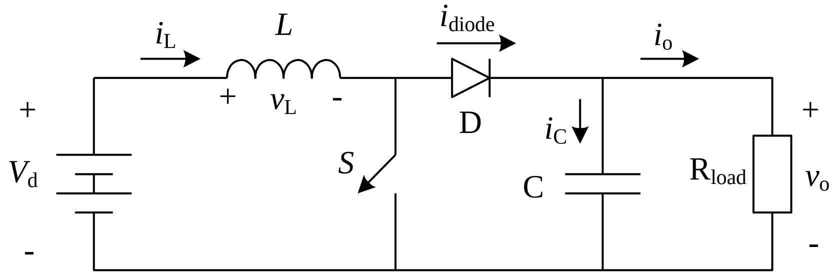
Solution of demonstration 4

Problem 1 (P7-7 in Undeland book)

In a step-up converter, consider all components to be ideal. The input voltage (V_d) varies between 8V and 16V. The output voltage (V_o) is regulated and kept constant at 24V. The switching frequency (f_{sw}) is 20kHz, the output capacitance is 470μF and the output power (P_o) is always greater than 5W.

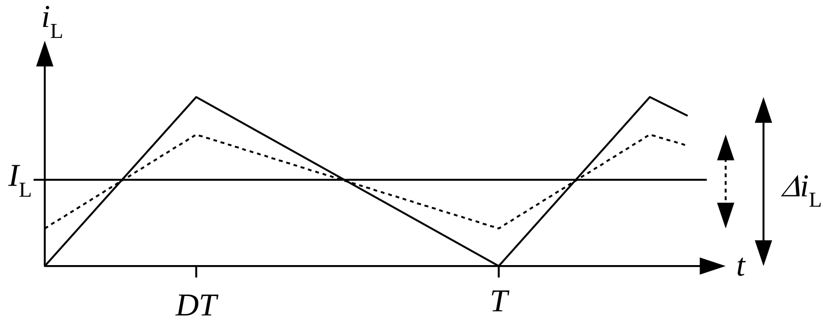


We assume that the converter is in steady state and that the output capacitor is so large that the output voltage can be treated as a DC voltage.

Find the minimum value of the inductor that makes the boost converter operate in CCM for the entire operating range.

Solution

We start by drawing the inductor current at the border between CCM and DCM. Since the converter is operating at the border the inductor current will only be zero at one point. This gives that the inductor voltage will be equal to the input voltage when the switch is on, which gives an increasing current. When the switch is off the diode will conduct and the inductor voltage will be the input voltage minus the output voltage. Since the output voltage is greater than the input voltage the inductor voltage is negative which gives a decreasing current. The output voltage must be greater than the input voltage otherwise the average of the inductor voltage will not be zero.



From the figure it can be seen that the converter is operating in CCM if $I_L \geq \Delta i_L/2$. If the average value of the inductor current is equal to half the peak-to-peak ripple in the inductor current, the converter will be operating at the border between DCM and CCM, see the solid line in the figure. If the ripple is reduced there will be a margin to the border, see the dashed line.

We now calculate the inductance value required to operate the converter at the border between CCM and DCM. This can be done by first calculate the three quantities that characterize the figure, namely D, I_L and Δi_L.

First we find an expression for the duty ratio. We do this by calculating the average inductor voltage.

$$V_L = \frac{1}{T_{sw}} \int_0^{T_{sw}} v_L dt = \frac{1}{T_{sw}} (V_d D T_{sw} + (V_d - V_o)(1 - D) T_{sw}) = V_d + V_o(D - 1) = 0$$

$$V_o = \frac{V_d}{1 - D} \quad \rightarrow \quad D = \frac{V_o - V_d}{V_o}$$

The average input current (I_L) can be calculated by using the fact that we have a loss less converter with only ideal components.

$$P_d = P_{load} \quad \rightarrow \quad I_L = I_d = \frac{P_{load}}{V_d}$$

The peak-to-peak ripple in the inductor current can be calculated by using:

$$v_L = L \frac{di_L}{dt} = L \frac{\Delta i_L}{\Delta t}$$

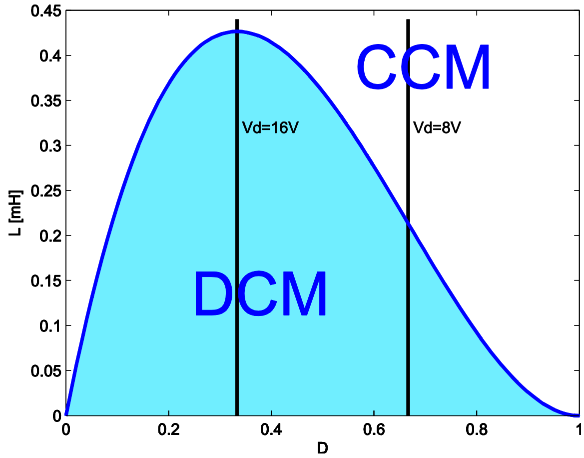
During the period $0 \leq t \leq DT_{sw}$ is the inductor voltage constant and equal to the input voltage. The equation above can then be rewritten as:

$$\Delta i_L = \frac{V_d D T_{sw}}{L} = \frac{V_d D}{L f_{sw}}$$

At the border between continuous and discontinuous conduction mode, the average current equals half the current ripple. This gives:

$$L = \frac{V_d D T_{sw}}{2 I_L} = \frac{V_d^2 D}{2 P_{load} f_{sw}} = \frac{V_o^2 (1 - D)^2 D}{2 P_{load} f_{sw}} = \frac{V_o^2}{2 P_{load} f_{sw}} (D - 2D^2 + D^3)$$

For this equation the maximum inductance for the given operation interval must be found. We must find the maximum of this function because the function gives the inductance value required for operating the converter at the border for a given operating point. Because this value is the smallest value we can have in order to operate the converter in CCM within the interval. From the equation for the peak-to-peak ripple in the inductor current it can be seen that if the inductance value is reduced, the ripple will increase. Therefore we must search for the maximum inductance within the operation interval given, see the figure below for how the required inductance varies as function off the duty cycle on the border between CCM and DCM.



The lowest output power gives the highest inductance, this gives that $P_o = 5W$. For the given operating conditions ($V_o = 24V$ and V_d varies between 8V and 16V), the following expression for the duty-ratio applies:

$$D = \frac{V_o - V_d}{V_o}$$

where D varies between 2/3 and 1/3. The equation for the inductance with respect of the duty ratio can now be derived as:

$$\frac{dL}{dD} = \frac{V_{load}^2}{2P_{load}f_{sw}}(1 - 4D + 3D^2) = 0$$

$$\frac{1}{3} - \frac{4}{3}D + D^2 = 0 \quad \rightarrow \quad \left(D - \frac{2}{3}\right)^2 = \frac{-1}{3} + \frac{4}{9} = \frac{1}{9}$$

$$D = \frac{2}{3} \pm \frac{1}{3} = \begin{cases} 1/1 \\ 1/3 \end{cases}$$

We now have to cheek which point that is the maximum:

D =	0	<	1/3	<	1
dL/dD =		+	0	-	0
L =	0	Increasing	Max	Decreasing	0

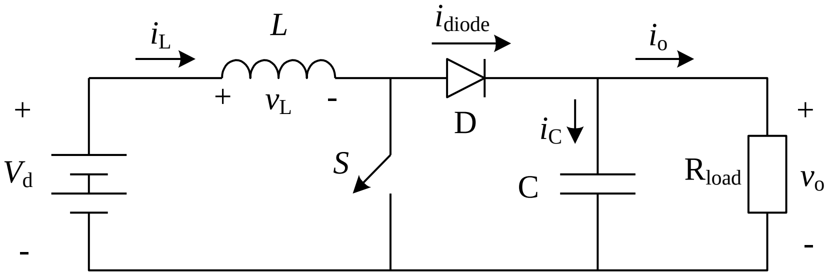
From the table above, it can be seen that D=1/3 gives the maximum.

We now calculate the minimum inductance required for operating the converter in CCM when $V_d = 8V - 16V$, $V_o = 24V$ (regulated), f_{sw}=20kHz, C=470μF and P_o ≥5W

$$L_{min} = \frac{V_o^2(1-D)^2 D}{2P_{load}f_{sw}} = \frac{24V^2(1-1/3)^2 \cdot 1/3}{2 \cdot 5W \cdot 20kHz} = 0.43mH$$

Problem 2 (P7-8 in Undeland book)

A step-up converter has the following specifications: $V_d = 12V$, $V_o = 24V$, $I_o = 0.5A$, $f_{sw} = 20kHz$, $C = 470\mu F$ and $L = 150\mu H$.



Calculate the peak-to-peak output voltage ripple.

Solution for Problem 7.8

We start by assuming that the converter is operating in steady-state and in CCM and that the output capacitor is so large that the output voltage can be treated as a DC voltage. First we have to find an expression for the duty ratio. We do this by calculating the average inductor voltage.

$$V_L = \frac{1}{T_{sw}} \int_0^{T_{sw}} v_L dt = \frac{1}{T_{sw}} (V_d D T_{sw} + (V_d - V_o)(1 - D) T_{sw}) = V_d + V_o(1 - D) = 0$$

$$V_o = \frac{V_d}{1 - D} \quad \rightarrow \quad D = \frac{V_o - V_d}{V_o} = \frac{24V - 12V}{24V} = 0.5$$

The average input current we calculate by knowing that we have a loss less converter consisting of only ideal components.

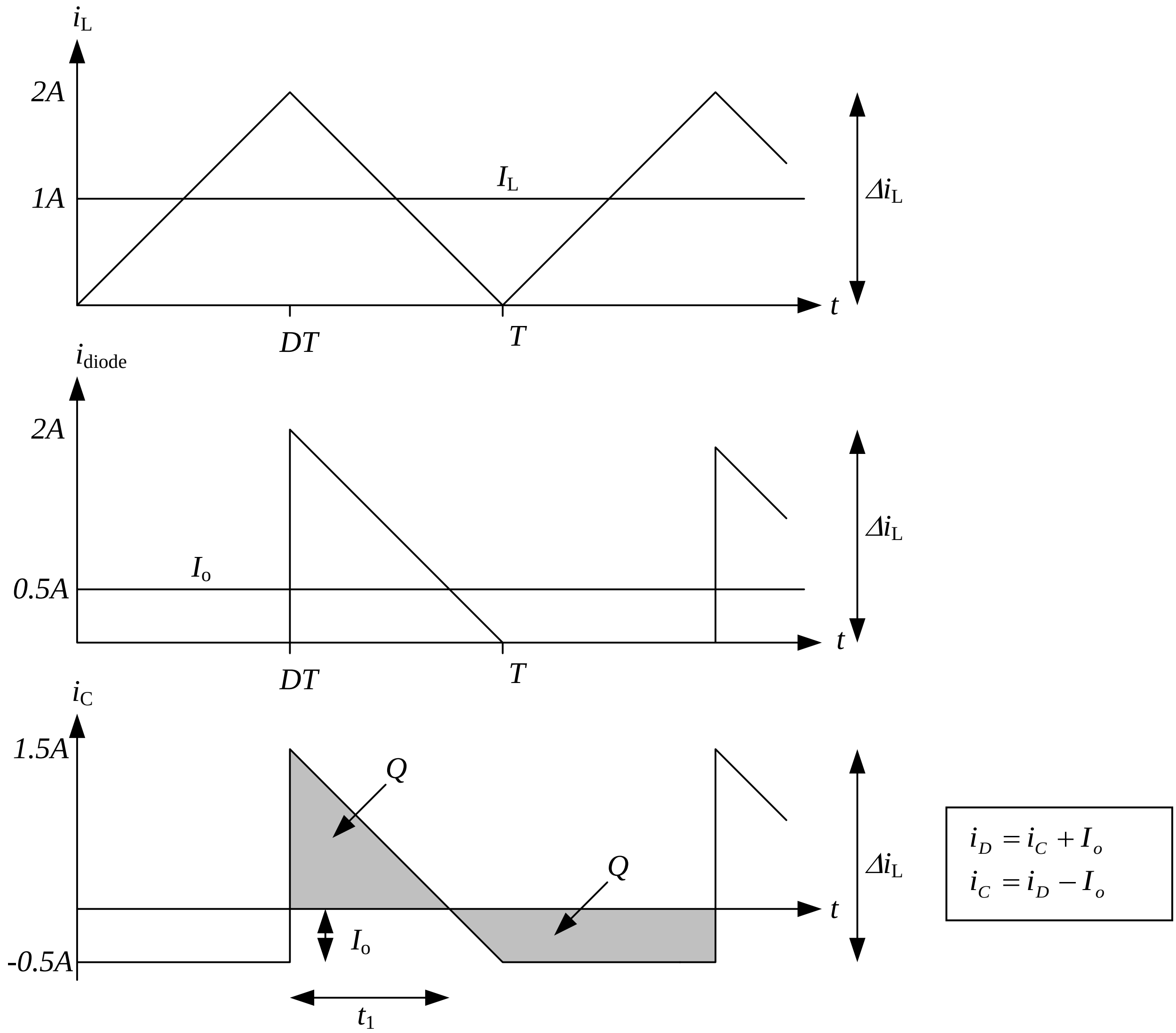
$$P_d = P_o \quad \rightarrow \quad I_L = \frac{V_o I_o}{V_d} = \frac{V_o I_o}{V_o(1 - D)} = \frac{I_o}{(1 - D)} = \frac{0.5A}{(1 - 0.5)} = 1A$$

The assumption of CCM has to be checked. The peak-to-peak ripple in the inductor current we calculate by studying the inductor voltage during the on-time of the switch.

$$\Delta i_L = \frac{V_d D T}{L} = \frac{V_d D}{L f_{sw}} = \frac{12V \cdot 0.5}{150\mu H \cdot 20kHz} = 2A$$

We see that the average current equals half the current ripple. This gives that the converter is operating on the border between CCM and DCM so the assumption of CCM operation is OK.

Now we plot the inductor current, the diode current and the capacitor current. The output current is a DC current, since we have assumed a constant output voltage.



For the voltage across the capacitor we know that it can be expressed as:

$$v_c(t) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{t_0+t} i_c(t)dt$$

This gives that the voltage at time *t* is the voltage at time t_0 plus the integral of the current from the time t_0 to *t*. From the figure it can be seen that from time DT, the capacitor current is positive until time $t = DT + t_1$. This gives a capacitor voltage that increases from time $t = DT$ until time t=$DT + t_1$. The peak-to-peak ripple in the output voltage can then be calculated by

$$\Delta v_o = v_c(DT + t_1) - v_c(DT) = \frac{1}{C}\underbrace{\int_{DT}^{DT+t_1} i_c(t)dt}_{Charge} = \frac{Q}{C}$$

The integral of the capacitor current is equal to the charge put into the capacitor; the charge is marked in the figure above with Q. From this figure it is clearly seen that the charge is equal to the area under the capacitor current, marked with grey in the figure. The two areas must be equal in the figure since the average capacitor current must be zero in steady state.

The output ripple can be expressed as:

$$\Delta v_o = \frac{Q}{C} = \frac{1}{C} \cdot \frac{\Delta i_L - I_o}{2} t_1$$

To solve the equation above, the time $t_1$has to be calculated. During the time interval t_1 we know that the derivative of the diode current is:

$$\frac{di_{diode}}{dt} = \frac{di_L}{dt} = \frac{V_d - V_o}{L} = \text{constant}$$

Which gives us:

$$\frac{di_{diode}}{dt} \cdot t_1 = I_o - \Delta i_L \quad \rightarrow \quad t_1 = \frac{I_o - \Delta i_L}{di_{diode}/dt} = \frac{(\Delta i_L - I_o) \cdot L}{V_o - V_d}$$

The expression for the time t_1 is entered in the expression for the voltage ripple and the final answer can be calculated.

$$\Delta v_o = \frac{1}{C} \frac{\Delta i_L - I_o}{2} \frac{\Delta i_L - I_o}{V_o - V_d} L = \frac{L(\Delta i_L - I_o)^2}{2C(V_o - V_d)} = \frac{150\mu H \cdot (2A - 0.5A)^2}{2 \cdot 470\mu F(24V - 12V)} = 29.9mV$$

The approximation that the output voltage is a pure DC voltage was OK. Also, note that we have a significantly higher voltage ripple compare to the buck-converter.