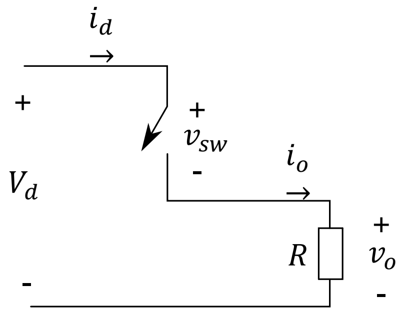
Solution of demonstration 2

Problem 1

In the circuit below, the ideal switch is turned on and off with a duty cycle of D=0.6 at 20 kHz.



V_d= 15V
f_{sw}= 20 kHz
D = 0.6
R = 5 Ω

(a) Calculate the average output voltage V_o.
(b) Sketch the output current i_o.
(c) Calculate the average output current I_o.
(d) Calculate the power dissipation in the resistor R.

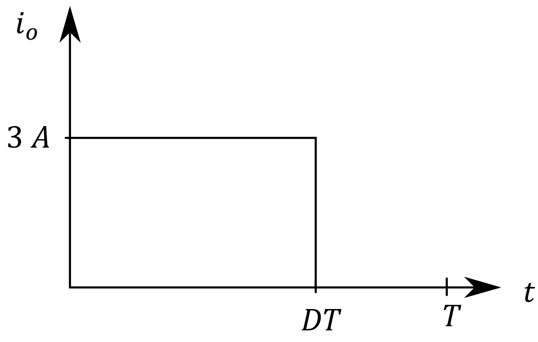
Solution

(a) When the switch is on, $v_0 = V_d$, and when the switch is of, $v_0 = 0$. The average value can then be calculated according to the definition as

$$V_o = \frac{1}{T}\int_0^T v_o dt = \frac{1}{T}\int_0^{DT} V_d dt + \frac{1}{T}\int_{DT}^{T} 0\, dt = \frac{V_d}{T}[t]_0^{DT} = \frac{V_d}{T}(DT - 0) = DV_d = 0.6 \cdot 15 = 9V$$

(b) When the switch is on, the output current is

$$i_o = \frac{V_d}{R} = \frac{15}{5} = 3A$$



(c) The average output current can be calculated similar to the average output voltage

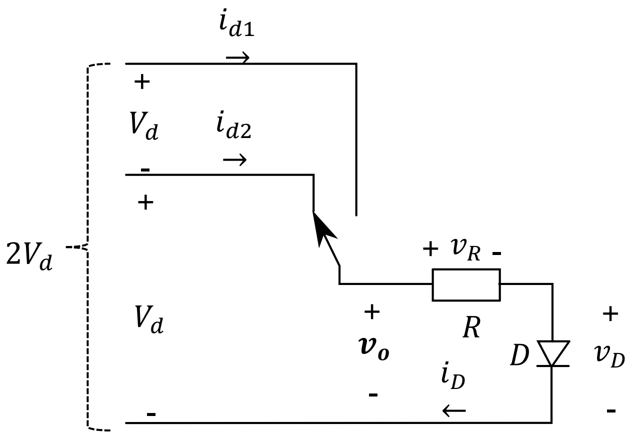
$$I_o = \frac{1}{T}\int_0^T i_o dt = \frac{V_d}{RT}(DT - 0) = \frac{DV_d}{R} = \frac{0.6 \cdot 15}{5} = 1.8A$$

(d) For an AC current, the rms value is equal to the value of the direct current that would produce the same average power dissipation in a resistive load. Therefore we can use the rms current to calculate the power dissipation in the resistor R.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T {i_0}^2\, dt}$$

$$P_o = RI_{rms}^2 = R\frac{1}{T}\int_0^T {i_0}^2\, dt \;= R\frac{1}{T}\int_0^{DT}\left(\frac{V_d}{R}\right)^2 dt = R\cdot\frac{3^2}{T}(DT-0) = R\cdot 5.4 = 27W$$

Problem 2



V_d= 15V
f_{sw}=20kHz
D = 0.6
R = 5 Ω
V_f = 0.82V (diode forward voltage drop)

(a) Calculate the average diode voltage V_D.
(b) Sketch the diode current i_D.
(c) Calculate the average diode current I_D.
(d) Calculate the power dissipation in the diode D.

Solution

(a) When the diode is conduction, the forward voltage drop V_f is constant

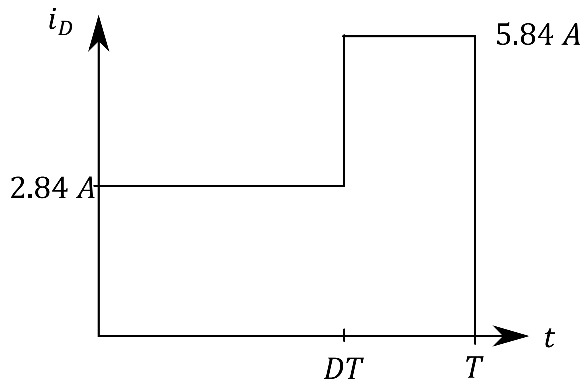
$$V_D = \frac{1}{T}\int_0^T v_D\, dt = \frac{1}{T}\int_0^T v_f\, dt = \frac{v_f}{T}\int_0^T dt = \; v_f = 0.82V$$

(b) During the time interval, $0 < t < DT$

$$i_D = \frac{V_d - v_D}{R} = \frac{15 - 0.82}{5} = 2.84A$$

and during the time interval, $DT < t < T$

$$i_D = \frac{2V_d - v_D}{R} = \frac{30 - 0.82}{5} = 5.84A$$



(c)

$$I_D = \frac{1}{T}\int_0^T i_D\,dt = \frac{V_d - v_D}{RT}(DT - 0) + \frac{2V_{d-}v_D}{RT}(T - DT)$$

$$= \frac{0.6 \cdot (15 - 0.82)}{5} + \frac{0.4 \cdot (2 \cdot 15 - 0.82)}{5} = 4.04\ A$$

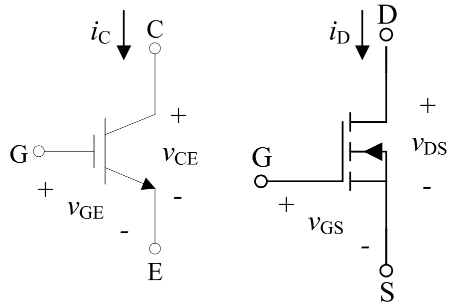
(d) We start from the definition of average power again

$$P_D = \frac{1}{T}\int_0^T p_D\,dt = \frac{1}{T}\int_0^T v_D i_D\,dt = V_f\frac{1}{T}\int_0^T i_D\,dt$$

Let's stop again, considering that the forward voltage drop is constant while the diode conducts, one can observe that the integral on the right hand side corresponds to the definition of the average current. So,

$$P_D = V_f I_{D(AVG)} = 0.82 \cdot 4.04 = 3.62W$$

Problem 3

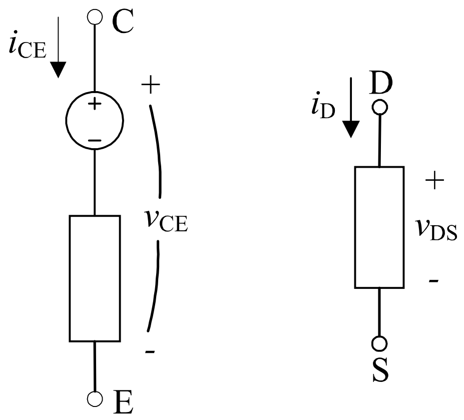


(a) Sketch the voltages (v_{CE} and v_{DS}) over the two switching devices as function of the current trough the devices (i_C and i_D).

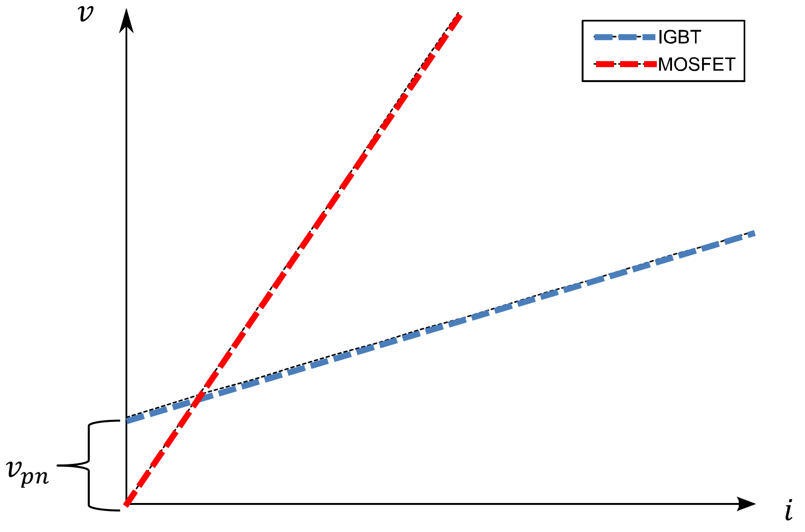
(b) Derive a formula, for each switching device, that can be used to calculate its conduction losses.

Solution

(a) The two symbols correspond to an IGBT (left hand side) and a MOSFET (right hand side). Their respective equivalent circuit when conducting can be illustrated as below. The major difference is that the IGBT is a bipolar component due to its PN-junction.



Due to the PN-junction, there is a voltage drop across the IGBT as soon as it starts to conduct. Further, the on-resistance, R_{ON}, of the IGBT is generally lower than the on-resistance, $R_{DS(on)}$, of the MOSFET, why the slope of the MOSFET curve is steeper.



(b) For an arbitrary semiconductor (IGBT, MOSFET, BJT), the voltage drop over it when it is conducting can be expressed as:

$$v_{cond} = V_{pn} + R_{cond} i_{cond}$$

From this basic definition can the conduction losses be calculated as:

$$P_c = \frac{1}{T_{sw}} \int_0^{T_{sw}} i_{cond}\left(V_{pn} + R_{cond} i_{cond}\right) dt = \frac{V_{pn}}{T_{sw}} \int_0^{T_{sw}} i_{cond}\, dt + \frac{R_{cond}}{T_{sw}} \int_0^{T_{sw}} i_{cond}^2\, dt$$
$$= V_{pn} I_{cond(AVG)} + R_{cond} I_{cond(RMS)}^2$$

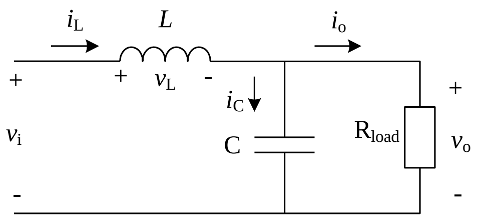
This gives that both the average current and RMS-current through the switch is needed if the power dissipation is to be calculated. However, for a MOSFET which is a unipolar device without any constant voltage drop ($V_{pn} = 0$), the power dissipation can be simplified to:

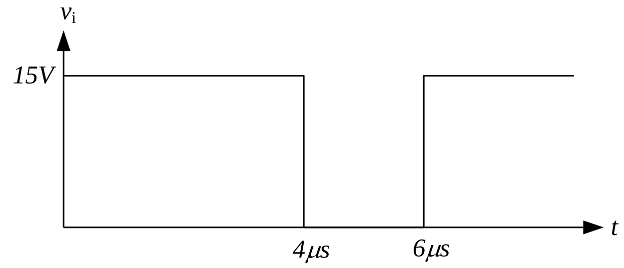
$$P_{c-MOSFET} = R_{DS(on)} I_{FET(RMS)}^2$$

Depending on the on-resistance, R_{ON}, and the current through the IGBT it is possible that conduction losses can be approximated as:

$$P_{c-IGBT} \approx V_{pn} I_{cond(AVG)}$$

Problem 4 (P3-8 in Undeland book)





The applied voltage (v_i) is repetitive and the system is in steady state. Assume that the capacitance is very large, the inductor has a value of $L = 5\mu H$ and that the load consumes 250W (P_{load})

(a) Calculate the average output voltage V_o.
(b) Calculate the average output current and the rms-value of the capacitor current.

The average for an arbitrary function is defined as:

$$F_{AVG} = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt$$

We start by assuming that the output voltage is a pure DC voltage ($v_o = V_o$). This can be done since the capacitor on the output is very large. It is able to supply the load with energy (in combination with the inductor) during the time that the input voltage equals to zero.

What do we mean with steady state in power electronics? The circuit is considered to be in steady state when the waveforms are repeated with a time period (T_{sw}) that depends on the specific nature of that circuit ($f(t) = f(t + T_{sw})$). We know that for the inductor and capacitor the following equations hold:

$$v_L(t) = L\frac{di_L(t)}{dt} \quad \rightarrow \quad i_L(T_{sw} + t_0) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{T_{sw}+t_0} v_L(t)dt$$

$$i_c(t) = C\frac{dv_c(t)}{dt} \quad \rightarrow \quad v_c(T_{sw} + t_0) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{T_{sw}+t_0} i_c(t)dt$$

But since the system is operating in steady state we know that $i_L(T_{sw} + t_0) = i_L(t_0)$ and $v_c(T_{sw} + t_0) = v_c(t_0)$. This results in:

$$0 = \frac{1}{C}\int_{t_0}^{T+t_0} i_C(t)dt \quad \text{and} \quad 0 = \frac{1}{L}\int_{t_0}^{T+t_0} v_L(t)dt$$

From this we see that the average of the inductor voltage and the average of the capacitor current must be zero in steady state (compare with the average formula). The voltage over the inductor can be derived from the input voltage and from basic circuit analysis. The average inductor voltage, which we know is zero can then be calculated as:

$$V_L = \frac{1}{6\mu s}\int_0^{4\mu s}(15V - V_o)\,dt + \frac{1}{6\mu s}\int_{4\mu s}^{6\mu s}(-V_o)\,dt = \frac{1}{6\mu s}(15V \cdot 4\mu s - V_o \cdot 6\mu s) = 0$$

In the equation above, there is only one unknown namely the quantity that we are searching for. The output voltage can be calculated as:

$$V_o = 15V \cdot \frac{4\mu s}{6\mu s} = 10V$$

b) Calculate the average output current and the rms-value of the capacitor current.

The output current can be easily calculated since we have a resistive load and we have assumed that the output voltage is a pure DC voltage.

$$I_o = \frac{P_o}{V_o} = \frac{250W}{10V} = 25A$$

The RMS for an arbitrary function is defined as:

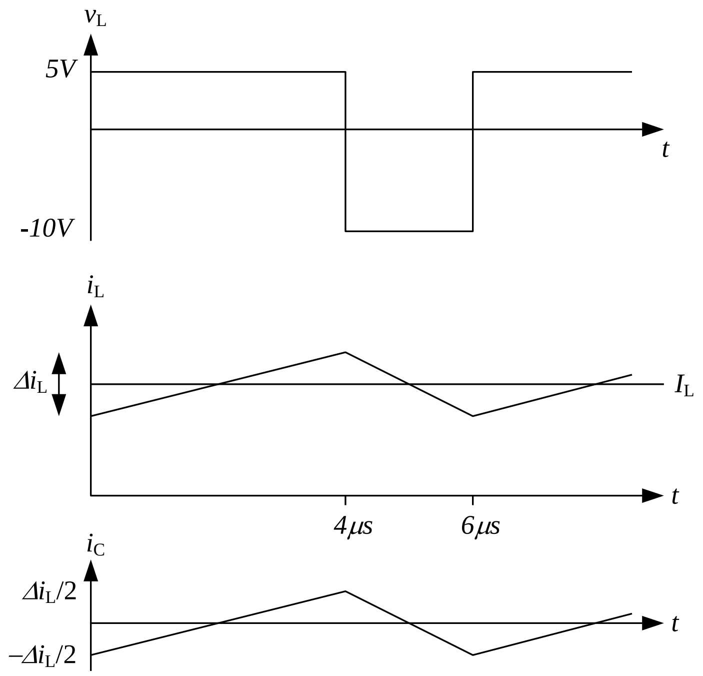
$$F_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} f(t)^2 dt}$$

We know that the inductor current (i_L) consists of two parts, i_C and i_o. Since the converter is operating in steady-state, the average current must be zero $(I_C = 0)$, since the output voltage is a DC-voltage, the output current must also be pure DC-current $(i_o = I_o)$ due to the purely resistive load. The average capacitor current can be calculated as:

$$I_C = \frac{1}{T_{sw}}\int_0^{T_{sw}}(i_L(t) - I_o)dt = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_L(t)dt - I_o = 0 \quad \rightarrow \quad \frac{1}{T_{sw}}\int_0^{T_{sw}} i_L(t)dt = I_o = I_L$$

The average inductor current must be equal to the output current. This gives that the ripple current in the inductor must go through the capacitor $\left(i_C = i_{L(ripple)} = i_L - I_L\right)$.

The inductor voltage and current and the capacitor current can be plotted as:



Since v_L and L are constant, the derivative of the inductor current must be constant. This results in a linear increase and decrease of the inductor current.

We have a new unknown quantity, Δi_L, that must be calculated. Due to the fact that the inductor voltage is constant from 0μs to 4μs, the ripple current can be calculated as:

$$v_L = L\frac{\Delta i_L}{\Delta t} \rightarrow \Delta i_L = \frac{v_L \Delta t}{L} = \frac{5V \cdot 4\mu s}{5\mu H} = 4A$$

The rms-value of the capacitor current can be calculated according to the definition

$$I_{C(RMS)} = \sqrt{\frac{1}{6\mu}\int_0^{6\mu} i_c^2(t)dt}$$

where $i_c(t)$ can be expressed as two linear functions, for the time interval $0 < t < 4\mu s$

$$i_{c1}(t) = k_1 \cdot t + m_1 = \frac{\Delta i_L}{4\mu s}t - \frac{\Delta i_L}{2}$$

and for the time interval $4\mu s < t < 6\mu s$

$$i_{c2}(t) = k_2 \cdot t + m_2 = -\frac{\Delta i_L}{2\mu s}t + m_2$$

where m_2 can be found easily at $t = 5\mu s$

$$i_{c2}(t = 5\mu s) = 0 = -\frac{\Delta i_L}{2\mu s}5\mu s + m_2$$

$$m_2 = \frac{5\Delta i_L}{2}$$

So

$$I_{C(RMS)} = \sqrt{\frac{1}{6\mu}\int_0^{6\mu} i_c^2(t)dt} = \sqrt{\frac{1}{6\mu}\int_0^{4\mu} i_{c1}^2(t)dt + \frac{1}{6\mu}\int_{4\mu}^{6\mu} i_{c2}^2(t)dt} =$$

$$= \sqrt{A+B}$$

then

$$A = \frac{1}{6\mu}\int_0^{4\mu} i_{c1}^2(t)dt = \frac{1}{6\mu}\int_0^{4\mu}\left(\frac{\Delta i_L}{4\mu}t - \frac{\Delta i_L}{2}\right)^2 dt =$$

$$= \frac{1}{6\mu}\int_0^{4\mu}\left(\left(\frac{\Delta i_L}{4\mu}\right)^2 t^2 - 2\left(\frac{\Delta i_L}{4\mu}\right)\left(\frac{\Delta i_L}{2}\right)t + \left(\frac{\Delta i_L}{2}\right)^2\right)dt =$$

$$= \frac{1}{6\mu}\int_0^{4\mu}\left(\left(\frac{\Delta i_L}{4\mu}\right)^2 t^2 - \left(\frac{\Delta i_L^2}{4\mu}\right)t + \left(\frac{\Delta i_L}{2}\right)^2\right)dt =$$

$$= \frac{1}{6\mu}\left[\left(\frac{\Delta i_L}{4\mu}\right)^2\frac{t^3}{3} - \left(\frac{\Delta i_L^2}{4\mu}\right)\frac{t^2}{2} + \left(\frac{\Delta i_L}{2}\right)^2 t\right]_0^{4\mu} = \frac{1}{6\mu}\left(\left(\frac{4}{4\mu}\right)^2\frac{4\mu^3}{3} - \left(\frac{4^2}{4\mu}\right)\frac{4\mu^2}{2} + \left(\frac{4}{2}\right)^2 4\mu\right) =$$

$$= \frac{1}{6\mu}\left(\frac{64\mu}{3} - 32\mu + 16\mu\right) = \frac{16}{18}$$

And

$$B = \frac{1}{6\mu}\int_{4\mu}^{6\mu} i_{c2}^2(t)dt = \frac{1}{6\mu}\int_{4\mu}^{6\mu}\left(-\frac{\Delta i_L}{2\mu}t + \frac{5\Delta i_L}{2}\right)^2 dt =$$

$$= \frac{1}{6\mu}\int_{4\mu}^{6\mu}\left(\left(\frac{\Delta i_L}{2\mu}\right)^2 t^2 - 2\left(\frac{\Delta i_L}{2\mu}\right)t\left(\frac{5\Delta i_L}{2}\right) + \left(\frac{5\Delta i_L}{2}\right)^2\right)dt =$$

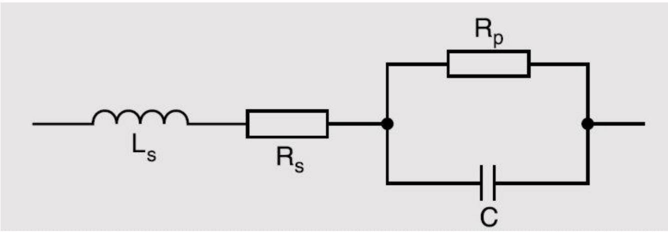
$$= \frac{1}{6\mu}\left[\left(\frac{\Delta i_L}{2\mu}\right)^2\frac{t^3}{3} - \left(\frac{\Delta i_L}{2\mu}\right)\left(\frac{5\Delta i_L}{2}\right)t^2 + \left(\frac{5\Delta i_L}{2}\right)^2 t\right]_{4\mu}^{6\mu} =$$

$$= \frac{1}{6\mu}\left(\left(\left(\frac{4}{2\mu}\right)^2 \frac{6\mu^3}{3} - \left(\frac{4}{2\mu}\right)\left(\frac{5\cdot 4}{2}\right)6\mu^2 + \left(\frac{5\cdot 4}{2}\right)^2 6\mu\right)\right.$$
$$\left. - \left(\left(\frac{4}{2\mu}\right)^2 \frac{4\mu^3}{3} - \left(\frac{4}{2\mu}\right)\left(\frac{5\cdot 4}{2}\right)4\mu^2 + \left(\frac{5\cdot 4}{2}\right)^2 4\mu\right)\right)$$
$$= \frac{1}{6\mu}\left((288\mu - 720\mu + 600\mu) - \left(\frac{256}{3}\mu - 320\mu + 400\mu\right)\right) =$$
$$= \frac{8}{18}$$

Finally

$$I_{C(RMS)} = \sqrt{A+B} = \sqrt{16/18 + 8/18} = \sqrt{24/18} = 1.15A$$

Why is the RMS-current through a capacitor necessary to know? All capacitors can only handle a certain amount of RMS-current due to heating of the internal resistances. An equivalent circuit is shown below where C and Rs are the dominating terms.



The operating voltage and especially the working temperature have a significant influence on the working life of electrolytic capacitors.

However, the pure series resistance (from e.g. connecting wires) is not of great interest since it does not contribute to the losses in a significant extent, instead is the equivalent series resistance (ESR) more useful. The ESR sums up the all losses in the capacitor (resistive and dielectric losses). The ESR is temperature and frequency dependent.

The parameter often given in the datasheets for a capacitor is the *dissipation factor* (tan δ) which relates capacitance at a certain frequency to the *ESR*. By definition is the dissipation factor the ratio between the ESR and the reactance of the capacitor.

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Outline Drawing

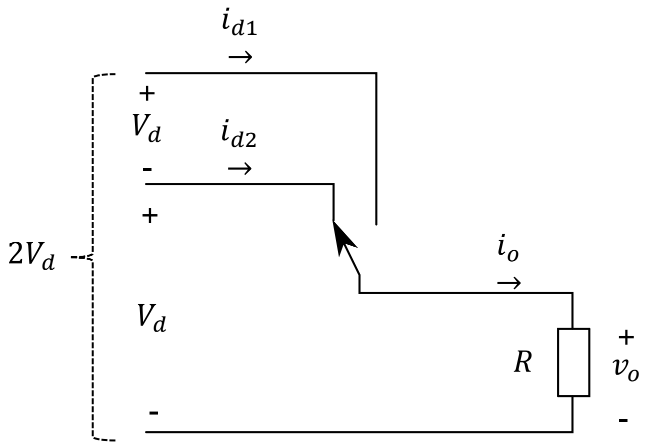
Vent (except ϕ5) Sleeve ϕd ± 0.05 copper clad steel wire (tinned)

Photo

ELNA PART NO. / WV (V)	CAP. (µF)	SIZE (ϕ x L) (mm)	tan δ	IMPEDANCE (Ω) 20°C	IMPEDANCE (Ω) -10°C	Rip Cur. (mArms)
RJH-10V471MG4	470	8 x 15	0.19	0.13	0.29	617
RJH-10V471MH3	470	10 x 12.5	0.19	0.10	0.23	625
RJH-10V561MH4	560	10 x 16	0.19	0.080	0.18	825
RJH-10V681MG5	680	8 x 20	0.19	0.095	0.21	800

Problem 5

In the circuit below, the switch connects the resistor to the voltage, V_d, during the time interval $0 < t < DT$. The rest of the time period, T, it connects the resistor to the voltage $2V_d$.



V_d= 15V
f_{sw}= 20 kHz
D = 0.6
R = 5 Ω

(a) Calculate the average output voltage V_o.
(b) Sketch the output current i_o.
(c) Calculate the average output current I_o.
(d) Calculate the power dissipation in the resistor R.

Solution

(a) The average output voltage is calculated according to the definition of an average value

$$V_o = \frac{1}{T}\int_0^T v_o dt = \frac{1}{T}\int_0^{DT} V_d dt + \frac{1}{T}\int_{DT}^{T} 2V_d dt = \frac{V_d}{T}(DT - 0) + \frac{2V_d}{T}(T - DT)$$

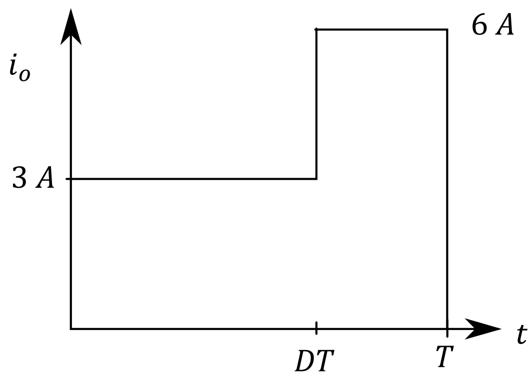
$$= V_d D + 2V_d(1 - D) = 15 \cdot 0.6 + 2 \cdot 15 \cdot 0.4 = 21V$$

(b) During the time interval, $0 < t < DT$

$$i_o = \frac{V_d}{R} = \frac{15}{5} = 3A$$

and during the time interval, $DT < t < T$

$$i_o = \frac{2V_d}{R} = \frac{30}{5} = 6A$$



(c) The average output current can be calculated similar to the average output voltage

$$I_o = \frac{1}{T}\int_0^T i_o dt = \frac{V_d}{RT}(DT - 0) + \frac{2V_d}{RT}(T - DT) = \frac{DV_d}{R} + \frac{(1 - D)2V_d}{R}$$

$$= \frac{0.6 \cdot 15}{5} + \frac{0.4 \cdot 2 \cdot 15}{5} = 4.2A$$

(d) From the previous problem we know that the (average) power dissipation can be calculated with the rms current

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_o^2\,dt} = \sqrt{\frac{1}{T}\int_0^{DT} i_o^2\,dt + \frac{1}{T}\int_{DT}^{T} i_o^2\,dt} = \sqrt{\frac{1}{T}\int_0^{DT}\left(\frac{V_d}{R}\right)^2 dt + \frac{1}{T}\int_{DT}^{T}\left(\frac{2V_d}{R}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\left(\frac{V_d}{R}\right)^2 (DT-0) + \frac{1}{T}\left(\frac{2V_d}{R}\right)^2 (T-DT)} = \sqrt{D\left(\frac{V_d}{R}\right)^2 + (1-D)\left(\frac{2V_d}{R}\right)^2}$$

$$= \sqrt{0.6\left(\frac{15}{5}\right)^2 + 0.4\left(\frac{2\cdot 15}{5}\right)^2} = 4.45A$$

The power dissipation in the resistor is

$$P_o = RI_{rms}^2 = 5\cdot 4.45^2 = 99W$$