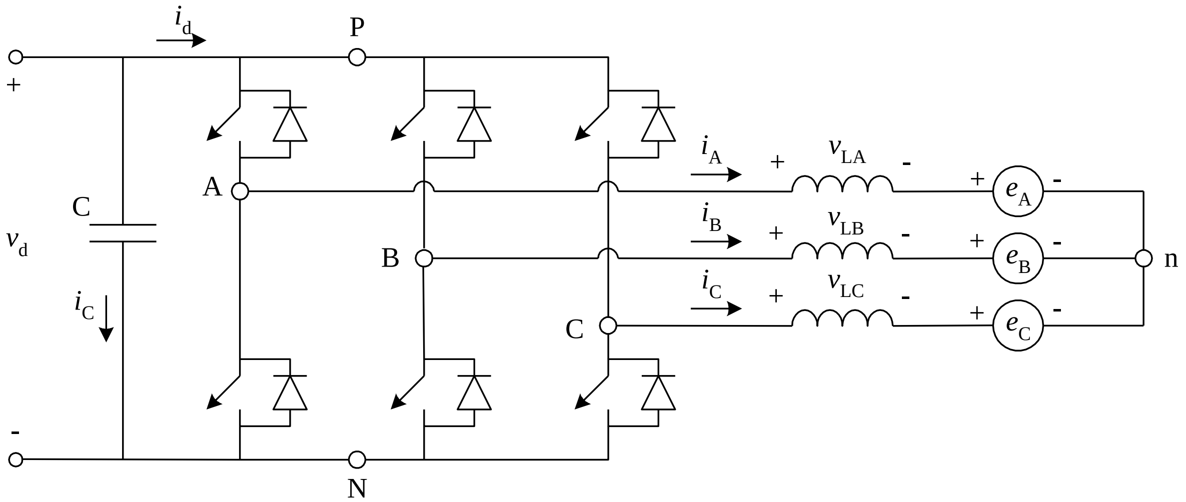
Solution of demonstration 8

Problem 1 (P8-7 in Undeland book)

Consider the problem of ripple in the output current of a three-phase square-wave inverter. $V_{LL(1)} = 200V$ at a frequency of 52Hz and the load is a three-phase ac motor with $L = 100mH$. Assume the back-emf has the same amplitude and phase as the output fundamental voltage.



Calculate the peak ripple current.

Solution

We should calculate the peak current ripple through the inductances due to the fact that it is the voltage over the inductor that forms the current. To be able to do this we must first start by finding the voltage over respective inductor: $V_{LA}(t)$, $V_{LB}(t)$ and $V_{LC}(t)$.

Therefore it can be a good idea to start by finding the voltage over the phases of the load $\left(V_{An}(t), V_{Bn}(t) \text{ and } V_{Cn}(t)\right)$. We start by drawing the voltages between the phases and N. Since the converter is operating as a square-wave inverter we know that the switches connected to P are on 50% of the period and the switches connected to N are on for the rest of the period. We also know that it is a three-phase inverter, which means that there is a 120° phase shift between the phases.

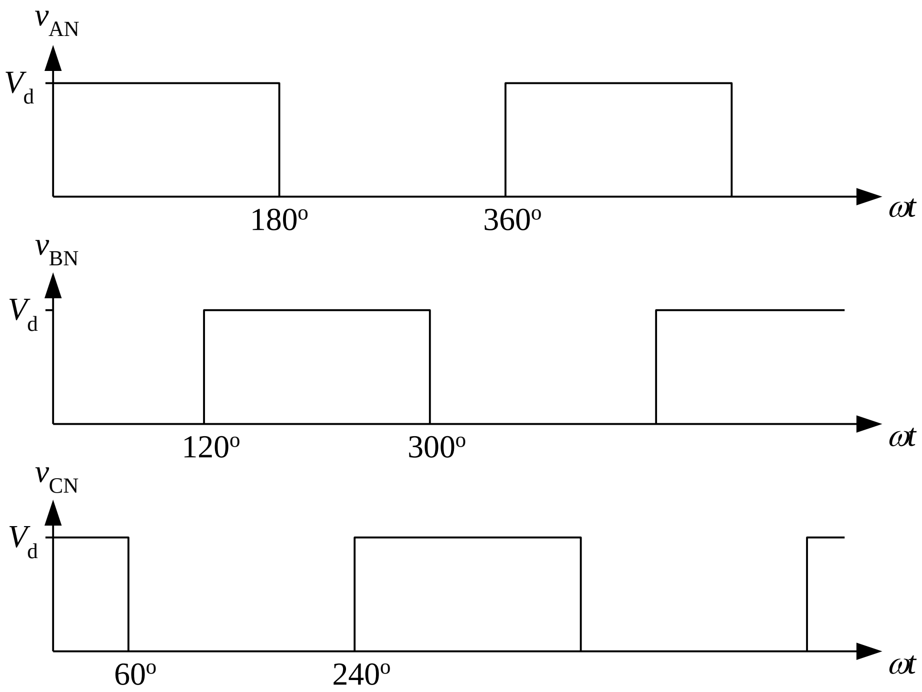
We know that in a three-phase system, the sum of all currents and voltage must be equal to zero. If the sum of the currents are zero, the sum of the current derivatives must also be zero.

$$i_A + i_B + i_C = 0 \qquad (1)$$

and

$$\frac{d(i_A + i_B + i_C)}{dt} = 0 \qquad (2)$$

The voltage in each phase leg (V_{AN}, V_{BN} and V_{CN}) can be drawn as:



The load is assumed to be balanced ($L_A = L_B = L_C$), which gives that the sum of the back-emf's must be equal to zero:

$$e_A + e_B + e_C = 0. \tag{3}$$

From the figure it can be seen that the phase-to-neutral voltage in each phase can be expressed as:

$$\begin{Bmatrix} v_{An} = v_{AN} - v_{nN} \\ v_{Bn} = v_{BN} - v_{nN} \\ v_{Cn} = v_{CN} - v_{nN} \end{Bmatrix} \tag{4}$$

The voltage over the load is also the sum of the voltage over the load inductor plus the back-emf.

$$\begin{Bmatrix} v_{An} = L\frac{di_A}{dt} + e_A \\ v_{Bn} = L\frac{di_B}{dt} + e_B \\ v_{Cn} = L\frac{di_C}{dt} + e_C \end{Bmatrix} \tag{5}$$

We now sum up the phase voltages over the load (equation (5) are summed together):

$$v_{An} + v_{Bn} + v_{Cn} = L\frac{di_A}{dt} + e_A + L\frac{di_B}{dt} + e_B + L\frac{di_C}{dt} + e_C = e_A + e_B + e_C + L\left(\frac{di_A}{dt} + \frac{di_B}{dt} + \frac{di_C}{dt}\right) = 0$$

Hence, it is proven that the sum of all phase-to-neutral voltages equals to zero. If equation (4) is solved for v_{nN} and summed together, it is concluded that:

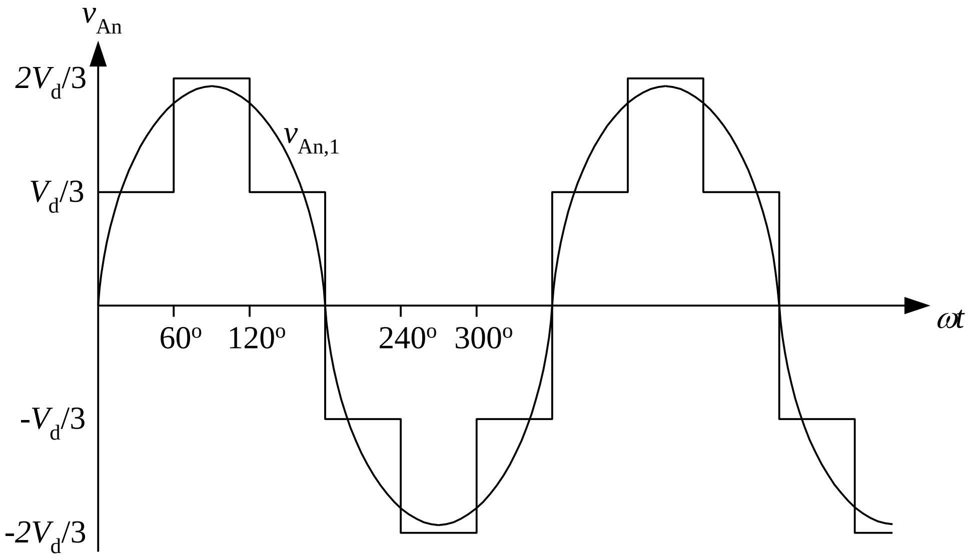
$$3v_{nN} = v_{AN} - v_{An} + v_{BN} - v_{Bn} + v_{CN} - v_{Cn} = v_{AN} + v_{BN} + v_{CN} - \underbrace{(v_{An} + v_{Bn} + v_{Cn})}_{=0} =$$

$$3v_{nN} = v_{AN} + v_{BN} + v_{CN} \tag{6}$$

We can now calculate the voltage over one phase of the load as (equations (4) and (6) together gives)

$$v_{An} = v_{AN} - v_{nN} = v_{AN} - \frac{v_{AN} + v_{BN} + v_{CN}}{3} = \frac{2}{3} v_{AN} - \frac{1}{3}(v_{BN} + v_{CN})$$

We now plot the voltage over the load for phase A and the fundamental of it



We know that the back-emf voltage ($e_A = e_B = e_C$) is a sinusoidal voltage with the same frequency as the fundamental. We also know that:

$$v_{An} = L\frac{di_A}{dt} + e_A$$

The equation above is a linear equation which can be separated into two components; the fundamental component and the ripple component. This gives:

$$v_{An} = v_{An(1)} + v_{An(ripple)} = L\frac{d\left(i_{A(1)} + i_{A(ripple)}\right)}{dt} + e_A = L\frac{di_{A(1)}}{dt} + e_A + L\frac{di_{A(ripple)}}{dt}$$

The fundamental voltage can then be expressed as:

$$v_{An(1)} = L\frac{di_{A(1)}}{dt} + e_A$$

Or expressed with vectors:

$$\bar{V}_{An} = j\omega_1 L\bar{I}_A + \bar{E}_A$$

The ripple component can be expressed as:

$$v_{An(ripple)} = v_{An} - v_{An(1)} = L\frac{di_{A(ripple)}}{dt}$$

If the fundamental is known, the differential equation above can be solved for the ripple current. So we have to calculate the fundamental. The figure gives us that the function is odd and half-wave (a_n=0 for all values of n) which gives:

$$b_1 = \frac{4}{\pi}\int_0^{\pi/2} v_{An}(\theta)\sin(\theta)\,d\theta = \frac{4}{\pi}\int_0^{\pi/3} \frac{V_d}{3}\sin(\theta)\,d\theta + \frac{4}{\pi}\int_{\pi/3}^{\pi/2} \frac{2V_d}{3}\sin(\theta)\,d\theta$$

$$= \frac{4V_d}{3\pi}[-\cos(\theta)]_0^{\pi/3} + \frac{8V_d}{3\pi}[-\cos(\theta)]_{\pi/3}^{\pi/2} = \frac{4V_d}{\pi}\left(\frac{1}{3} - \frac{1}{3}\frac{1}{2} + \frac{2}{3}\frac{1}{2} - 0\right) = \frac{4V_d}{2\pi} = \frac{2V_d}{\pi}$$

The fundamental is sinusoidal and can be expressed as:

$$V_{An(1)} = \frac{2}{\pi}V_d\sin(\theta)$$

We also know that the solution for the differential equation can be written as:

$$i_{A(ripple)}(t) = i_{A(ripple)}(t_0) + \frac{1}{L}\int_{t_0}^{t} v_{An(ripple)}(\xi)d\xi$$

By performing a variable substitution from time to angle, the integral can be written as:

$$i_{A(ripple)}(\theta) = i_{A(ripple)}(\theta_0) + \frac{1}{\omega L}\int_{\theta_0}^{\theta} v_{An(ripple)}(\xi)d\xi$$

$$= i_{A(ripple)}(\theta_0) + \frac{1}{\omega L}\int_{\theta_0}^{\theta}\left(v_{An}(\xi) - v_{An(1)}(\xi)\right)d\xi$$

The next step is to figure out how the ripple current will look like. We know that the average of the ripple current must be zero. This due to the fact that the phase currents will have the same shape with a phase shift of 120° between them.

$$i_A + i_B + i_C = 0 \Rightarrow$$

$$0 = \frac{1}{T}\int_0^T i_A + i_B + i_C dt = \frac{1}{T}\int_o^T i_{A,1} + i_{B,1} + i_{C,1} + i_{A,ripple} + i_{B,ripple} + i_{C,ripple} dt =$$

$$= \frac{1}{T}\int_o^T \underbrace{i_{A,1} + i_{B,1} + i_{C,1}}_{three-phase system=0} dt + \frac{1}{T}\int_o^T i_{A,ripple}dt + \frac{1}{T}\int_o^T i_{B,ripple}dt + \frac{1}{T}\int_o^T i_{C,ripple}dt = I_{A,ripple} + I_{B,ripple} + I_{C,ripple}$$

But due to the symmetry of the phases $I_{A,ripple} = I_{B,ripple} = I_{C,ripple} = 0$

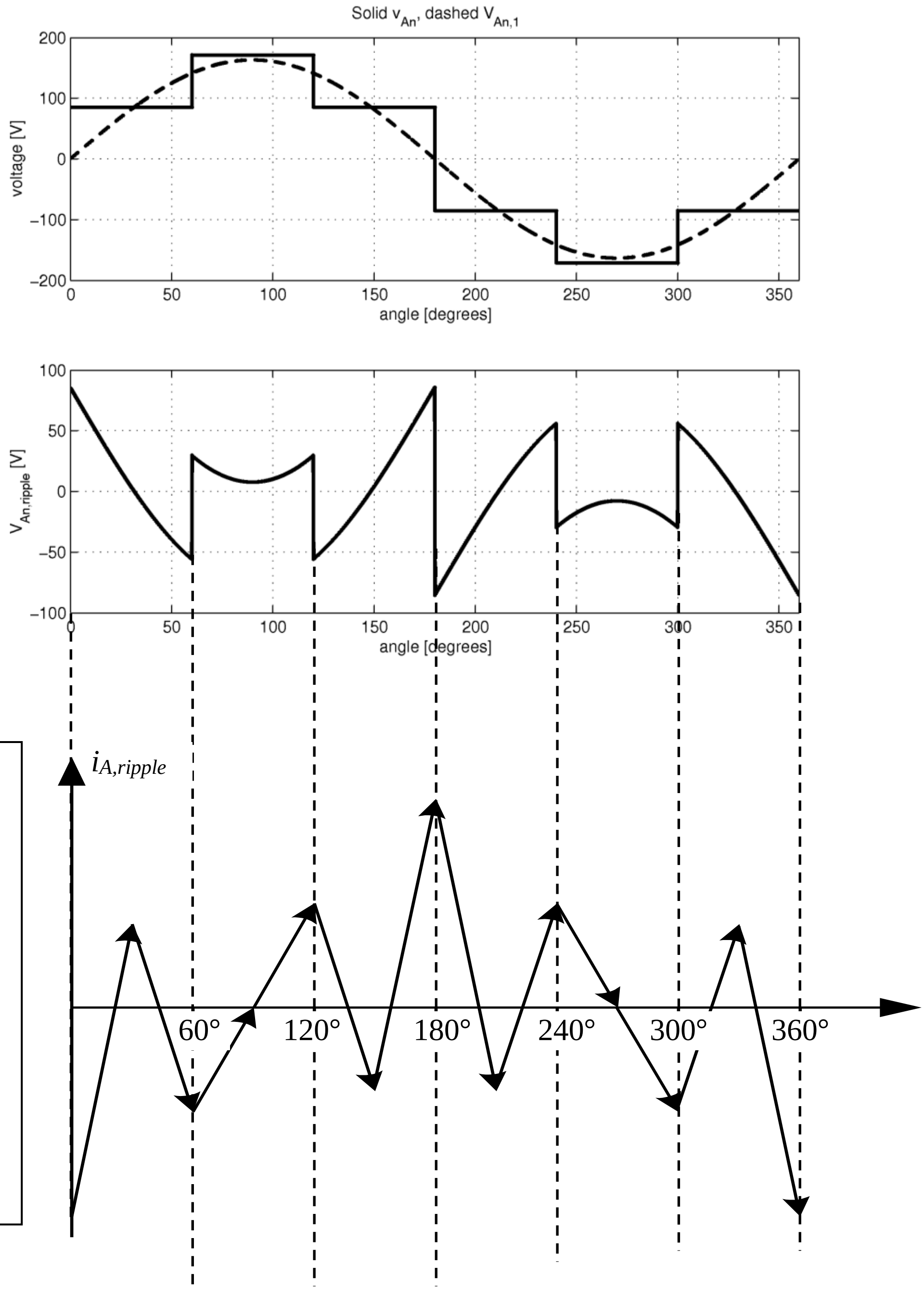
We know that $V_{LL(1)} = 200V$ with a frequency of 52Hz. This gives:

$$V_{An(1)} = \frac{1}{\sqrt{3}} V_{LL(1)} = \frac{1}{\sqrt{3}} \cdot 200V = 115.5V$$

$$v_{An(1)} = \frac{2}{\pi} V_d \sin(\omega t) \quad \rightarrow \quad V_d = \frac{\pi}{2} \hat{v}_{An(1)} = \frac{\pi}{2} \sqrt{2} V_{An(1)} = \frac{\pi}{2} \sqrt{2} \frac{1}{\sqrt{3}} V_{LL(1)} = \frac{\pi}{\sqrt{6}} 200V = 256.5V$$

The ripple voltage can now be plotted:

$$v_{An(ripple)} = v_{An} - v_{An(1)}$$



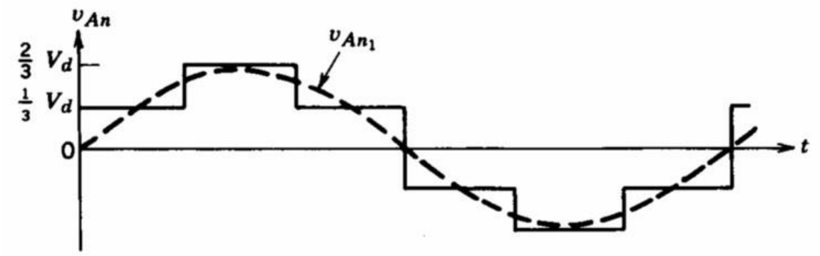
Once the ripple voltage is known, the ripple current can be sketched.

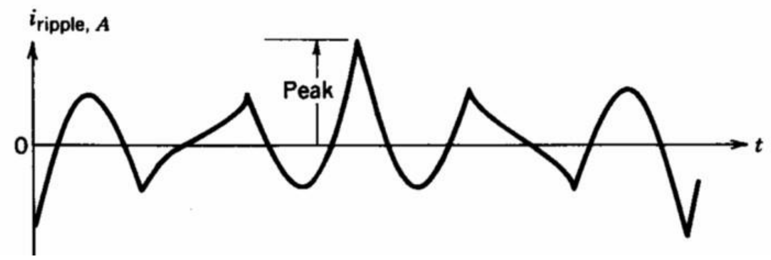
When the ripple voltage is positive we have an increasing ripple current and when the ripple voltage is negative we have a decreasing ripple

From the figures we see that we have symmetry lines at 90°, 180° and 270°. We see that the ripple voltage is odd around 180° ($v_{An,ripple}(180-\theta) = -v_{An,ripple}(180+\theta), \quad \theta \geq 0$). This leads to a ripple current that is an even function around 180° ($i_{A,ripple}(180-\theta)=i_{A,ripple}(180+\theta), \quad \theta \geq 0$). The areas around 180° show symmetry which give a current that will have the same shape.

We see that the ripple voltage is even around 90° ($v_{An,ripple}(90-\theta)=v_{An,ripple}(90+\theta), \quad 90\geq\theta\geq 0$). This gives that the ripple current will be odd around 90° ($i_{A,ripple}(90-\theta)=-i_{A,ripple}(90+\theta), \quad 90\geq\theta\geq 0$). This gives that the ripple current must be zero at 90°, due to that the average of the ripple current is zero.

Another way of getting the starting point in this problem is by using Fig.8-26a where the peak ripple current is defined. What we have done so far is basically a derivation of this figure.





Now we know a starting point for solving the differential equation $\left(i_{A,ripple}(90°)=0A\right)$ and we also know that the maximum ripple current occurs at $\theta = 180°$. We now calculate the peak ripple current:

$$\hat{i}_{A,ripple}=i_{A,ripple}(\pi)=i_{A,ripple}(\frac{\pi}{2})+\frac{1}{\omega L}\int_{\frac{\pi}{2}}^{\pi}v_{An,ripple}(\xi)d\xi=\frac{1}{\omega L}\int_{\frac{\pi}{2}}^{\pi}v_{An}(\xi)-v_{An,1}(\xi)d\xi=$$

$$=\frac{1}{\omega L}\int_{\frac{\pi}{2}}^{\frac{2}{3}\pi}\frac{2}{3}V_d-\frac{2}{\pi}V_d\sin(\xi)d\xi+\frac{1}{\omega L}\int_{\frac{2}{3}\pi}^{\pi}\frac{1}{3}V_d-\frac{2}{\pi}V_d\sin(\xi)d\xi=$$

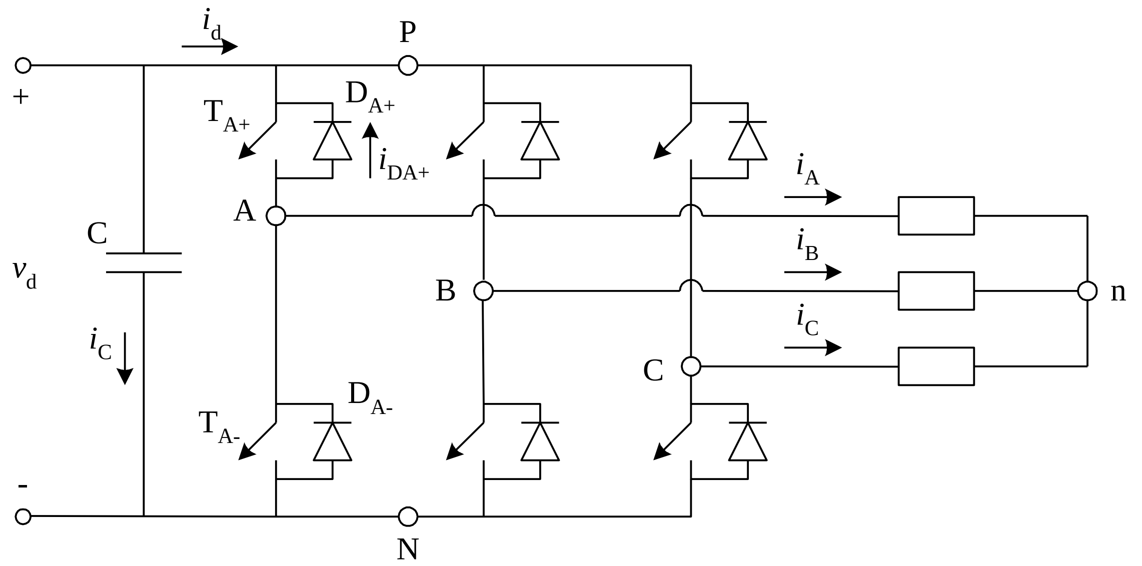
$$=\frac{V_d}{\omega L}\left[\frac{2}{3}\xi+\frac{2}{\pi}\cos(\xi)\right]_{\frac{\pi}{2}}^{\frac{2}{3}\pi}+\frac{V_d}{\omega L}\left[\frac{1}{3}\xi+\frac{2}{\pi}\cos(\xi)\right]_{\frac{2}{3}\pi}^{\pi}=$$

$$=\frac{V_d}{\omega L}\left(\frac{2}{3}(\frac{2}{3}\pi-\frac{\pi}{2})+\frac{2}{\pi}\cos(\frac{2}{3}\pi)-\frac{2}{\pi}\cos(\frac{\pi}{2})+\frac{1}{3}(\pi-\frac{2}{3}\pi)+\frac{2}{\pi}\cos(\pi)-\frac{2}{\pi}\cos(\frac{2}{3}\pi)\right)=$$

$$=\frac{V_d}{\omega L}\left(\pi(\frac{4}{9}-\frac{2}{6}+\frac{1}{3}-\frac{2}{9})+-\frac{2}{\pi}\cos(\frac{\pi}{2})+\frac{2}{\pi}\cos(\pi)\right)=\frac{V_d}{\omega L}\left(\pi\frac{2}{9}-\frac{2}{\pi}\right)=\frac{256.5}{2\pi 52\cdot 0.1}\left(\pi\frac{2}{9}-\frac{2}{\pi}\right)=0.48\ A$$

Problem 2 (P8-10 in Undeland book)

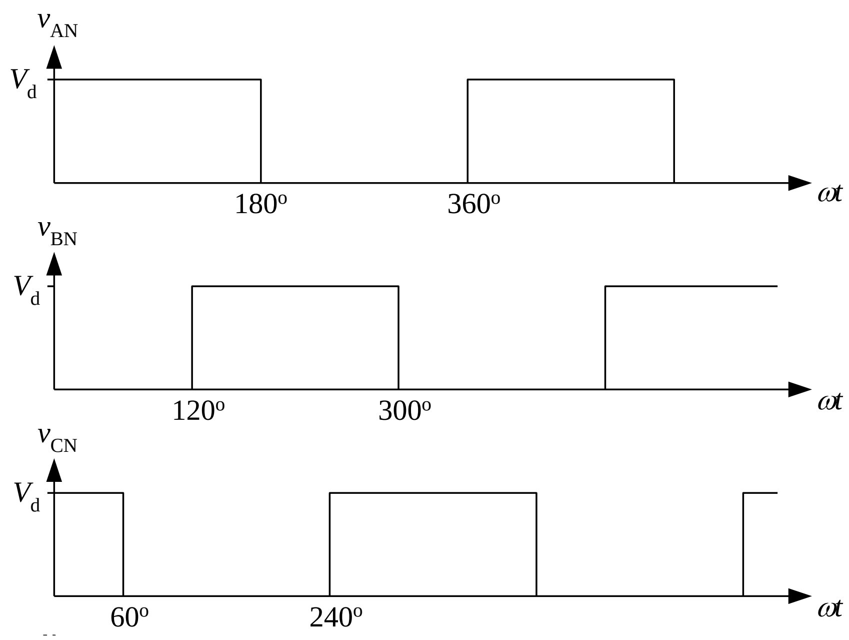
In the three-phase, square-wave inverter (see Fig 8-24a in Undeland), consider the load to be balanced and purely resistive with a load-neutral point *n*.



Draw the steady state waveforms for v_{An}, i_A, i_{DA+} and i_d, where i_{DA+} is the current through diode D_{A+}.

Solution

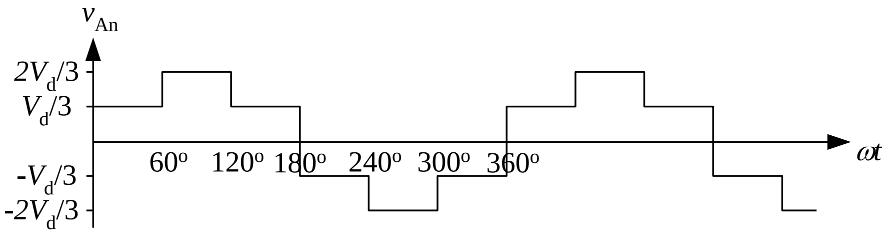
Each leg in the inverter is operated in square wave operation. This means that the switches connected to P are on 50% of the period and the switches connected to N are on for the rest of the period. We also know that it is a three-phase inverter, which means that there is a 120º phase shift between the phases.



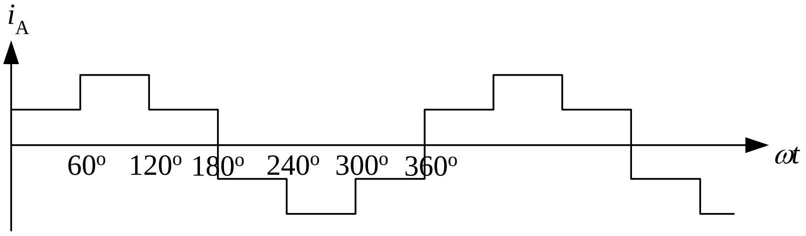
From the previous problem (8-7) we know that the voltage in each phase to the neutral point can be expressed as::

$$v_{An} = \frac{2}{3} v_{AN} - \frac{1}{3}(v_{BN} + v_{CN})$$

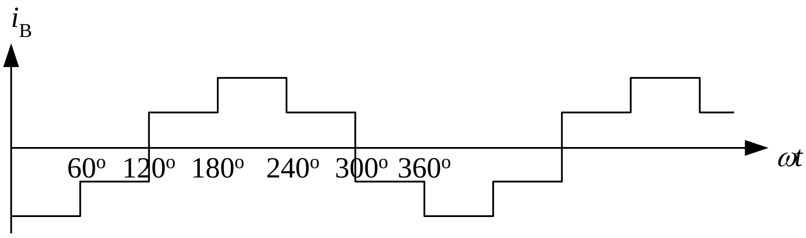
With help by the voltage in each leg $\left(V_{An}(t), V_{Bn}(t) \text{ and } V_{Cn}(t)\right)$, the voltage over the load in each phase can be drawn (e.g. the voltage between A and n).



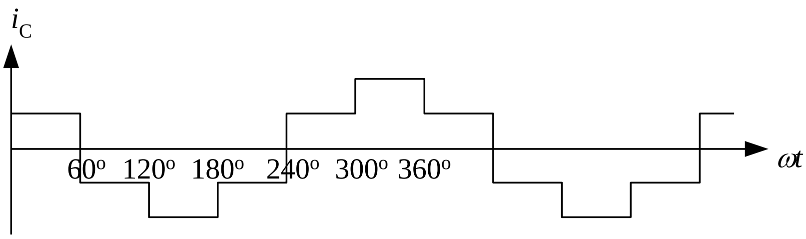
Note that the voltage in each phase has the same amplitude, but with the only difference that they are 120° phase shifted. Since we have a purely resistive load, the current has the same shape as the voltage.



$$i_A = \frac{v_{An}}{R}$$



$$i_B = \frac{2}{3R} v_{BN} - \frac{1}{3R}(v_{AN} + v_{CN})$$



$$i_C = \frac{2}{3R} v_{CN} - \frac{1}{3R}(v_{AN} + v_{BN})$$

The current i_d is the sum of the parts of the phase-currents when the switches connected to P are on. This gives a current that is constant. The current through diode D_{A+} is zero since the phase voltage is in phase with the current due to the purely resistive load.