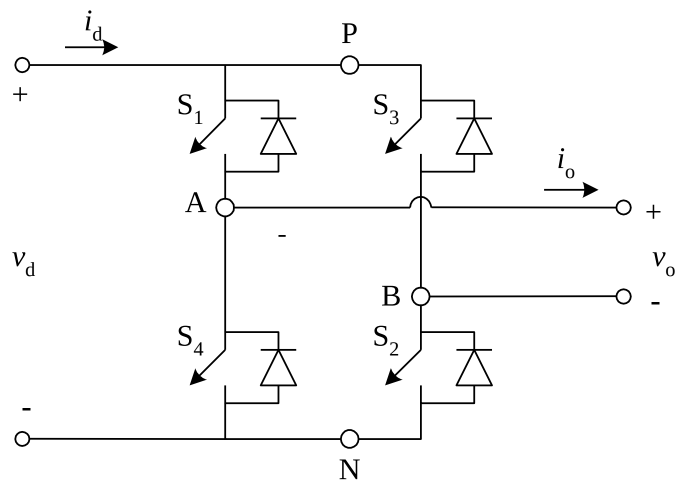
Solution of demonstration 7



Problem 1 (P8-1 in Undeland book)

In a single-phase full-bridge PWM inverter, the input dc voltage varies in a range between 295V and 325V. Because of the low distortion required in the output (v_o), the modulation index is lower than 1 ($m_a \leq 1.0$).

What is the highest voltage of the fundamental frequency ((1)) that can be possible obtained? What is the voltage rating that should be stamped on the nameplate?

Solution

From the definition of the amplitude modulation ratio we know that the modulation ratio can be expressed as:

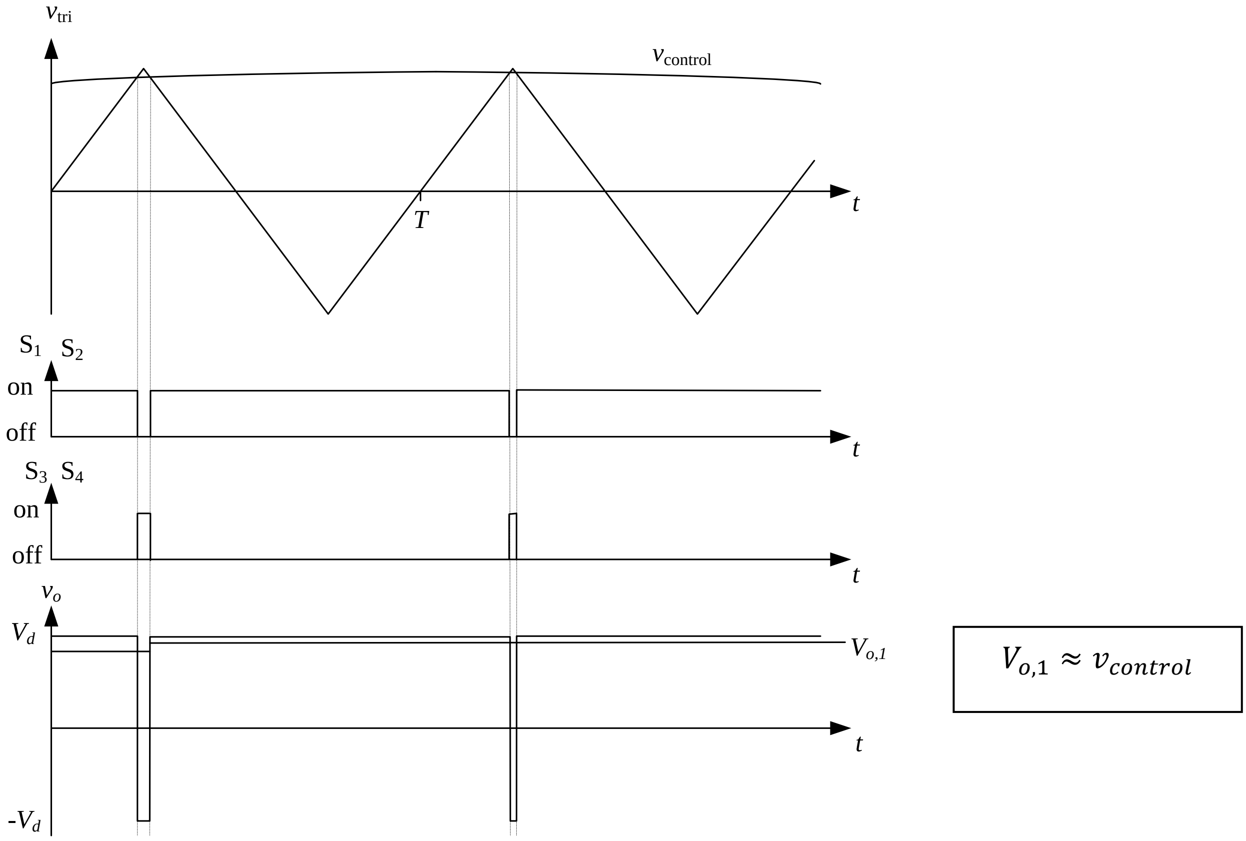
$$m_a = \frac{\hat{v}_{control}}{\hat{v}_{tri}}$$

We also know that this converter is made for creating an ac-voltage, therefore can the control voltage be expressed as:

$$v_{control} = \hat{v}_{control} \sin(\omega t - \phi)$$

We start by drawing the waveforms of the triangle voltage and the output voltage. We assume that the switching frequency is much greater than the fundamental output frequency $\left(f_{sw} \gg f_{fundamental}\right)$.

What happens when the control voltage ($\hat{v}_{control}$) gets closer to the peak value of the triangle wave ($\hat{v}_{tri}$)? Due to the limitation set in the task, the control voltage can only reach $\hat{v}_{control} = \hat{v}_{tri}$ since $m_a \leq 1.0$.



When $m_a = 1.0$ no switching will occur; the output voltage is equal to the input voltage since the switches are open all the time. This gives that the peak of the fundamental that can be produced in the inverter equals to the input voltage.

$$V_d = \hat{V}_{o(\,1)\,max}$$

Since the switching frequency is much greater than the fundamental frequency $\left(f_{sw} \gg f_{fundamental}\right)$, highest voltage of the fundamental frequency ((1)) that can be possible obtained is

$$V_{o(1)\,max} = \frac{1}{\sqrt{2}} V_{d,max} = \frac{1}{\sqrt{2}} 325V = 230V$$

The maximum rated output voltage must be:

$$V_{o(1)\,rated} = \frac{1}{\sqrt{2}} V_{d,min} = \frac{1}{\sqrt{2}} 295V = 209V$$

Note that when dealing with ac-quantities in an electric power context, it is always specified as an RMS-value unless other is specified. The rated voltage, the highest that can stamped on the nameplate, must be a voltage level that the inverter can deliver at all conditions. Therefore, the worst case scenario is for the lowest input dc voltage, $V_{d,min}$.

Problem 2

The single phase inverter above is operating in square wave mode with a load that consists of an inductor $(L = 50mH)$ in series with a sinusoidally shaped back-emf voltage source $\left(e_o = \sqrt{2} \cdot E_o \cdot \sin(\omega_1 t)\right)$. The output fundamental voltage is $V_{o(1)}$ has a frequency of 40Hz and the same amplitude and phase as the back-emf. The input dc voltage is kept constant at 300V.

2.1. Calculate the fundamental voltage $V_{o(1)}$.
2.2. Sketch the current (ripple) waveform to the load.
2.3. Calculate the peak value of the current.

Solution

1. Calculate the fundamental voltage, $V_{o(1)}$.

The output voltage fulfills both the odd and the half-wave conditions, it is therefore considered to be a odd quarter-wave symmetry.

For odd n:

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_d sin(n\theta)d\theta = -\frac{4}{\pi}\frac{V_d}{n}\left[cos(n\theta)\right]_0^{\frac{\pi}{2}} = \frac{4}{\pi}\frac{V_d}{n}\left(1 - \underbrace{cos\left(n\frac{\pi}{2}\right)}_{=0\ for\ odd\ n}\right) = \frac{4}{\pi}\frac{V_d}{n}$$

So the fundamental voltage, $V_{o(1)}$, can be calculated as

$$\hat{V}_{o(1)} = \frac{4V_d}{\pi} = \frac{4 \cdot 300}{\pi} = 382\ V$$
$$V_{o(1)} = \frac{1}{\sqrt{2}} \cdot 382V = 270\ V$$

2. Sketch the current (ripple) waveform to the load.
The voltage over the inductor can be expressed as

$$v_L(t) = v_0(t) - e_0(t) = v_0(t) - \hat{V}_{o(1)} \cdot sin(\omega_1 t)$$

The first thing we observe is that there is no fundamental component in the current, as the back-emf has both the same amplitude and phase as the fundamental from the square-wave. That’s why we are asked to sketch the current (ripple) waveform to the load. The current derivative can be expressed as

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

which gives that the current has a positive derivative when the voltage over the inductance is positive and a negative derivative when the voltage over the inductance is negative.

By plotting the voltage over the inductor, $v_L(t) = v_0(t) - e_0(t)$, we can see that there is an even symmetry around $\pi/2$ and $3\pi/2$. Further, there is an odd symmetry around $\theta = 0$ and $\theta = \pi$.
As there is an odd symmetry in the voltage, $v_L(\theta)$, around $\theta = \pi$, and we consider a steady state

$$\int_0^{\pi} v_L d\theta = -\int_{\pi}^{2\pi} v_L d\theta$$

The inductor must be magnetized and demagnetized equally amount during the two intervals. From that it follows that the current must be even around $\theta = \pi$. In the same manner, the current is odd around $\theta = \pi/2$ and $\theta = 3\pi/2$, where the voltage has its even symmetries.

We can now sketch the current wave form. There are four areas where the absolute value of the voltage over the inductor are the same (denoted A in the figure). During these interval, the current must increase or decrease the same Δi_L. There are two more areas where absolute value of the voltage are the same (denoted B in the figure). The current must increase or the decrease the same during this two interval as well.

3. Calculate the peak value of the current.
In the general case, the inductor current can be expressed as

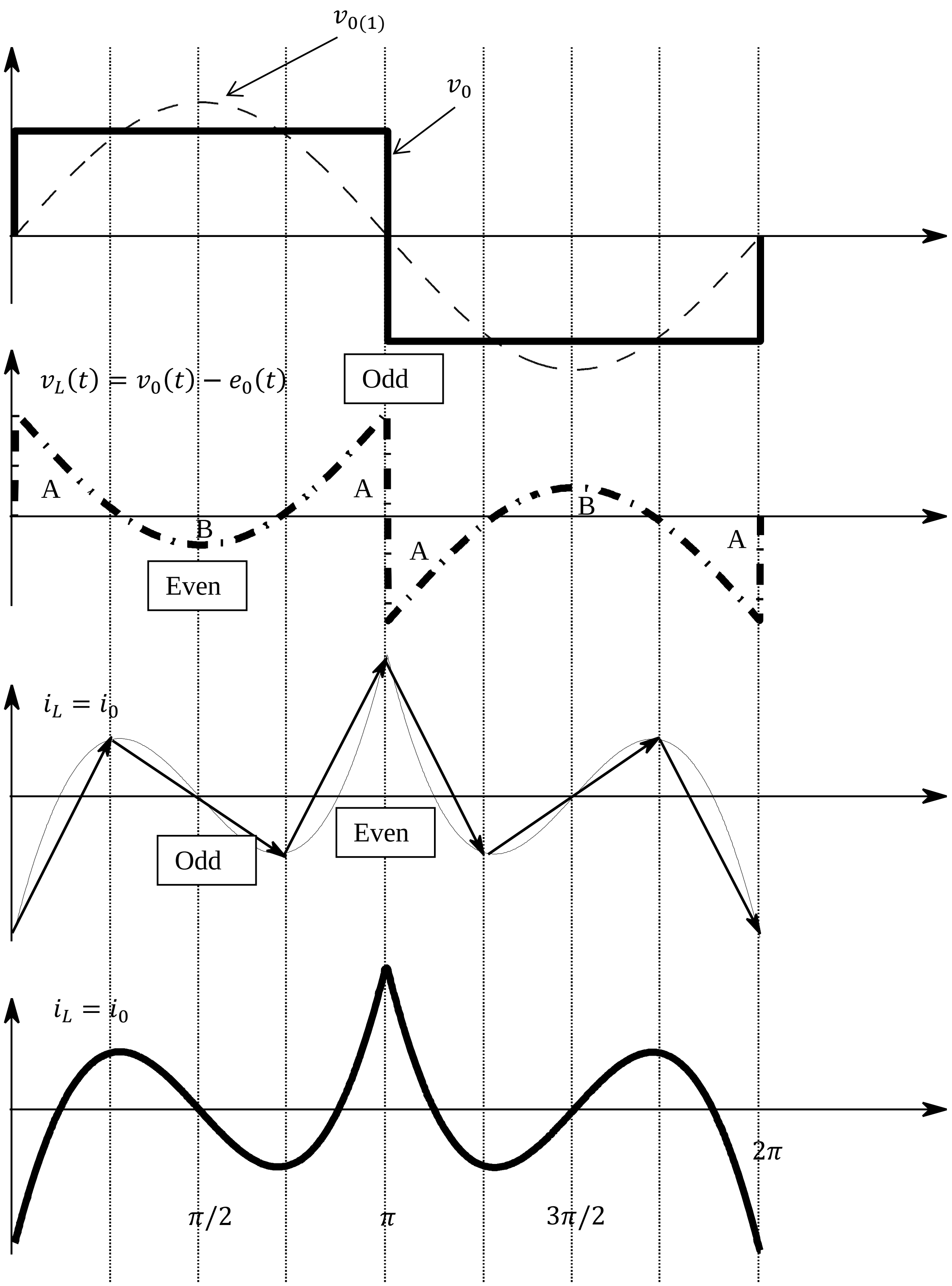
$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t_0+t} v_L \, dt$$

From the curve sketched previously, we observed that current wave form is odd around $\theta = \pi/2$, why $i_L(\pi/2) = 0$. We also observed that the current peak occurred at $\theta = \pi$. The peak value of the current can then be calculated as

$$i_{0-peak} = i_{L-peak} = i_L(\pi/2) + \frac{1}{\omega L}\int_{\pi/2}^{\pi} v_L(\omega t) \, d\omega t =$$

$$= 0 + \frac{1}{\omega L}\int_{\pi/2}^{\pi} \left(V_d - \hat{V}_{o(1)} \cdot sin(\omega t)\right) \, d\omega t = \frac{1}{\omega L}\int_{\pi/2}^{\pi} \left(V_d - \frac{4V_d}{\pi} \cdot sin(\omega t)\right) \, d\omega t =$$

$$= \frac{V_d}{\omega L}\left[\omega t + \frac{4}{\pi} cos(\omega t)\right]_{\pi/2}^{\pi} = \frac{V_d}{\omega L}\left(\pi + \frac{4}{\pi}(-1) - \frac{\pi}{2} - \frac{4}{\pi} \cdot 0\right) = \frac{300}{2\pi \cdot 40 \cdot 50m}\left(\frac{\pi}{2} - \frac{4}{\pi}\right) = 7.1 \, A$$

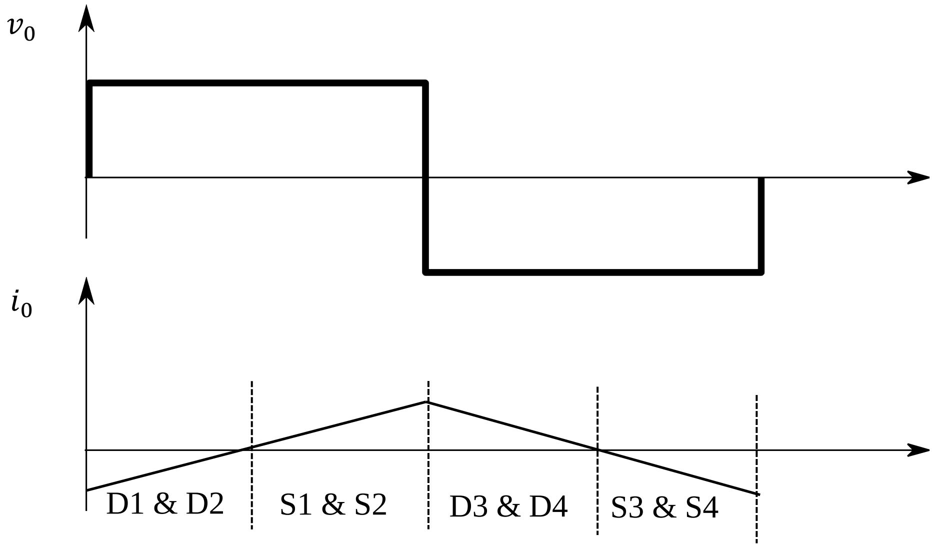

$v_{0(1)}$
v_0
$v_L(t) = v_0(t) - e_0(t)$
Odd
A
A
B
A
B
A
Even
$i_L = i_0$
Odd
Even
$i_L = i_0$
2π
$\pi/2$
π
$3\pi/2$

Problem 3

The single phase inverter above is operating in square-wave operation mode with a purely inductive load. For one switching period, draw the output voltage and the output current. Clearly mark which transistor/diode that conducts the current.

Solution

The voltage waveform is the same as in the previous tasks. Since the load is purely inductive, the current increases linearly when the output voltage is positive and decreases linearly when the output voltage is negative, the average must be zero.

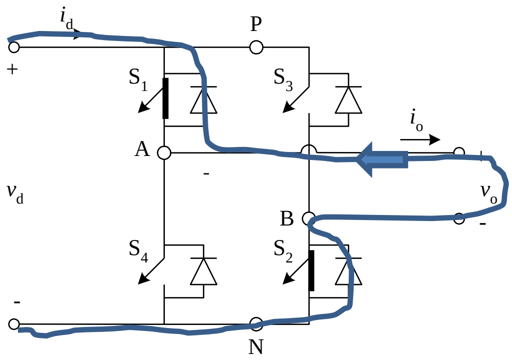


In the figure, we can observe four different intervals, or cases. The voltage is positive or negative at the same time as the current is negative or positive. The following table can be achieved using the procedure and the circuit diagrams on the next page.

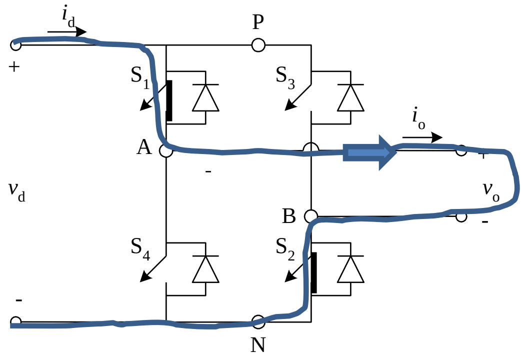
Case	v_0	i_0	Conducting devices
1	+	-	**D1, D2**
2	+	+	**S1, S2**
3	-	+	**D3, D4**
4	-	-	**S3,S4**

1) The voltage waveform gives which switch-pair that is ON and OFF respectively.
2) From the current waveform, we can draw the direction of the current.
3) Once the switch-states and the current directions are known, the current paths can be found by means of elimination.

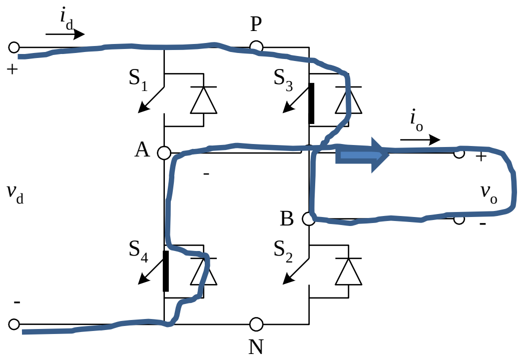
Case 1: $v_o >$ & $i_o < 0$



Case 2: $v_o >$ & $i_o > 0$



Case 3: $v_o <$ & $i_o > 0$



Case 4: $v_o <$ & $i_o < 0$