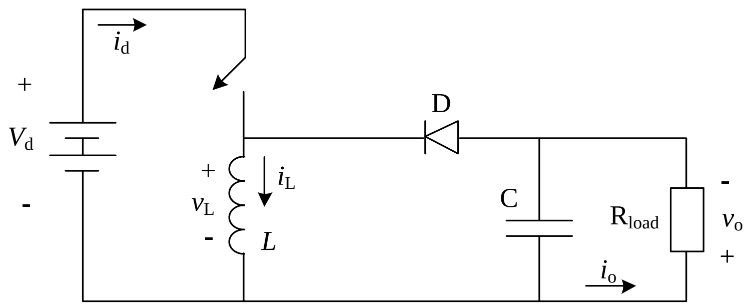
Solution of demonstration 5

Problem 1

For a Buck-boost converter calculate the voltage ripple (Δv_o).



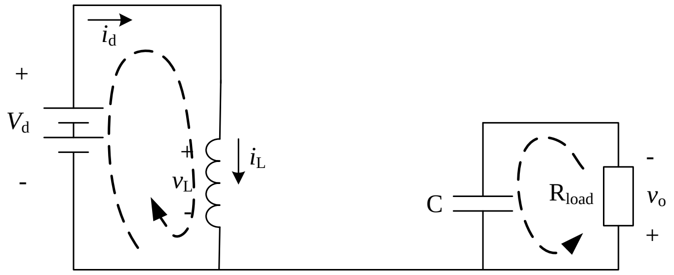
$V_d = 12V$ $\quad V_o = 15V$ $\quad f_{sw} = 20kHz$ $\quad C = 470\mu F$
$I_o = 2.5A$ $\quad L = 350\mu H$

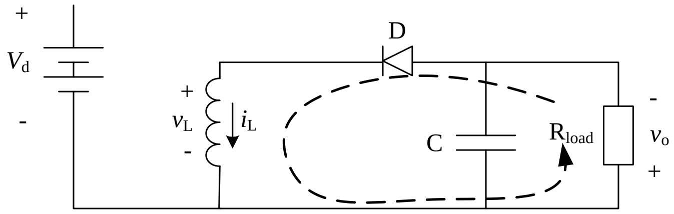
The converter is operating in steady state and the output voltage is considered to be a pure DC-voltage.

Solution

The main application of a buck/boost converter is in a regulated power supply where the voltage needs to be both increased and decreased. The converter can be derived by cascading a step-down and a step-up converter.

As in previous demonstrations, the fact that the average inductor voltage equals zero over one switching period is used to derive the ratio between input and output voltage. If CCM is assumed, a continuous current is flowing through the inductor. This gives that when the switch is on, the inductor is short circuited; the entire input current flows through the inductor and the diode is blocking. When the switch is off, the input is disconnected and the inductor current is freewheeling through the diode, see pictures below.





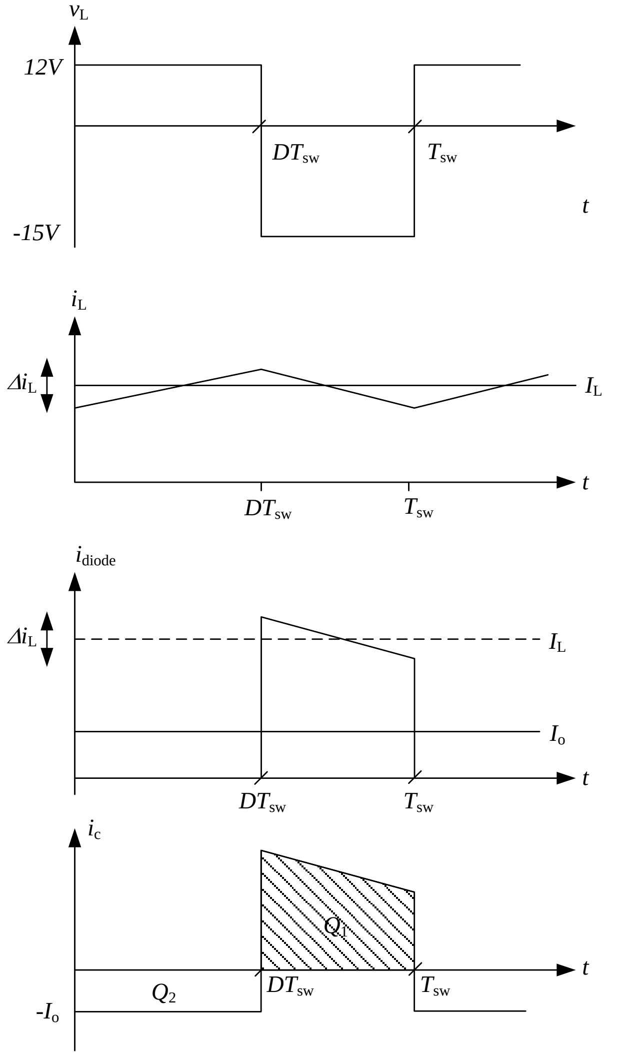
When the switch is turned on, the inductor voltage becomes: $v_L = V_d$. When the switch is off, the inductor voltage becomes: $v_L = -V_o$.

$$V_L = \frac{1}{T_{sw}} \int_0^{DT_{sw}} V_d dt + \frac{1}{T_{sw}} \int_{DT_{sw}}^{T_{sw}} -V_o dt = \frac{1}{T_{sw}} (V_d DT_{sw} - V_o T_{sw} + V_o DT_{sw}) = 0$$

$$V_d D = V_o(1-D) \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{1-D} \quad \rightarrow \quad \frac{15V}{12V} = \frac{D}{1-D} \quad \rightarrow \quad D = 0.556$$

$$P_o = P_d \quad \rightarrow \quad V_o I_o = V_d I_d \quad \rightarrow \quad I_o = \frac{V_d I_d}{V_o} = \frac{I_d}{V_o} \frac{V_o(1-D)}{D} \quad \rightarrow \quad \frac{I_o}{I_d} = \frac{(1-D)}{D}$$

The voltage over the inductor and the current through it can be drawn as:



$$\Delta i_L = \frac{V_d D}{f_{sw} L} = \frac{12V \cdot 0.556}{20kHz \cdot 350\mu H} = 0.95A$$

$$I_L = I_o + I_d = I_o + \frac{I_o D}{1-D}$$
$$= 2.5A + \frac{2.5A \cdot 0.556}{1-0.556} = 5.63A$$

$$I_L > \frac{\Delta i_L}{2}$$

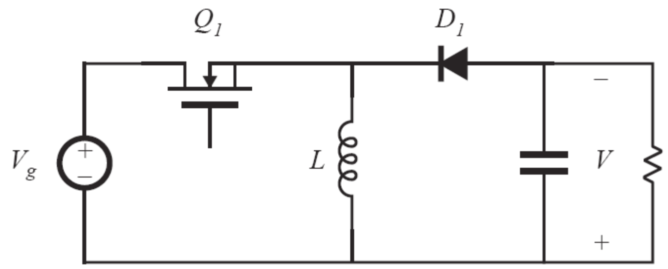
The converter is operating in CCM

When the capacitor current is positive, charge is added to the

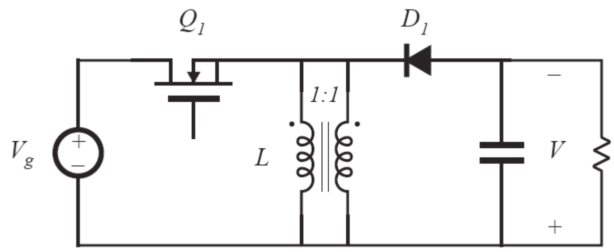
$$\Delta v_c = \frac{Q_2}{C} = \frac{I_o DT}{C} = \frac{2.5A \cdot 0.556}{470\mu F \cdot 20kHz} = 147mV$$

Derivation of Flyback from Buck/Boost

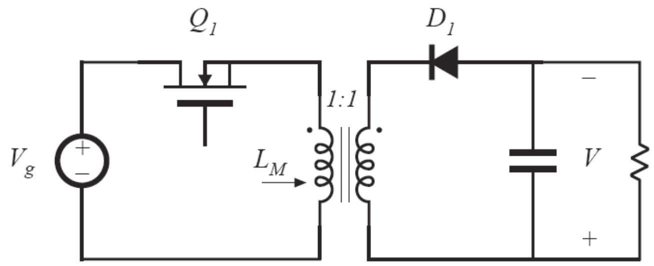
The schematic for a simple Buck/Boost converter is depicted below.



The derivation of a flyback converter comes from the buck-boost converter. The figure above depicts the basic buck-boost converter, with the switch realized using a MOSFET and diode. The converter is changed so that the inductor winding is constructed using two wires, with a 1:1 turns ratio. The basic function of the inductor is unchanged, and the parallel windings are equivalent to a single winding constructed of larger wire.

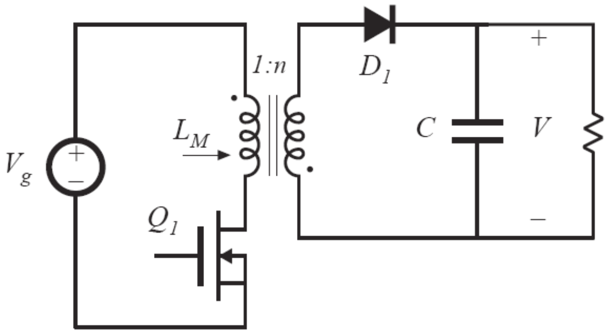


In next step, the connections between the two windings are broken. One winding is used while the transistor Q_1 conducts, while the other winding is used when diode D_1 conducts.

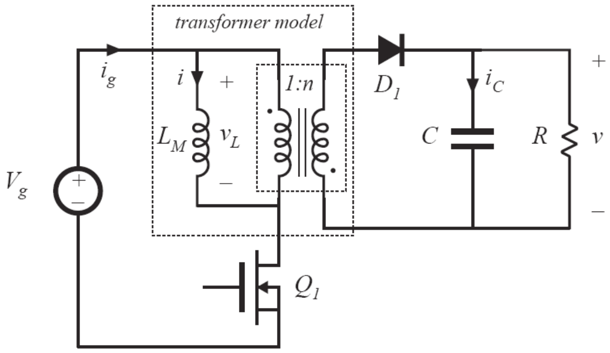


The total current in the two windings is unchanged from the circuit with connected windings; however, the current is now distributed between the windings differently. The magnetic fields inside the inductor in both cases are identical. Although the two-winding magnetic device is represented using the same symbol as the transformer, a more descriptive name is two-winding inductor. This device is sometimes also called a flyback transformer. Unlike the ideal transformer, current does not flow simultaneously in both windings of the Flyback transformer.

The Figure below illustrates the usual configuration of the flyback converter. The MOSFET source is connected to the primary-side ground, simplifying the gate drive circuit. The transformer polarity marks are reversed, to obtain a positive output voltage. A 1:*n* turns ratio, where *n* is an arbitrary selected integer, is introduced; this allows better converter optimization.

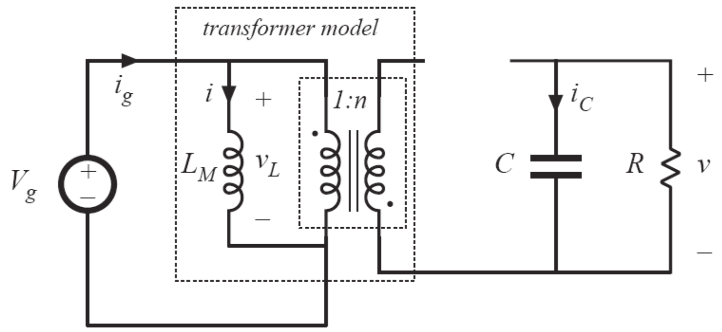


The behavior of most transformer-isolated converters can be adequately understood by modeling the physical transformer with a simple equivalent circuit consisting of an ideal transformer in parallel with the magnetizing inductance. The magnetizing inductance must then follow all of the usual rules for inductors; in particular, volt-second balance must hold when the circuit operates in steady-state. This gives that the average voltage applied across every winding of the transformer must be zero. The transformer above can be replaced with the equivalent circuit depicted below:

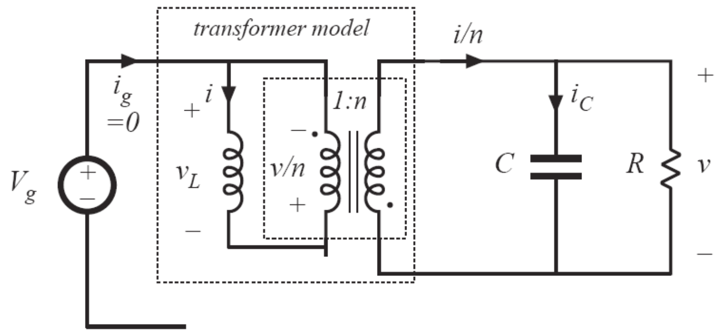


The magnetizing inductance L_M functions in the same manner as inductor *L* of the original buck-boost converter. When transistor Q_1 conducts, energy from the dc source V_g is stored in L_M. When diode D_1 conducts, this stored energy is transferred to the load, with the inductor voltage and current scaled according to the 1:*n* turns ratio.

With Switch Q_1 conducting

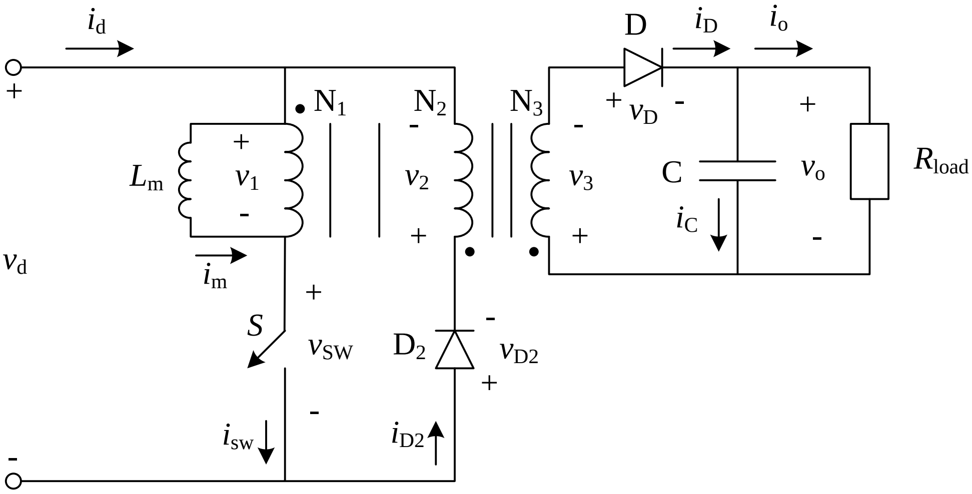


With switch Q_1 not conducting



Problem 2 (P10-2 in Undeland book)

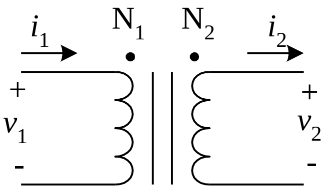
A flyback converter, with a protective winding (N_2), has a turn ratio $(N_1\!:N_2\!:N_3)$ equal to $(1\!:0.5\!:1)$.



For DCM, Derive the voltage transfer ratio V_o/V_d in terms of the load resistance R_{load}, switching frequency (f_{sw}) transformer inductance (L_m) and duty ratio (D).

Solution

What do the dots at the transformer windings mean? Let's consider an ideal transformer with two windings:



$$\frac{v_1}{N_1} = \frac{v_2}{N_2}$$

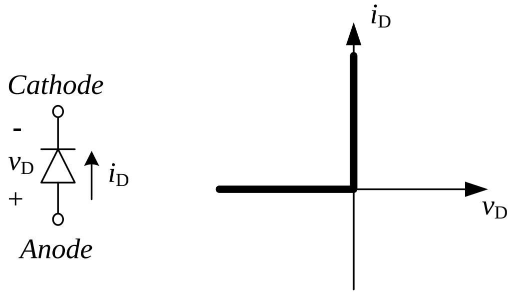
$$i_1 N_1 = i_2 N_2$$

If we apply a positive voltage at the dot then we get a positive voltage at the dot on the other side. If we have a current going in at the dot we must then have a current going out from the dot on the other side. If not, we would put in power from both sides, which mean that no power is transferred. This is not possible since we are studying an ideal transformer.

The converter is operating in DCM which means that the transformer is completely demagnetized each period; the magnetizing current (i_m) is equal to zero in the beginning of each period.

As usual, it is assumed that the output voltage is a pure DC voltage and that the converter is operating in steady state.

An ideal diode is characterized by the fact that a positive voltage makes the diode fully conducting and that a negative voltage makes the diode block. It can be illustrated in the following schematic:



The switch is closed at time zero$(t = 0)$, at this point $(i_m = 0)$ due to DCM operation. From the schematic it can be seen that the voltage over winding N_1 is equal to the input voltage $(v_1 = V_d)$. This gives that the magnetizing current will start to increase linearly. A positive voltage is applied at the dot and therefore the voltage on the other windings will also be positive.

$$\left.\begin{aligned} \frac{v_1}{N_1} = \frac{v_2}{N_2} \quad &\rightarrow \quad v_2 = V_d \frac{N_2}{N_1} = 0.5V_d \\ 0 = -v_2 - v_{D2} - V_d \end{aligned}\right\} \rightarrow \; v_{D2} = -v_2 - V_d = -1.5V_d$$

$$\left.\begin{aligned} \frac{v_1}{N_1} = \frac{v_3}{N_3} \quad &\rightarrow \quad v_3 = V_d \frac{N_3}{N_1} = V_d \\ 0 = v_3 + v_D + V_o \end{aligned}\right\} \rightarrow \; v_D = -v_3 - V_o = -V_d - V_o$$

Both diode voltages are negative and consequently in blocking state. This gives that there will be no current through winding N_2 and N_3 and consequently no current through winding N_1 either. This comes from the fact that if there is a current through one winding there must be a current through at least one other winding. The only thing that will happen during the time the switch is closed is that the magnetizing current will increase linearly, the transformer is magnetized and energy is stored.

At a time $t = DT$ the switch is opened. At this time, a current is flowing through the magnetizing inductance that cannot be changed instantaneously. Therefore will the magnetizing inductance put up a voltage to drive this current which gives that $v_1 < 0$. The voltage put up by the magnetizing inductance will be so high that the magnetizing current starts to flow through winding N_1. However, this is only possible if there is a closed path through some of the other windings for the current, if not the voltage will be so high that the switch brakes down and starts to conduct again $(v_{sw} = V_d - v_1)$.

Let's consider the voltages over the diodes again when a negative voltage is applied over winding N_1.

$$\left.\begin{aligned} \frac{v_1}{N_1} = \frac{v_2}{N_2} \quad &\rightarrow \quad v_2 = v_1 \frac{N_2}{N_1} = 0.5v_1 \\ 0 = -v_2 - v_{D2} - V_d \end{aligned}\right\} \rightarrow \; v_{D2} = -v_2 - V_d = -V_d - 0.5v_1$$

$$\left.\begin{aligned} \frac{v_1}{N_1} = \frac{v_3}{N_3} \quad &\rightarrow \quad v_3 = v_1 \frac{N_3}{N_1} = v_1 \\ 0 = v_3 + v_D + V_o \end{aligned}\right\} \rightarrow \; v_D = -V_o - v_1$$

We know that when the voltage over a diode is positive it will conduct. Note that the voltage over winding N_1 is negative which gives that the diode path that demands the lowest value of v_1 is the path that will carry the magnetizing current.

If $v_1 < -2V_d$ then D_2 will conduct.

If $v_1 < -V_o$ then D will conduct.

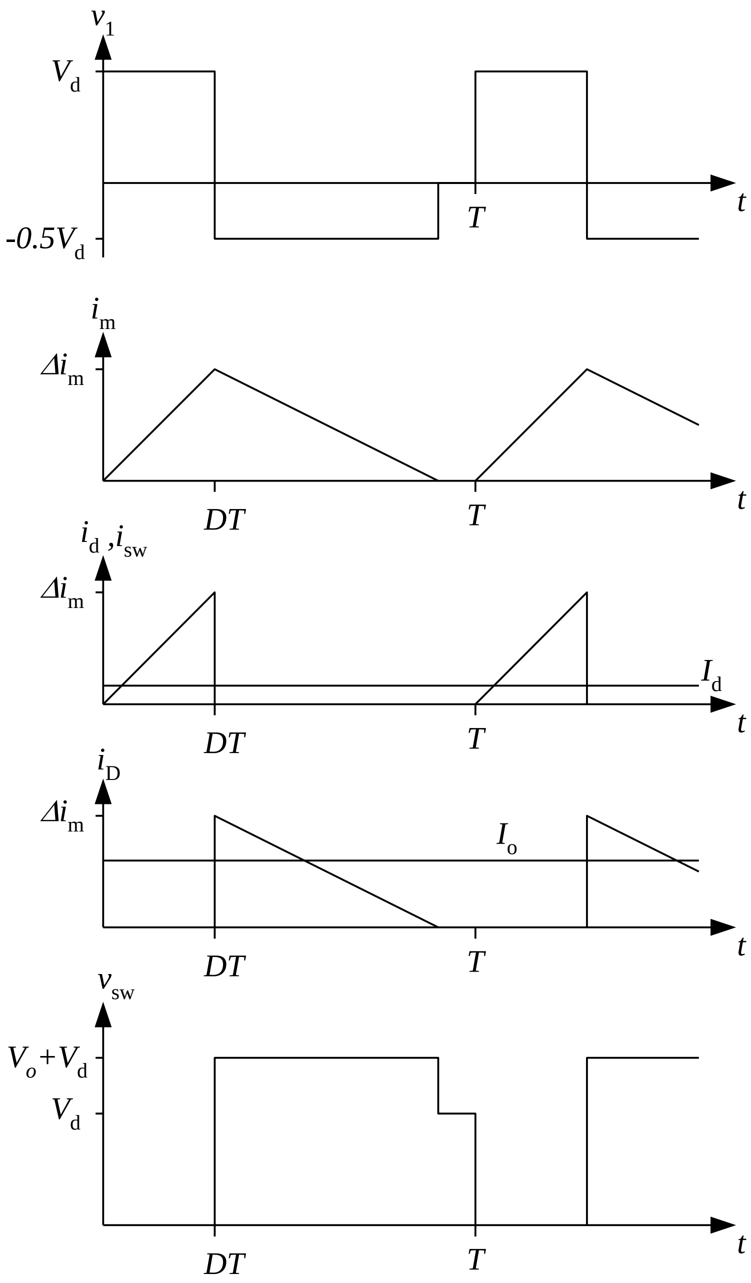
This means that the conduction of diode D_2 depends on how high the output voltage is; if $V_o < 2V_d$ then diode D will start to conduct and if $V_o > 2V_d$ then diode D_2 will start to conduct when the switch is opened. The output voltage is limited to twice the output voltage $(V_o = 2V_d)$ due to the fact that if the output voltage is higher than this no power can be transferred to the load side, which means that the capacitor will be discharged so that the output voltage gets lower than twice the input voltage$(2V_d)$.

In the following, it is therefore assume that the output voltage is lower than $2V_d$ which gives that diode D will start to conduct when the switch is opened. The voltage over winding v_1 can in this case be expressed as:

$$v_3 = -V_o \quad \rightarrow \quad \frac{v_1}{N_1} = \frac{v_3}{N_3} \quad \rightarrow \quad v_1 = \frac{N_1}{N_3} v_3 = -\frac{N_1}{N_3} V_o = -V_o$$

When the switch is non-conducting, the voltage over the magnetizing inductance is equal to the negative output voltage (constant voltage) which gives that the magnetizing current will decrease linearly to zero due to DCM. The voltage over this winding will only appear as long as there is energy stored in the inductor and there is a current flowing through it.

When all energy in the magnetizing inductance is dissipated, no voltage will appear over L_m and consequently neither over v_1. No voltage is neither applied from the secondary side since the diode on the secondary side (D) becomes reverse biased. This forces the voltage over the transformer to become zero during this phase. The waveforms in the converter can now be drawn as below.


v1
Vd
-0.5Vd
T
t
im
Δim
DT
T
t
id ,isw
Δim
Id
DT
T
t
iD
Δim
Io
DT
T
t
vsw
Vo+Vd
Vd
DT
T
t

We now calculate the voltage transfer ratio. This is done by assuming an ideal transformer and establishing that all energy stored in the transformer during on-time is transferred to the load during off-time. This gives:

$$W_{load} = W_{Lm} = \frac{1}{2} L i^2 = \frac{1}{2} L_m \Delta i_m^2$$

We calculate Δi_m by analyzing the voltage over the magnetizing inductance. During the on-time of the switch is the voltage constant and equal to the input voltage. This gives:

$$\Delta i_m = \frac{V_d DT}{L_m} = \frac{V_d D}{L_m f_s} \Rightarrow$$

$$W_{load} = \frac{L_m}{2} \left(\frac{V_d D}{L_m f_s} \right)^2$$

The output power is in steady state

$$P_o = W_{load} f_s = \frac{V_o^2}{R_{load}} \Rightarrow$$

$$\frac{L_m}{2} \left(\frac{V_d D}{L_m f_s} \right)^2 f_s = \frac{V_o^2}{R_{load}} \Rightarrow \frac{V_o^2}{V_d^2} = \frac{1}{2} \frac{D^2}{L_m f_s} R_{load} \Rightarrow$$

$$\frac{V_o}{V_d} = \sqrt{\frac{R_{load}}{2 L_m f_s}} D \text{ for } V_o \leq 2V_d$$

Extra problems based on problem 2

2.1. In the previous case $R_{load} = R_1, V_o = 0.5V_d, D = 0.3$. What happens if the load is changed to $R_{load} = 16R_1$ while the input voltage and the duty ratio are kept constant?

Solution

At first, the converter is operating at the point we assumed in the previous task when we drew the waveforms of the flyback converter ($R_{load} = R_1,\ V_o = 0.5V_d,\ D = 0.3$). Suddenly, the load is changed momentarily to a large value $R_{load} = 16R_1$.

Since the duty ratio and the input voltage are the same as before, the same amount of energy will be transferred to the load side each period just after the load resistance is changed. But since the load resistance is increased and the output voltage just after the change is the same as before the change (the voltage over a capacitor cannot change instantaneously) the load will not consume all energy. This means that the energy not consumed in the load each period will charge the capacitor which gives an increase in the output voltage.

We use the input /output voltage ratio obtained in previous task to calculate the new steady state output voltage if the resistance is increased 16 times.

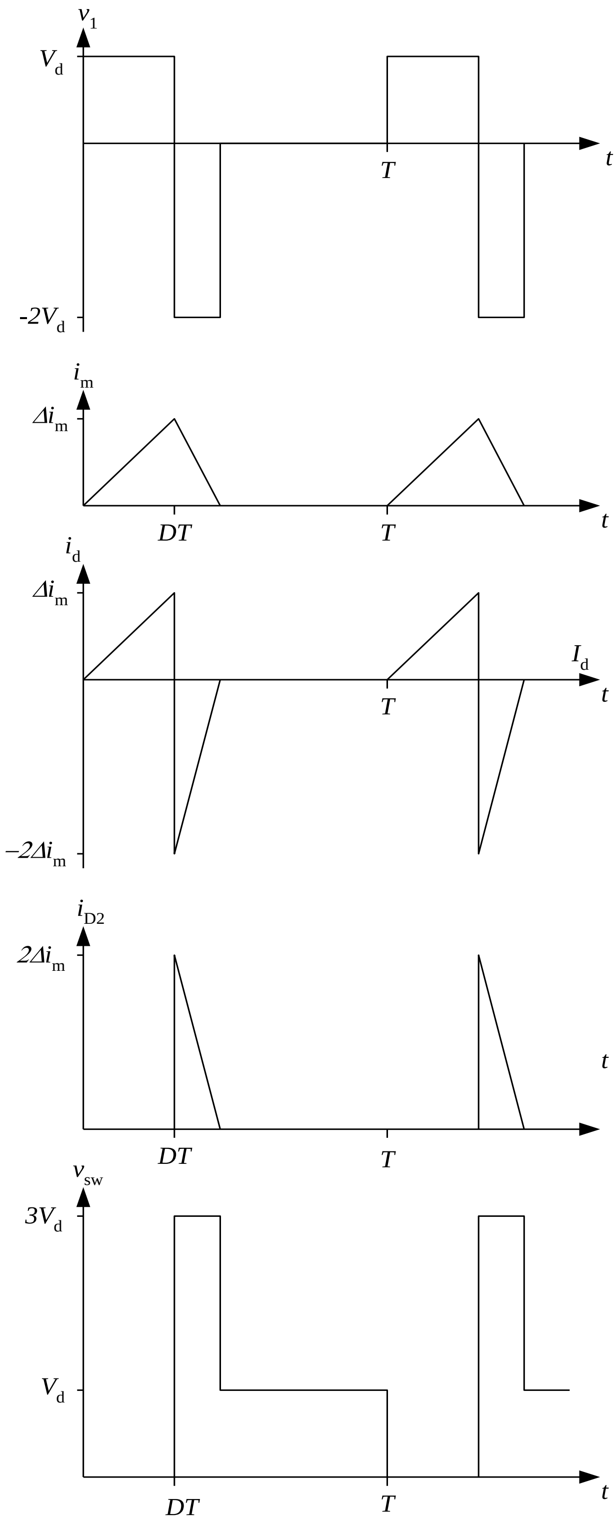
$$\left.\begin{aligned} D\sqrt{\frac{R_l}{2L_m f_s}} = \frac{V_{o1}}{V_d} = \frac{0.5V_d}{V_d} = 0.5 \\ D\sqrt{\frac{16R_l}{2L_m f_s}} = 4D\sqrt{\frac{R_l}{2L_m f_s}} = \frac{V_{o2}}{V_d} \end{aligned}\right\} \rightarrow \frac{V_{o2}}{V_d} = 4\cdot 0.5 \quad \rightarrow \quad V_{o2} = 2\cdot V_d$$

But from the previous task it is also known that the output voltage cannot be higher than twice the input voltage due to the winding N_2 and diode D_2. This gives that the output voltage will be limited to twice the input voltage by winding N_2 and diode D_2. So the energy needed to keep the output voltage at twice the input voltage will be transferred to the load each period. The remaining energy stored in the magnetizing inductance will be transferred back to the source through winding N_2 and diode D_2.

2.2. Draw the waveforms for the no load case, with $D = 0.3$.

Solution

For this case we know that the output voltage will be equal to the maximum output voltage, twice the input voltage.



$$\frac{v_1}{N_1} = \frac{v_2}{N_2} \Rightarrow$$
$$v_1 = \frac{N_1}{N_2} v_2 = \frac{1}{0.5} v_2 = 2v_2$$

$$i_{D2} N_2 = i_m N_1 \Rightarrow$$
$$i_{D2} = i_m \frac{N_1}{N_2} = i_m \frac{1}{0.5} = 2i_m$$

When the switch is off and $i_m > 0$
$$v_{sw} = V_d + V_o = V_d + 2V_d = 3V_d$$
when $i_m = 0$, $v_{sw} = V_d$

2.3. In what way does winding N_2 and diode D_2 limit the duty ratio in steady state?

Solution

We know that the turn ratio $(N_1\!:N_2\!:N_3)$ equal to $(1\!:0.5\!:1)$ limits the output voltage to twice the input voltage. This gives that the maximum voltage we can have to decrease the magnetizing current is twice the input voltage. But we only use the input voltage to increase the magnetizing current. This gives that we only need half the time to discharge the transformer that we use to charge it in steady state. This gives:

$$D_{max} + 0.5D_{max} = 1 \quad \rightarrow \quad D_{max} = \frac{2}{3}$$

If we have a duty ratio higher than this in steady state, the transformer will be saturated; the magnetizing current will increase all the time.

Note that this is independent if the converter is operating in DCM or in CCM.

2.4. Calculate when the flyback converter changes from DCM to CCM, assume that $V_o < 2V_d$.

Solution

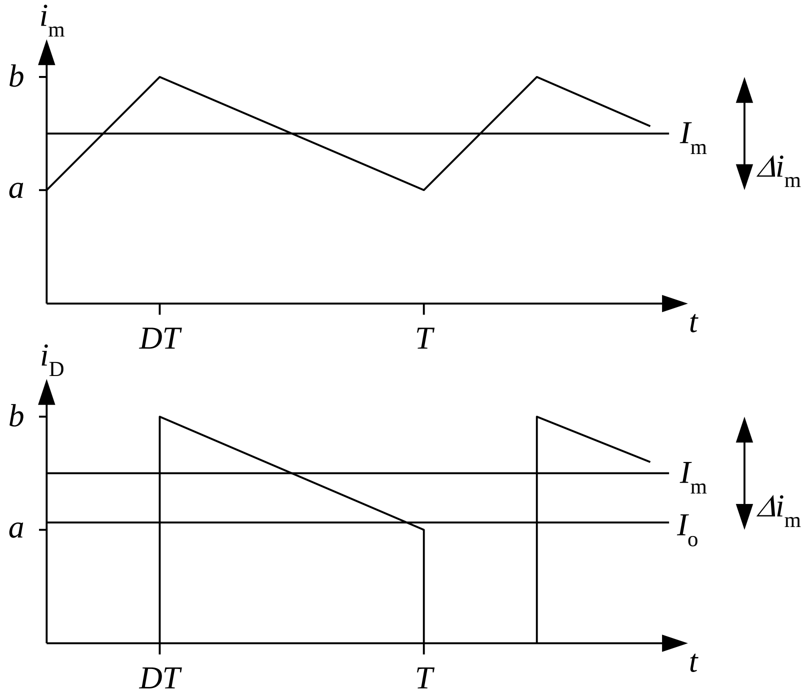
We assume CCM operation and steady state as usual. To find the input/output voltage relation we calculate the average magnetizing inductance voltage, which we know shall be zero in steady state.

$$V_{l(AVG)} = DV_d - V_o(1 - D) = 0 \quad \rightarrow \quad V_o = \frac{D}{1 - D}V_d$$

We also know that if the converter is operating in CCM, the current ripple must be smaller than twice the average magnetizing current ($\Delta i_m = 2I_m$). From previous tasks, we know that the current ripple can be expressed as:

$$\Delta i_m = \frac{V_d D T_{sw}}{L_m} = \frac{V_d D}{L_m f_{sw}}$$

Now we have to calculate the average magnetizing current (I_m). But at first we draw the magnetizing current (i_m) and the diode current (i_D) for CCM:



From the figure we see that the average magnetizing current can be calculated as:

$$I_m = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_m(t)dt = \frac{1}{T_{sw}}\int_0^{DT_{sw}} i_m(t)dt + \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T}DT + \frac{a+b}{2T}(1-D)T = \frac{a+b}{2}$$

$$I_o = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_D(t)dt = \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T}(1-D)T = \frac{a+b}{2}(1-D) = I_m(1-D)$$

This is a simple relation between the average output current and the average magnetizing current. The border between CCM and DCM can now be calculated as:

$$\Delta i_m = 2I_m \quad \rightarrow \quad \frac{V_d D}{L_m f_{sw}} = \frac{2I_o}{(1-D)}$$

If solving for D and considering that for CCM, the current ripple has to be smaller than twice the average magnetizing current, the final answer is obtained:

$$D(1-D) < \frac{2I_o L_m f_{sw}}{V_d}$$