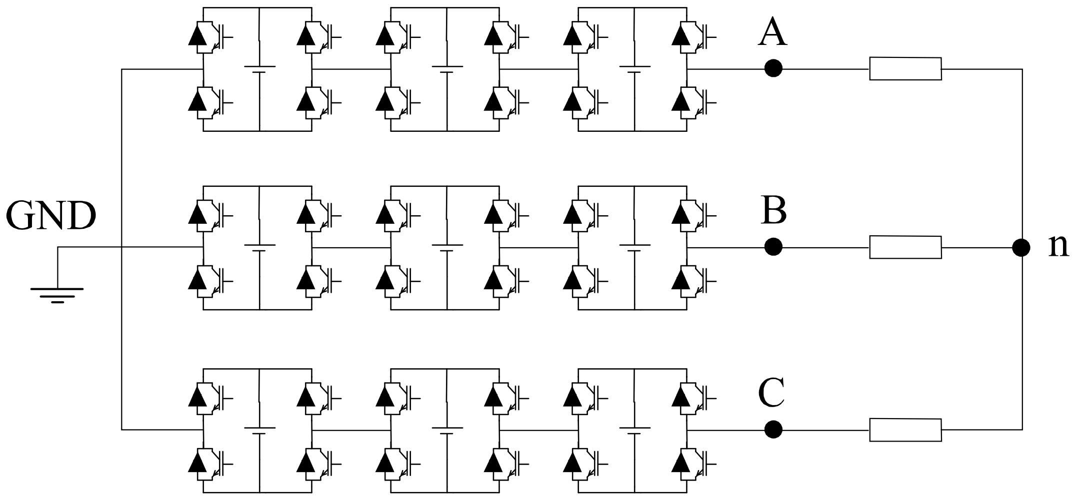
Solution of demonstration 9

Problem 1

Consider the cascaded three phase full bridge inverter with a purely resistive load below. Each H-bridge is connected to an ideal DC-source, V_d. The switching angle is 10°, 20° and 50° respectively.



a) Draw the waveform of voltages v_{AGND}, v_{AB}, v_{nGND} and v_{An}.

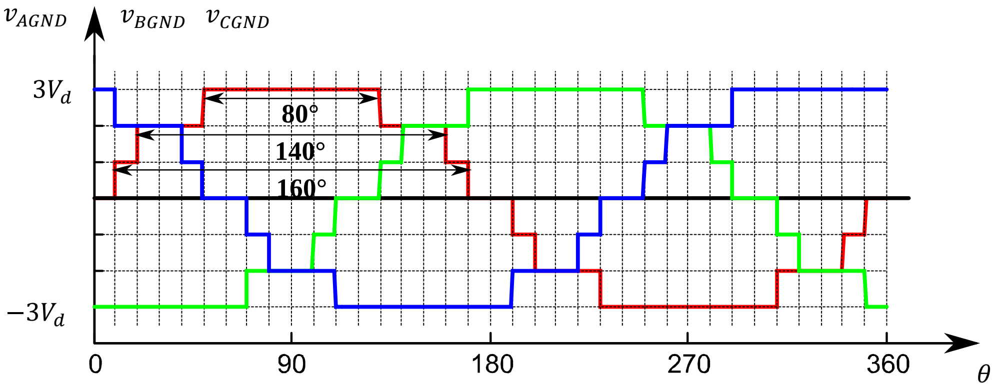
b) Calculate the THD of the phase voltage v_{An}.
Perform the same calculation for the phase voltage v_{An} in Problem 2 in Demonstration 8 (P8-9 in Undeland book).
Compare these two values.

c) Calculate the RMS value and average value of the DC-source currents in phase A.
Compare the RMS value and average value.
Perform the same calculation and comparison for the DC-source current in Problem 2 in demonstration 8 (P8-10 in Undeland book).

Solution

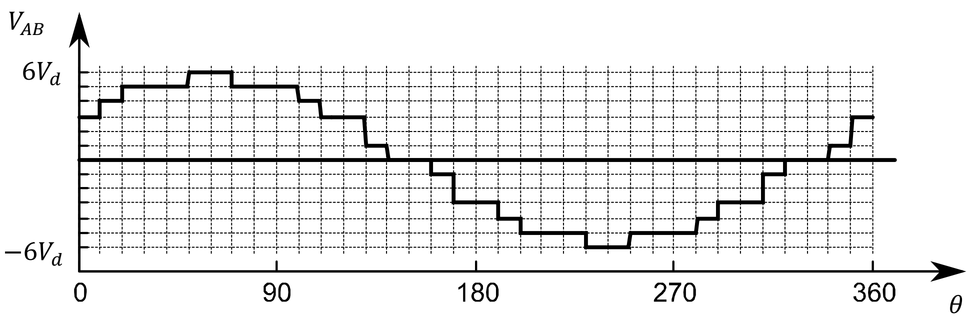
a) Draw the waveform of voltages v_{AGND}, v_{AB}, v_{nGND} and v_{An}.

We can directly draw the phase potentials with respect to GND, as the switching angle is 10°, 20° and 50° respectively



The line to line voltage, v_{AB}, can be drawn as the difference between the two phase potentials with respect to GND above as

$$v_{AB} = v_{An} - v_{Bn} = v_{AGND} - v_{BGND}$$



In a similar way as for an ordinary three phase inverter (2-level), we can express as set of equations for the three phase voltages

$$\begin{cases} v_{An} = v_{AGND} - v_{nGND} \\ v_{Bn} = v_{BGND} - v_{nGND} \\ v_{Cn} = v_{CGND} - v_{nGND} \end{cases}$$

If we sum the equations together and solve for v_{nGND}

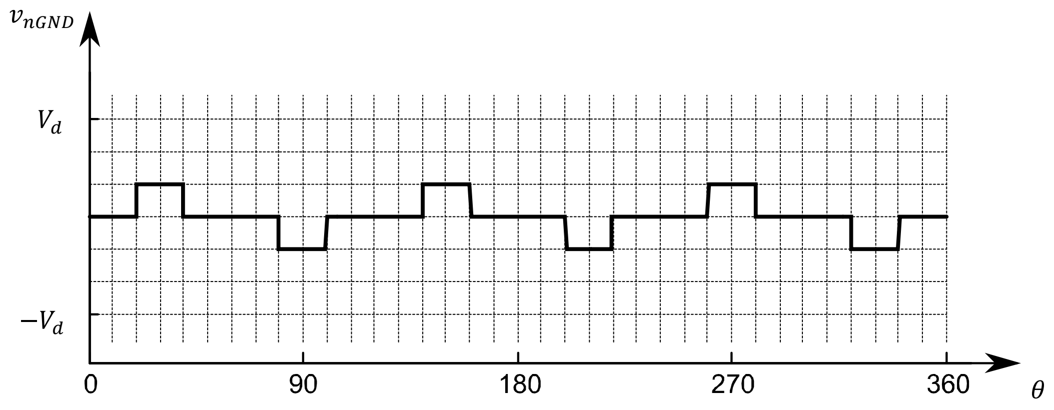
$$3v_{nGND} = v_{AGND} - v_{An} + v_{BGND} - v_{Bn} + v_{CGND} - v_{Cn} =$$

$$= v_{AGND} + v_{BGND} + v_{CGND} - \underbrace{(v_{An} + v_{Bn} + v_{Cn})}_{=0} =$$

$$3v_{nGND} = v_{AGND} + v_{BGND} + v_{CGND}$$

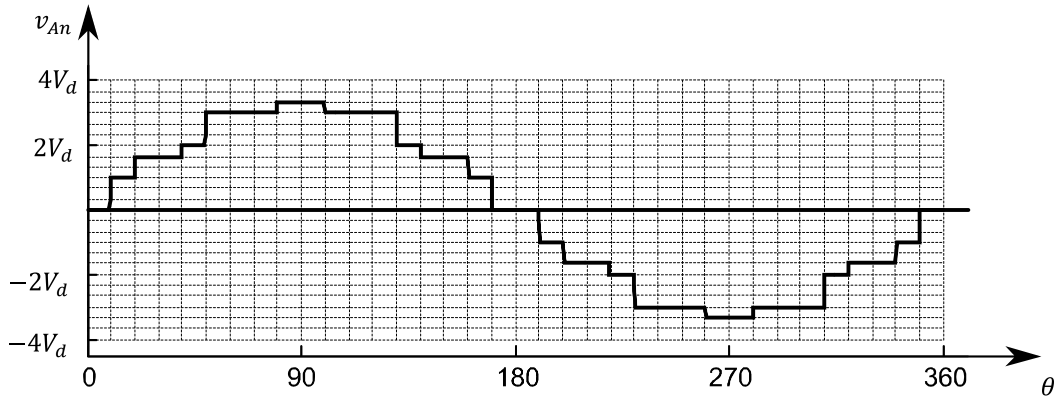
We can now draw the neutral to ground voltage

$$v_{nGND} = \frac{v_{AGND} + v_{BGND} + v_{CGND}}{3}$$



Finally, we can calculate

$$v_{An} = v_{AGND} - v_{nGND}$$



Please, observe the phase shift of 30° between the phase voltage and the line to line voltage (which was also shown in Demonstration 1).

b) Calculate the THD of the phase voltage v_{An}.
Perform the same calculation for the phase voltage v_{An} in Problem 2 in Demonstration 8 (P8-9 in Undeland book).
Compare these two values.

In terms of RMS values, the THD is defined as

$$\%THD = 100\frac{\sqrt{{V_{An}}^2 - {V_{An(1)}}^2}}{V_{An(1)}}$$

why we need to calculate the RMS value of the phase voltage as well as the fundamental of the phase voltage.

The phase voltage, v_{An}, is odd half wave so according to the formula sheet

$$a_n = 0$$

and

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(n\theta)\,d\theta$$

which applied to our case where we only need the fundamental component gives

$$\hat{V}_{An(1)} = \frac{4}{\pi}\int_{\pi/18}^{9\pi/18} f(t)\sin(\theta)\,d\theta =$$

$$= \frac{4}{\pi}\left(\int_{\pi/18}^{2\pi/18} V_d\sin(\theta)\,d\theta + \int_{2\pi/18}^{4\pi/18}\frac{5V_d}{3}\sin(\theta)\,d\theta + \int_{4\pi/18}^{5\pi/18} 2V_d\sin(\theta)\,d\theta + \right.$$
$$\left. + \int_{5\pi/18}^{8\pi/18} 3V_d\sin(\theta)\,d\theta + \int_{8\pi/18}^{9\pi/18}\frac{10V_d}{3}\sin(\theta)\,d\theta\right) =$$

$$= \frac{4V_d}{\pi}\left([-\cos(\theta)]_{\pi/18}^{2\pi/18} + \frac{5}{3}[-\cos(\theta)]_{2\pi/18}^{4\pi/18} + 2[-\cos(\theta)]_{4\pi/18}^{5\pi/18} + 3[-\cos(\theta)]_{5\pi/18}^{8\pi/18}\right.$$
$$\left. + \frac{10}{3}[-\cos(\theta)]_{8\pi/18}^{9\pi/18}\right) =$$

$$= \frac{4V_d}{\pi}(0.579 + 1.407 + 0.24 + 0.289 + 0.045)$$
$$\rightarrow\ V_{An(1)} = 2.311V_d$$

The RMS value of the phase voltage can be calculated according to the definition as

$$V_{An(RMS)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} {v_{An}}^2\ d\theta}$$

and applied to our case

$$V_{An(RMS)} = \sqrt{\frac{1}{\pi/2}\int_0^{\pi/2} {v_{An}}^2\ d\theta} =$$

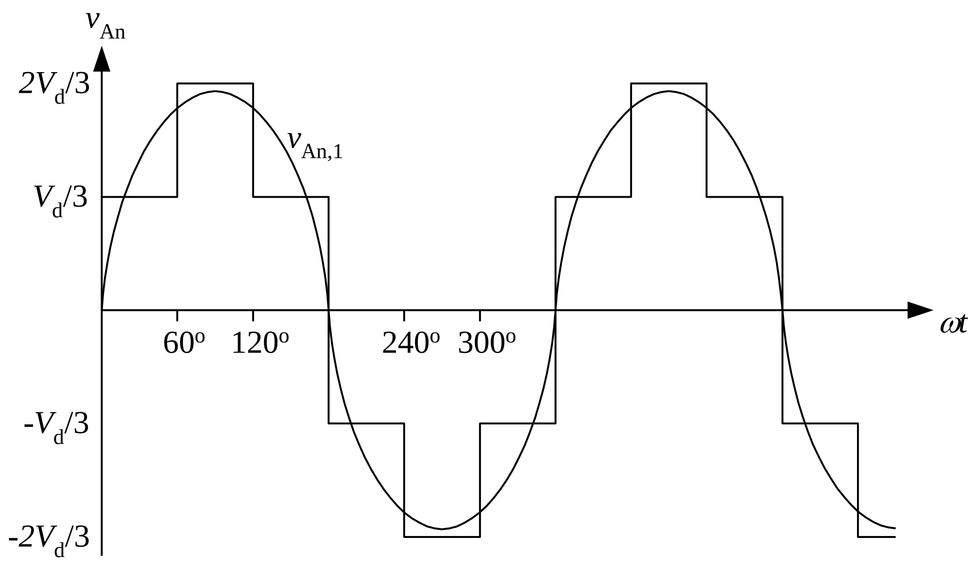
$$= \sqrt{\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)^2\frac{\pi}{18} + (3V_d)^2\frac{3\pi}{18} + (2V_d)^2\frac{\pi}{18} + \left(\frac{5V_d}{3}\right)^2\frac{2\pi}{18} + (V_d)^2\frac{\pi}{18}\right)} =$$

$$= V_d\sqrt{146/27}$$

$$\%THD = 100\frac{\sqrt{\left(V_d\sqrt{146/27}\right)^2 - (2.311V_d)^2}}{2.311V_d} = 11.03\%$$

In Demo 8 Problem 2, we got the following relation between the DC-link voltage and the fundamental phase voltage

$$V_d = \frac{\pi}{2}\sqrt{2}V_{An(1)} \rightarrow\ V_{An(1)} = \frac{V_d 2}{\pi\sqrt{2}}$$



The total RMS-value of V_{An} in Demo 8 Problem 2 is then

$$V_{An-RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} {v_{An}}^2 d\theta} = \begin{Bmatrix} since\ we \\ integrate\ the\ square \end{Bmatrix} = \sqrt{\frac{4}{2\pi}\int_0^{\pi/2} {v_{An}}^2 d\theta} =$$

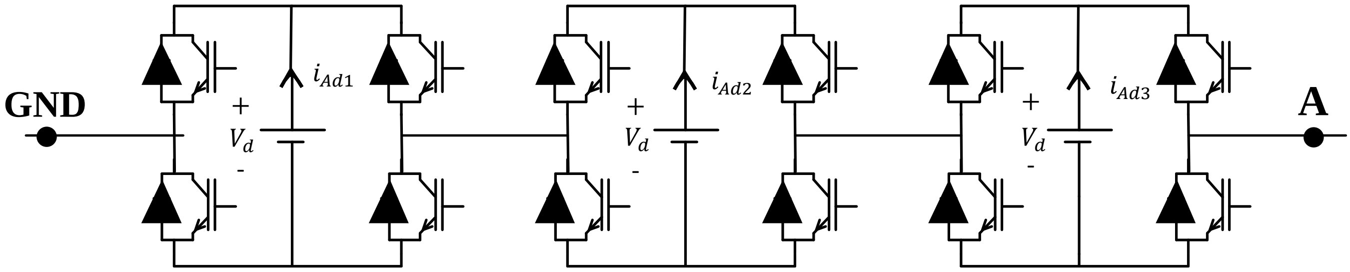
$$= \sqrt{\frac{2}{\pi}\left(\left(\frac{V_d}{3}\right)^2\frac{\pi}{3} + \left(\frac{2V_d}{3}\right)^2\frac{\pi}{6}\right)} = V_d\sqrt{6/27}\ V$$

$$\%THD = 100 \frac{\sqrt{\left(V_d\sqrt{6/27}\right)^2 - \left(\frac{V_d 2}{\pi\sqrt{2}}\right)^2}}{\left(\frac{V_d 2}{\pi\sqrt{2}}\right)} = 100\sqrt{\frac{\pi^2}{9} - 1} = 31.1\%$$

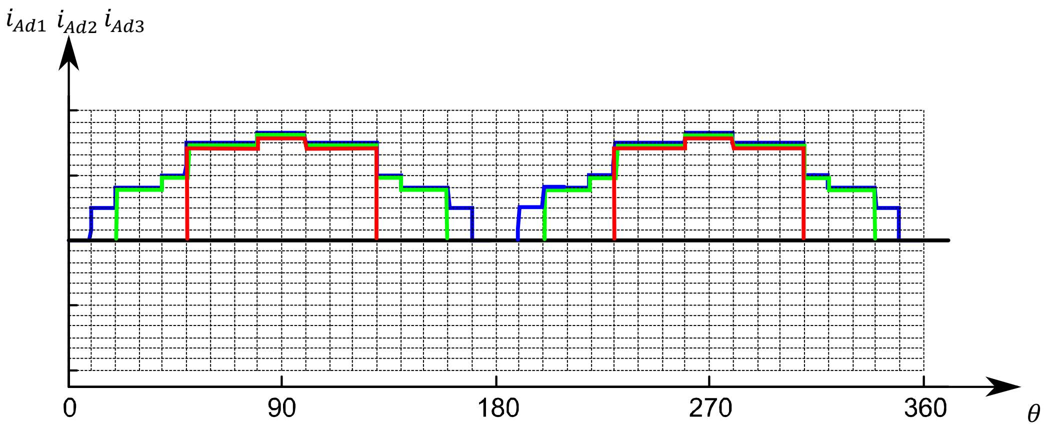
The total harmonic distortion in the output voltage is considerably lower for the cascaded three phase multilevel inverter compared to the ordinary three phase inverter that is operated in square wave mode. Note that both converters are switched with the fundamental frequency. See slides from lecture for the difference between fundamental switching and PWM.

**c) Calculate the RMS value and average value of the DC-source currents in phase A.
Compare the RMS value and average value.
Perform the same calculation and comparison for the DC-source current in Problem 2 in demonstration 8 (P8-10 in Undeland book).**

Since each phase consists of three H-bridges, which connect the DC-sources during different intervals, the DC-currents from the three sources are not the same. In phase A, we can denote three source currents, i_{Ad1}, i_{Ad2} and i_{Ad3} as



Due to the resistive load, the phase current will have the same shape as the phase voltage. The currents from the DC-sources will always be positive as active power is supplied to the load. If the same source is assumed to supply during the same interval during a full period (both half periods are the same), the three different source currents can be drawn as



The RMS value of the source connected during the 160° intervals can be calculated the same way as $V_{An(RMS)}$ previously

$$i_{Ad1(RMS)} = \frac{V_{An(RMS)}}{R} = \frac{V_d}{R}\sqrt{{}^{146}/_{27}} = \frac{V_d}{R}2.33$$

while the RMS values of the two remaining currents can be calculated by removing the intervals where the sources are not connected.

$$i_{Ad2(RMS)} = \frac{1}{R}\sqrt{\frac{1}{\pi/2}\int_0^{\pi/2} v_{An}{}^2 \; d\theta} =$$

$$= \frac{1}{R}\sqrt{\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)^2\frac{\pi}{18} + (3V_d)^2\frac{3\pi}{18} + (2V_d)^2\frac{\pi}{18} + \left(\frac{5V_d}{3}\right)^2\frac{2\pi}{18}\right)} = \frac{V_d}{R}\sqrt{{}^{143}\!/_{27}} = \frac{V_d}{R}2.30$$

$$i_{Ad3(RMS)} = \frac{1}{R}\sqrt{\frac{1}{\pi/2}\int_0^{\pi/2} {v_{An}}^2\ d\theta} =$$

$$= \frac{1}{R}\sqrt{\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)^2\frac{\pi}{18} + (3V_d)^2\frac{3\pi}{18}\right)} = \frac{V_d}{R}\sqrt{{}^{343}\!/_{81}} = \frac{V_d}{R}2.06$$

The average DC-source currents can be calculated in a similar manner

$$i_{Ad1(AVG)} = \frac{1}{R}\frac{1}{\pi/2}\int_0^{\frac{\pi}{2}} v_{An}\ d\theta =$$

$$= \frac{1}{R}\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)\frac{\pi}{18} + 3V_d\frac{3\pi}{18} + 2V_d\frac{\pi}{18} + \left(\frac{5V_d}{3}\right)\frac{2\pi}{18} + V_d\frac{\pi}{18}\right) = \frac{V_d}{R}\frac{56}{27} = \frac{V_d}{R}2.07$$

$$i_{Ad2(AVG)} = \frac{1}{R}\frac{1}{\pi/2}\int_0^{\frac{\pi}{2}} v_{An}\ d\theta =$$

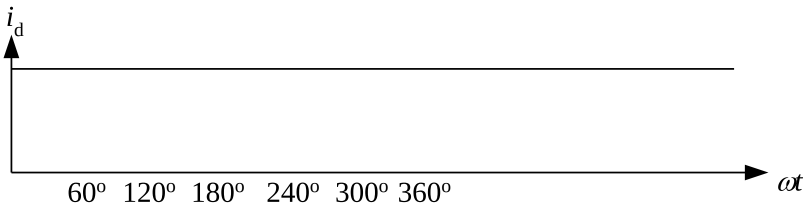
$$= \frac{1}{R}\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)\frac{\pi}{18} + 3V_d\frac{3\pi}{18} + 2V_d\frac{\pi}{18} + \left(\frac{5V_d}{3}\right)\frac{2\pi}{18}\right) = \frac{V_d}{R}\frac{53}{27} = \frac{V_d}{R}1.96$$

$$i_{Ad3(AVG)} = \frac{1}{R}\frac{1}{\pi/2}\int_0^{\frac{\pi}{2}} v_{An}\ d\theta =$$

$$= \frac{1}{R}\frac{2}{\pi}\left(\left(\frac{10V_d}{3}\right)\frac{\pi}{18} + 3V_d\frac{3\pi}{18}\right) = \frac{V_d}{R}\frac{37}{27} = \frac{V_d}{R}1.37$$

Note that there is a difference between the RMS values and the average values and only the DC-component of a current from a DC-source contributes to the active power. The ripple corresponds to reactive power and will cause extra conduction losses.

In Problem 8.10 in Demonstration 8, we concluded that the source current is a pure DC-current. The average and the RMS value of the current is therefore the same.



In this demonstration, we have shown that it is possible to lower the THD in the output voltage when using a cascaded three phase full bridge inverter. At the same time the ripple current from the DC-sources was increased compared to the ripple current in the DC-source of an ordinary two level inverter. Observe that the switch intervals were not chosen to minimize the THD for the multilevel inverter in this demonstration.

In more general terms, the multilevel converters can offer a greater degree of freedom compared with the two level converters. It is possible to utilize this extra degree of freedom to:

- Minimize THD in the AC voltage and/or current.
- Cancel some specific harmonics on the AC and/or DC-side. See selective harmonic elimination in slide 34 in lecture 11.
- Minimize zero-sequence (v_{nGND}).

  etc, etc…