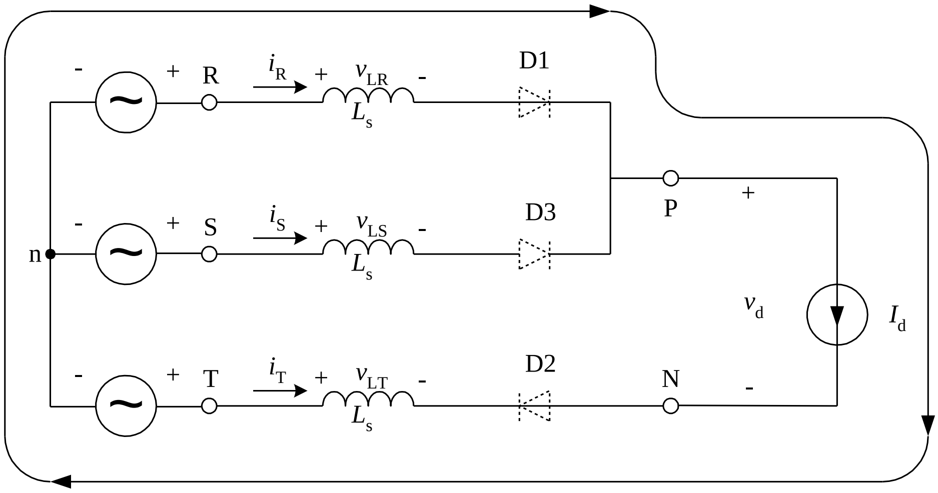
a) Obtain the expression for the commutation angle u

We assume that we have the case where the dc-current is going through diode D1 and D2 (phases R and T) and it shall commutate from D1 to D3 (from R phase to S phase), see figure below.

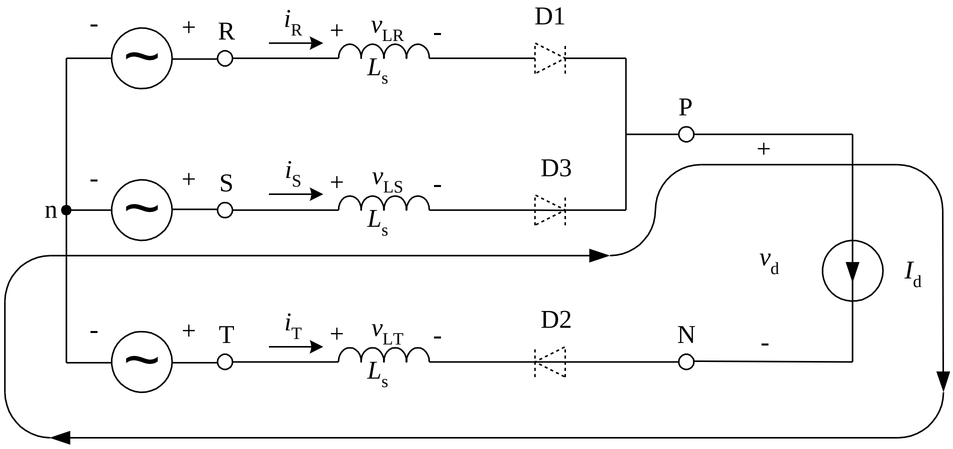


We now draw the circuit before and after the commutation.

Before



After

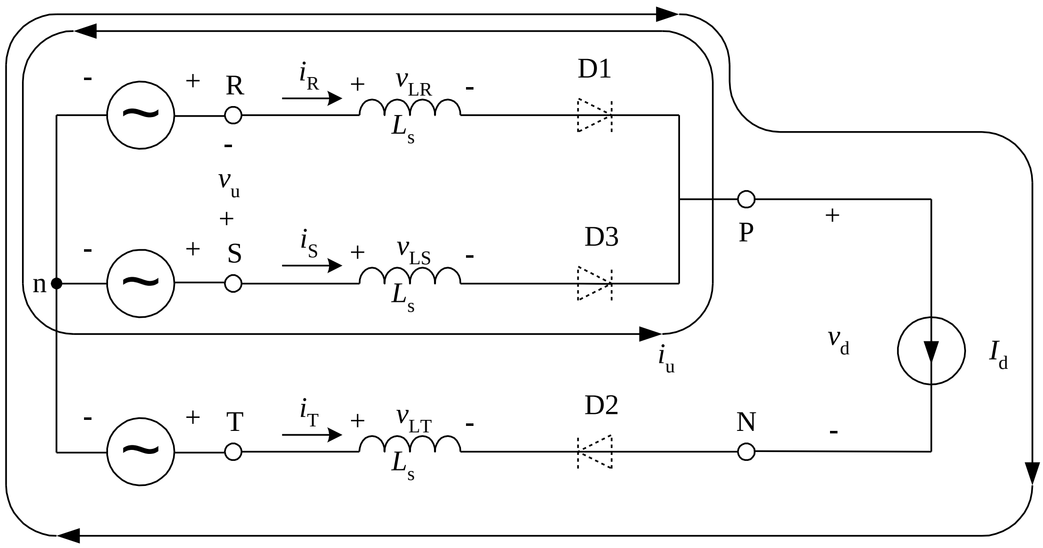


The commutation begins when the voltage from phase R equals to the voltage in phase S ($v_{Rn} = v_{Sn}$) which occurs at $t = 0$ according to our definition. At this time, diode D3 starts to conduct and since diode D1 also conducts (the inductor keeps it conducting) a short circuit will occur between phase R and S.

We see that before we have a current through the inductor in the R phase and we shall move this current to the S phase. To do this we need to reduce the current through the inductor in the R phase and increase the current through the inductor in the S phase. This in such way that the sum of the current from phase R and S equals to the load current ($i_R + i_S = I_d$). Since we have a current source as load, the load current must be constant.

From the basic definition of the current through an inductor, it is known that in order to change the current through an inductor a voltage needs to be applied over the inductor. The circuit can now be drawn during the commutation, including the commutation voltage (v_u) and the commutation current(i_u).

During



From the circuit above, it is seen that the commutation voltage will increase the commutation current and thereby increase the current in the S phase and decrease it in the R phase:

$$i_S = i_u$$
$$i_R = I_d - i_u$$

From this definition it can be seen that the commutation is over when the commutation current has increased to the dc current.

$$i_u = I_d \quad \rightarrow \quad \begin{cases} i_S = I_d \\ i_R = 0 \end{cases}$$

The figure also gives that the commutation voltage is equal to the voltage between phase S and phase R (line-to-line voltage):

$$v_u = v_{Sn} - v_{Rn} = \sqrt{2} \cdot V_{LL} \cdot \sin(\omega t)$$

According to the problem definition, this voltage can be approximated with a straight line (linearly increasing voltage). This is a good approximation for small angles around zero, due to that the first order Taylor series of sinus is almost the value of sinus itself.

$$\sin(x) \approx x$$

For this problem can the commutation voltage be expressed as:

$$v_u = \sqrt{2} \cdot V_{LL} \cdot \omega t$$

But the figure also gives that:

$$v_u = v_{LS} - v_{LR} = L_S \frac{di_S}{dt} - L_S \frac{di_R}{dt} = L_S \frac{di_u}{dt} - L_S \frac{d(I_d - i_u)}{dt} = 2L_S \frac{di_u}{dt} - L_S \frac{dI_d}{dt} = 2L_S \frac{di_u}{dt} \quad \rightarrow$$
$$v_u = 2\omega L_S \frac{di_u}{d\omega t}$$

This differential equation can be solved by multiplying both sides with $d\theta = d\omega t$ and integrating. The integration limits for the commutation current comes from the fact that the commutation current increases from 0 to I_d and the commutation angle ranges from 0 to u.

$$\int_0^u v_u d\theta = \int_0^{I_d} 2\omega L_s di_u = 2\omega L_s \int_0^{I_d} di_u = 2\omega L_s\big(I_d - i_u(0)\big) \quad \rightarrow \quad \int_0^u v_u d\theta = 2\omega L_s I_d$$

$$I_d = \frac{1}{2\omega L_S}\int_0^u v_u d\theta = \frac{1}{2\omega L_S}\int_0^u \big(\sqrt{2}\cdot V_{LL}\cdot\theta\big)d\theta = \frac{\sqrt{2}\cdot V_{LL}}{2\omega L_S}\left[\frac{1}{2}\theta^2\right]_0^u = \frac{\sqrt{2}\cdot V_{LL}}{4\omega L_S}u^2 \quad \rightarrow$$

$$u = \sqrt{\frac{2\sqrt{2}\cdot\omega L_S}{V_{LL}}I_d}$$

The angle u is consequently the angle when the commutation is finished.

But the dot-paper is still not completed, we still don't have an expression for the phase-to-neutral-voltage during commutation. The figure gives us that the voltages in the circuit can be expressed as:

$$v_{Pn} = v_{Rn} - v_{L_R} = v_{Rn} - L_S\frac{d(I_d - i_u)}{dt} = v_{Rn} + L_S\frac{di_u}{dt}$$

$$v_{Pn} = v_{Sn} - v_{L_S} = v_{Sn} - L_S\frac{di_S}{dt} \quad = \quad v_{Sn} - L_S\frac{di_u}{dt}$$

By adding these equations together we get

$$2\cdot v_{Pn} = v_{Sn} - L_S\frac{di_u}{dt} + v_{Rn} + L_S\frac{di_u}{dt} = v_{Sn} + v_{Rn} \quad \rightarrow \quad v_{Pn} = \frac{v_{Sn} + v_{Rn}}{2}$$

We see that the voltage between P and n is equal to the average of the S and R phase voltage, now we can finish the drawing in the dot paper

b) For $V_{LL} = 208V@60Hz$, $L_s = 2mH$ and $I_d = 10A$, compare the results in task (a) with the more realistic case where a sinusoidal commutation voltage occurs.

We shall now compare the result from (a) with the real case where a sinusoidal commutation voltage occurs. Let's calculate the real solution by inserting the real line-to-line voltage in the differential equation

$$v_u = \sqrt{2} \cdot V_{LL} \sin(\theta)$$

$$I_d = \frac{1}{2\omega L_s} \int_0^u v_u d\theta = \frac{1}{2\omega L_s} \int_0^u \sqrt{2} V_{LL} \sin(\theta)\, d\theta = \frac{\sqrt{2} V_{LL}}{2\omega L_s} [-\cos(\theta)]_0^u = \frac{\sqrt{2} V_{LL}}{2\omega L_s} (1 - \cos(\theta)) \quad \rightarrow$$

$$u = \arccos\left(1 - \frac{\sqrt{2} \cdot \omega \cdot L_s \cdot I_d}{V_{LL}}\right)$$

If numerical values are inserted:

$$u = \sqrt{\frac{2 \cdot \sqrt{2} \cdot 2\pi \cdot 60Hz \cdot 2mH \cdot 10A}{208V}} = 0.32rad = 18.35°$$

$$u = \arccos\left(1 - \frac{\sqrt{2} \cdot 2\pi \cdot 60Hz \cdot 2mH \cdot 10A}{208V}\right) = 18.43°$$

The agreement is quite good since the commutation angle is low.