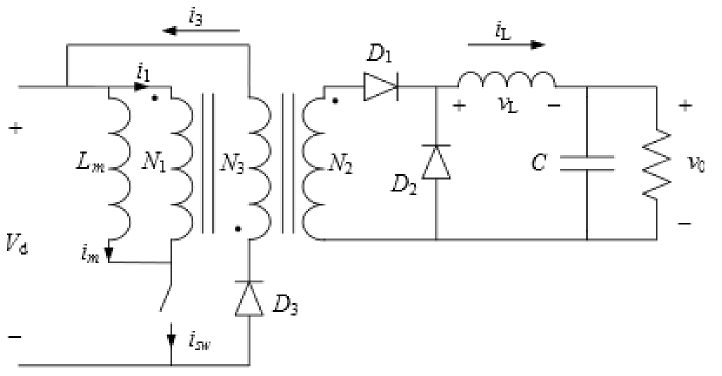
Solution of demonstration 6

Problem 1 Forward converter



N_1: N_3: $N_2 = 1: 1: 1$

$V_d = 50V$

$D = 0.4V$ $L_m = 22\mu H$

$C = 470\mu F$ $f = 200kHz$

$R_{load} = 1\Omega$ $L_{out} = 7.5\mu H$

1.1. Derive an expression for the ratio of the input and output voltage.
1.2. Plot the currents through all three diodes and the switch.
1.3. Calculate the power that is circulating to magnetize the transformer.
1.4. Derive an expression for the maximum allowed duty cycle.
1.5. Assume that each winding consists of 10 turns, $N_1 = N_2 = 10$, and calculate the maximum number of turns in the third winding, N_3, when the duty cycle is adjusted to achieve an output voltage of 30V.

Solution

1.1. Derive an expression for the ratio of the input and output voltage.

We start by assuming that the forward converter is operating in steady-state and CCM (continuous conduction mode). When the switch is turned on, D1 becomes forward biased and D2 reverse biased:

$$v_L = \frac{N_2}{N_1}V_d - V_o \qquad 0 < t < DT$$

When the switch is turned off, the inductor current, i_L, circulates through the diode D2:

$$v_L = -V_o \qquad DT < t < T$$

We know that the average voltage over the inductor must be zero when operating in steady-state, the average inductor voltage can be expressed as:

$$V_L = \frac{1}{T_{sw}}\int_0^{DT_{sw}} \frac{N_2}{N_1}V_d - V_o dt + \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} -V_o dt = \frac{1}{T_{sw}}\left(\frac{N_2}{N_1}V_d DT_{sw} - V_o DT_{sw} - V_o T_{sw} + V_o DT_{sw}\right) = 0$$

The ratio of the input and output voltage becomes then:

$$\frac{V_0}{V_d} = \frac{N_2}{N_1}D$$

1.2.Plot the currents through all three diodes and the switch.

We start by calculating the inductance average inductor current, I_L , and the current ripple in the inductor, Δi_L, in order to define the current waveform of the inductor current, i_L

$$I_L = I_C + I_0 = 0 + I_0$$

$$I_L = I_0 = \frac{V_0}{R_{load}} = \left\{V_0 = V_d \frac{N_2}{N_1} D = 50 \frac{1}{1} 0.4 = 20\right\} = \frac{20}{1} = 20\ A$$

When the switch is on:

$$\Delta i_L = \frac{v_L \Delta t}{L} = \frac{\left(\frac{N_2}{N_1} V_d - V_o\right) DT}{L} = \frac{(50-20)0.4}{7.5\mu \cdot 200k} = 8\ A$$

It can then be confirmed that the forward converter is operating in CCM as:

$$\frac{\Delta i_L}{2} < I_L \qquad\qquad \frac{8}{2} = 4 < 20$$

Further when the **switch is on**, there is a current into the dot-side of the first winding:

$$i_{sw} = i_m + i_1 = \{i_1 N_1 = i_2 N_2\} = i_m + \frac{i_2 N_2}{N_1} = i_m + i_2 = i_m + i_L$$

as the diode, D_2, is reverse biased by the voltage over the second winding, $V_2 = \frac{N_2}{N_1} V_d = 50\ V$
Consequently, $i_{D2} = 0A$ and $i_{D1} = i_L$ when the switch is on. The current in the magnetizing inductance, i_m, starts from zero when the switch is turned on and increases linearly up to Δi_m when the switch is turned off:

$$\Delta i_m = \frac{v_{Lm} \Delta t}{L_m} = \frac{V_d \Delta t}{L_m} = \frac{50 \cdot 0.4}{22\mu \cdot 200k} = 4.55\text{A}$$

When the **switch is off**, the current in the switch is zero:

$$i_{sw} = 0$$

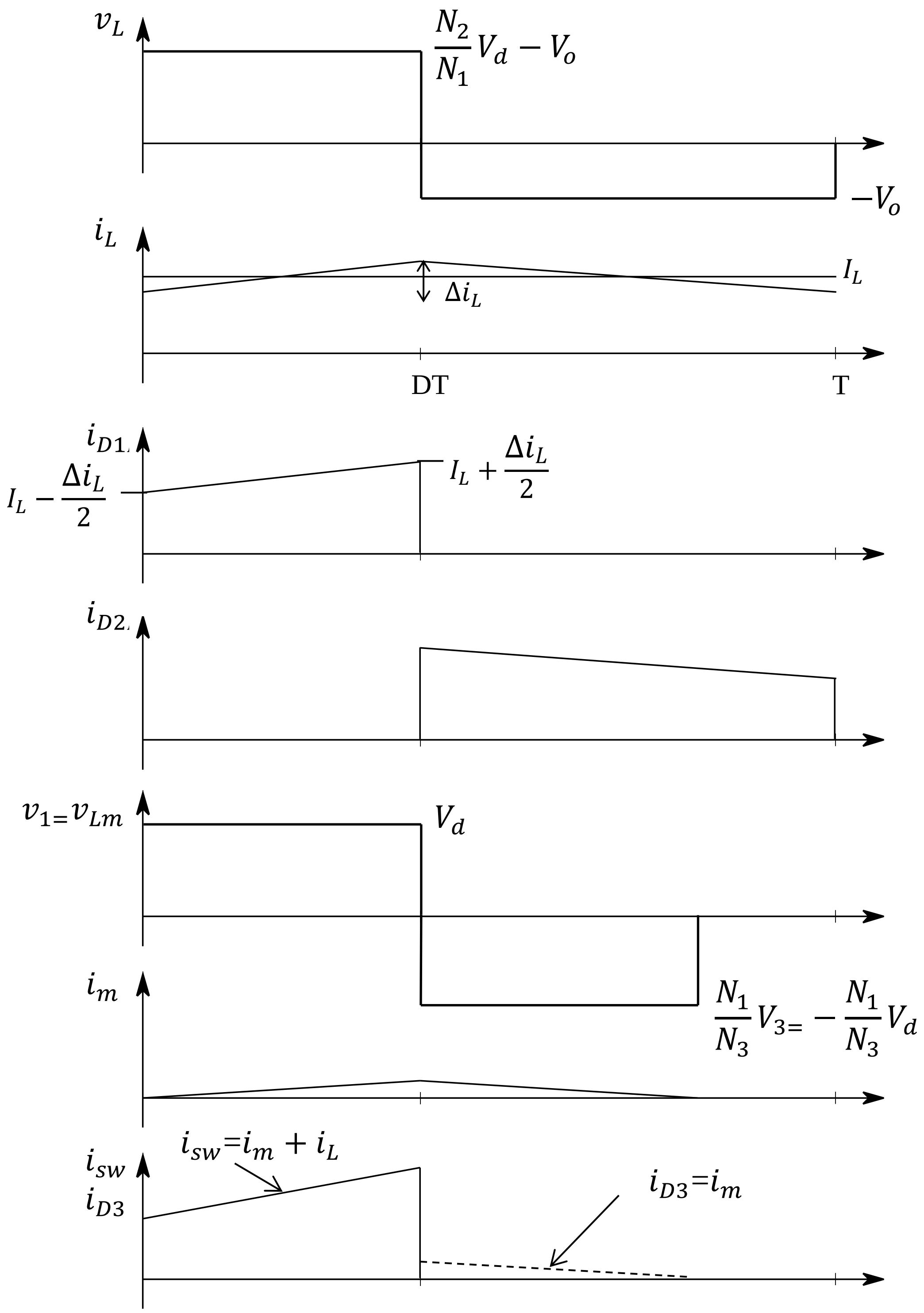
but magnetic energy has been stored in the magnetizing inductance, L_m, and the current can't be interrupted instantaneously. Consequently, the voltage needed to maintain the current in the magnetizing inductance, L_m, will be put up over the first winding by the magnetizing inductance. There will be a current into the side without a dot in the first winding, N_1. According to the theory of ideal transformers (see Demonstration 4), there must be a current out from another winding on the side without a dot. The diode, D_1, will block a current out from a dot in the second winding, N_2.Thereby, the current in the first winding, N_1, must be linked to the third winding, N_3, and the demagnetize the magnetizing inductance, L_m, towards the source.

The current in the diode, D_3 ,is then

$$i_{D3} = i_3 = -i_1 \frac{N_1}{N_3} = i_m \frac{N_1}{N_3} = i_m$$

where the minus sign is due to defined current directions.

We can now plot the waveforms:



1.3. Calculate the power that is circulating to magnetize the transformer.

In the formula sheet, we can found:

$$W_{Lm} = \frac{1}{2} L i^2$$

which corresponds to the circulating energy required to magnetize the transformer. The corresponding power can be calculated as:

$$P_{Lm} = W_{Lm} f_{sw} = \frac{1}{2} L_m \Delta i_m^2 f_{sw} = \frac{1}{2} L_m \left(\frac{v_{Lm} \Delta t}{L_m} \right)^2 f_{sw} =$$
$$= \frac{1}{2} \frac{1}{L_m} (v_{Lm} \Delta t)^2 f_{sw} = \frac{1}{2 \cdot 22\mu} \cdot (50 \cdot 0.4)^2 \cdot 200k = 45W$$

The circulating power is more than 10% of the transferred power to the load

$$P_o = \frac{V_o^2}{R_{load}} = \frac{20^2}{1} = 400\ W$$

The circulating power is of no use and will create losses. To minimize the circulating power and thereby the losses, the transformer should be designed with an high magnetizing inductance, L_m, as is seen in the final expression of the circulating power:

$$P_{Lm} = \frac{1}{2} \frac{1}{L_m} (v_{Lm} \Delta t)^2 f_{sw}$$

The energy to magnetize a transformer of a forward converter should not be confused with the energy stored in a transformer of a flyback convert. In the forward converter, the energy is just circulating from the source and back to the source again, while it is stored during one part of the switching period and released to the load in the other part of a switching period in case of a flyback converter.

1.4. Derive an expression for the maximum allowed duty cycle.

The magnetizing inductance must be fully demagnetized each switching cycle, L_m. Considering that the current is zero before the switch is turned on, the maximum allowed duty cycle can be found by the fact that the average voltage of the magnetizing inductance must be zero in order to fully demagnetize.

$$0 = V_{Lm} = \frac{1}{T}\int_0^T v_{Lm} dt = \frac{1}{T}\int_0^{D_{max}T} V_d dt + \frac{1}{T}\int_{D_{max}T}^{T} -\frac{v_3}{N_3} N_1 dt =$$

$$= \frac{1}{T}\int_0^{D_{max}T} V_d dt + \frac{1}{T}\int_{D_{max}T}^{T} -\frac{V_d}{N_3} N_1 dt = \frac{1}{T}\left(V_d D_{max}T + (T - D_{max}T)\left(-\frac{V_d}{N_3}N_1\right)\right) =$$

$$= V_d D_{max} - (1 - D_{max})\left(\frac{V_d}{N_3}N_1\right) = 0$$

$$\Rightarrow \quad D_{max} = (1 - D_{max})\left(\frac{N_1}{N_3}\right)$$

which can be rewritten as

$$D_{max} = \frac{1}{1 + N_3/N_1} = \frac{1}{1 + 1/1} = 0.5$$

1.5. Assume that each winding consists of 10 turns, $N_1: N_2: N_3 = 10: 10: 10$, and calculate the maximum number of turns in the third winding, N_3, when the duty cycle is adjusted to achieve an output voltage of 30V.

The turns ratio $N_1: N_2$ is still 1:1 and a the new duty cycle can be found

$$\frac{V_0}{V_d} = \frac{N_2}{N_1} D \quad \Rightarrow \quad D = \frac{V_0}{V_d}\frac{N_1}{N_2} = \frac{30}{50} = 0.6$$

The new average conductor current can be calculated

$$I_L = I_0 = \frac{V_0}{R_{load}} = \frac{30}{1} = 30\ A$$

and the current ripple in the inductor

$$\Delta i_L = \frac{v_L \Delta t}{L} = \frac{\left(\frac{N_2}{N_1}V_d - V_o\right)DT}{L} = \frac{(50-30)0.6}{7.5\mu \cdot 200k} = 16\ A$$

The assumption of CCM is still valid, since

$$\frac{16}{2} = 8 < 40$$

The previous expression of D_{max} can be rewritten as

$$N_3 = (1 - D)\left(\frac{N_1}{D}\right) = N_1\frac{1 - D}{D} = 10\frac{1 - 0.6}{0.6} = 6.67\ turns$$

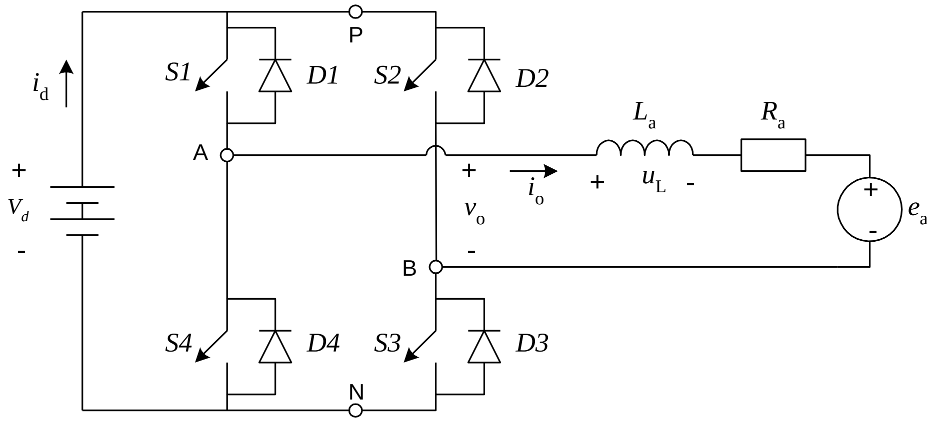
The number of turns must be chosen as a discrete number and 6 turn is the maximum that we can choose.

$$D_{max} = \frac{1}{1 + N_3/N_1} = \frac{1}{1 + 6/10} = 0.625$$

$$D_{max} = \frac{1}{1 + N_3/N_1} = \frac{1}{1 + 7/10} = 0.59$$

Problem 2 (P7-18 in Undeland book)

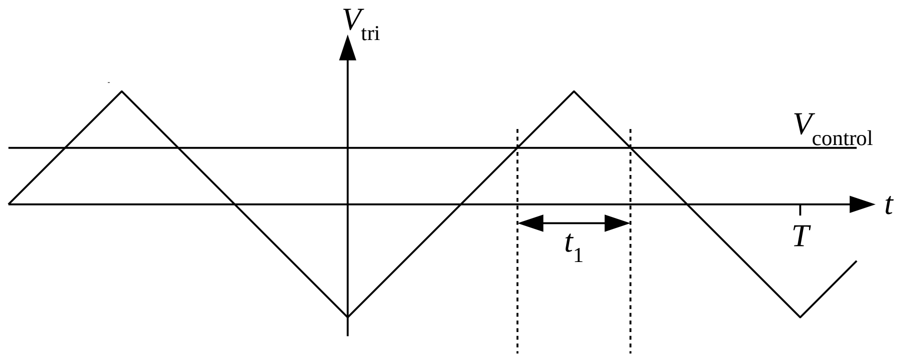
In a full-bridge DC/DC converter using PWM bipolar voltage switching, $v_{control} = 0.5\hat{V}_{tri}$. Obtain V_o and I_d in terms of given V_d and I_o. Assume $i_o \cong I_o$.



By Fourier analysis, calculate the amplitude of the switching-frequency harmonics in the output voltage (v_o) and in the input current (i_d).

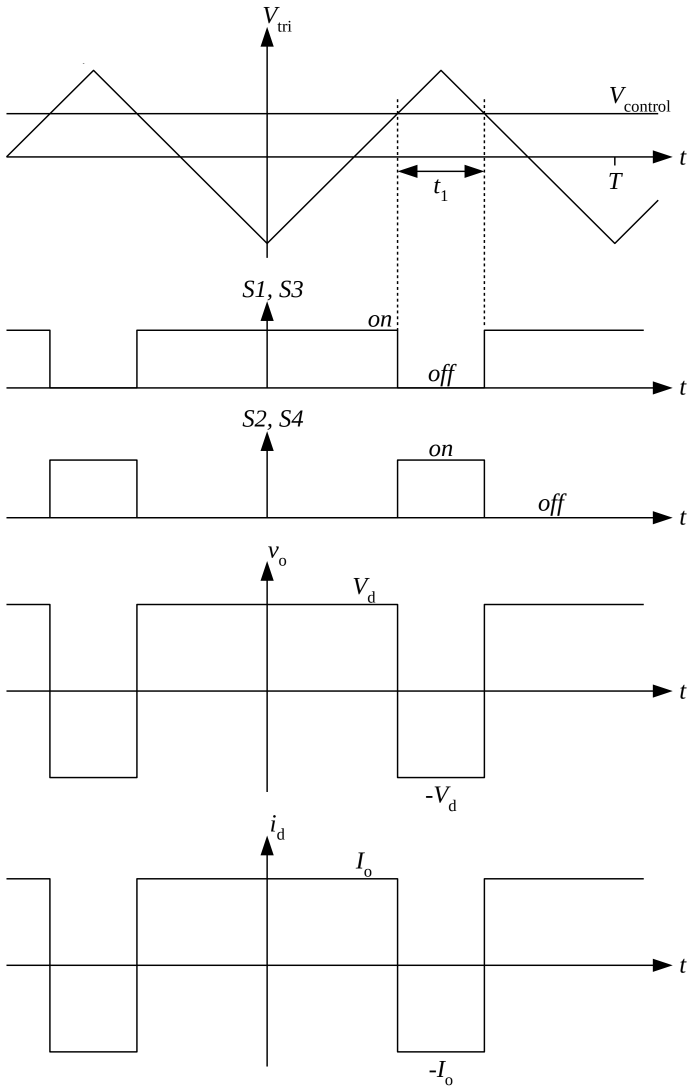
Solution

The control of a full-bridge converter is done by comparing a triangular reference waveform, which has the same frequency as the switching frequency, with a reference voltage, see figure below.



When the triangle voltage exceeds the reference voltage, one pair of switches is turned on while the other pair of switches is turned off. As the triangle voltage falls under the reference voltage again, the switches changes position again.

We start by plotting the voltages and current waveforms for the converter. Note that since the output current is assumed to be a constant DC-current $(i_o \cong I_o)$, a purely resistive load is assumed. As a consequence of the constant load current, the input current (i_d) must change polarity for each switching cycle, see figure on next page.



From the figure we see that the time t_1 can be expressed as

$$t_1 = \frac{T}{2\hat{V}_{tri}}\left(\hat{V}_{tri} - v_{control}\right)$$

This can be tested by inserting some time values at which we know the value.

$$v_{control} = \hat{V}_{tri} \quad \rightarrow \quad t_1 = 0$$
$$v_{control} = -\hat{V}_{tri} \quad \rightarrow \quad t_1 = T$$

We now calculate the average output voltage

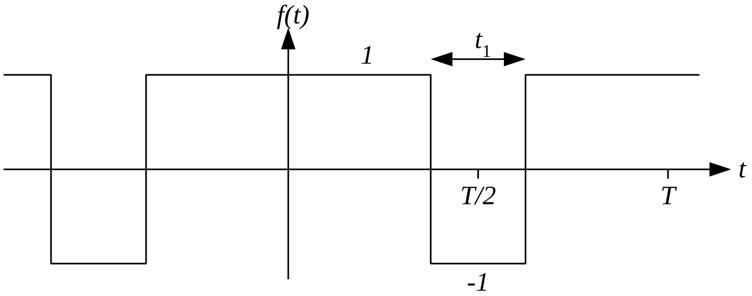
$$V_o = \frac{1}{T}\left(V_d(T - t_1) - V_d t_1\right) = V_d - \frac{2}{T}V_d t_1 = V_d - \frac{2}{T}V_d\frac{T}{2\hat{V}_{tri}}\left(\hat{V}_{tri} - v_{control}\right) = V_d\frac{v_{control}}{\hat{V}_{tri}}$$

$$V_o = V_d\frac{v_{control}}{\hat{V}_{tri}} = V_d\frac{0.5 \cdot \hat{V}_{tri}}{\hat{V}_{tri}} = 0.5 \cdot V_d$$

In the same way we calculate the average input current, since the shapes of the waveforms are the same the equation for the average will be the same

$$I_d = I_o \frac{v_{control}}{\hat{V}_{tri}} = 0.5 \cdot I_o$$

We now calculate the amplitudes of the switch harmonics, we do this for the waveform v_o/V_d, this due to that this waveform is identical with i_d/I_o which we also shall calculate. In this way we only have to calculate one Fourier series. We first plot the waveform



From the figure we see that the function $f(t)$ is an even function → $b_n = 0$ for all n:

$$a_n = \frac{4}{T}\int_{t_0}^{t_0+T/2} f(t)\cos(n\omega t)\,dt \qquad \text{(for all } n\text{)}$$

$$a_n = \frac{4}{T}\int_{0}^{\frac{T-t_1}{2}} \cos(n\omega t)\,dt - \frac{4}{T}\int_{\frac{T-t_1}{2}}^{\frac{T}{2}} \cos(n\omega t)\,dt = \frac{4}{Tn\omega}\left(\left[\sin(n\omega t)\right]_0^{\frac{T-t_1}{2}} - \left[\sin(n\omega t)\right]_{\frac{T-t_1}{2}}^{\frac{T}{2}}\right) =$$

$$= \left[\omega = \frac{2\pi}{T}\right] = \frac{2}{n\pi}\left(\sin\left(n\omega\frac{T-t_1}{2}\right) + \sin\left(n\omega\frac{T-t_1}{2}\right) - \sin\left(n\omega\frac{T}{2}\right)\right) =$$

$$= \frac{2}{n\pi}\left(2\sin\left(n\pi - \frac{n\pi t_1}{2}\right) - \sin(n\pi)\right) = \frac{4}{n\pi}\sin\left(n\pi - n\pi\frac{\hat{V}_{tri} - v_{control}}{2\hat{V}_{tri}}\right) =$$

$$= \frac{4}{n\pi}\left(\underbrace{\sin(n\pi)}_{0 \text{ for all } n}\cos\left(n\pi\frac{\hat{V}_{tri} - v_{control}}{2\hat{V}_{tri}}\right) - \cos(n\pi)\sin\left(n\pi\frac{\hat{V}_{tri} - v_{control}}{2\hat{V}_{tri}}\right)\right) =$$

$$= \frac{-4\cos(n\pi)}{n\pi}\sin\left(n\pi\frac{\hat{V}_{tri} - v_{control}}{2\hat{V}_{tri}}\right) = \frac{-4(-1)^n}{n\pi}\sin\left(n\pi\frac{\hat{V}_{tri} - v_{control}}{2\hat{V}_{tri}}\right) = \frac{-4(-1)^n}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

The harmonics can now be calculated:

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$\left\lvert\frac{v_{o(n)}}{V_d}\right\rvert = \left\lvert\frac{i_{d(n)}}{I_o}\right\rvert$	0.9	0.64	0.30	0	0.18	0.21	0.13	0	0.10