Introduction to Electronic System Design

Laboratory assignment 6

A multirate system

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Goal

To combine and extend the knowledge from lab assignments 2 and 5 into one sampled system

Multirate system

A multirate system is a system that involves more than one sampling frequency

In a typical multirate system we read input samples at one clock frequency and output samples at an other clock frequency

This is just what we are going to do in of this assignment

Multirate system are also used in systems using oversampling where we trade sampling frequency for amplitude resolution

We can use fewer bits and/or simplier filters if we use a higher sampling frequency

Multirate system

What frequencies can be used?

Simple sampling frequency changes involves raising the sampling frequency by an integer factor I (interpolation)

or lowering it by an integer factor D (decimation)

We can combine these and change the sampling frequency by a rational number R=I/D

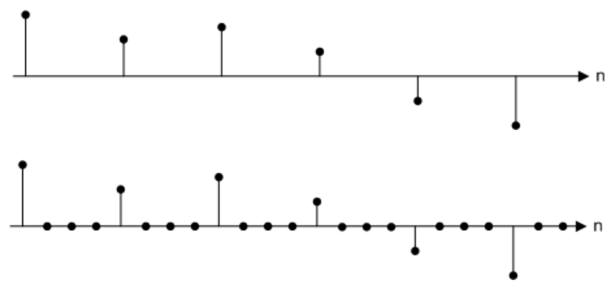
There is no simple way to do change the sampling frequency by some other factor, the factors should be integers

Interpolation

We can do interpolation in several ways

We will look at two ways

We can increase the sampling frequency by an integer factor I and set the new intermediate samples to zero



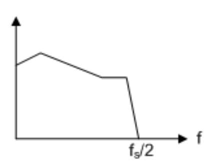
Sampling rate f_s

Sampling rate $4f_s$

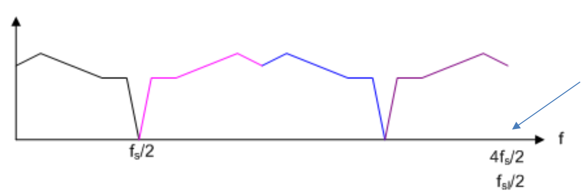
Interpolation cont.

What happens to the frequency spectra?

Our original spectra



When we increase the sampling rate we will get mirror images of this spectra

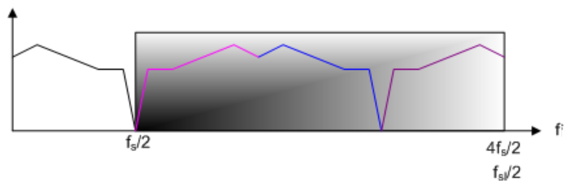


Our new $f_s/2$

Interpolation cont.

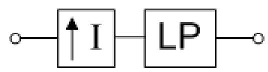
What happens to the frequency spectra cont.

These mirror images will have to be filtered away



This has to be done **after** the increase in sampling rate.

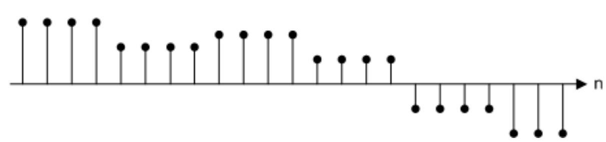
In this case with a four times increase in sampling frequency we will need a low pass filter with a cut off frequency of $f_{sI}/8=f_{sI}/2I$



For the filter to be realizable the filter cut off frequency must be somewhat lower than this.

Interpolation cont.

Instead of inserting zeros we could insert copies of the last sample as the intermediate samples when we increase the sampling rate



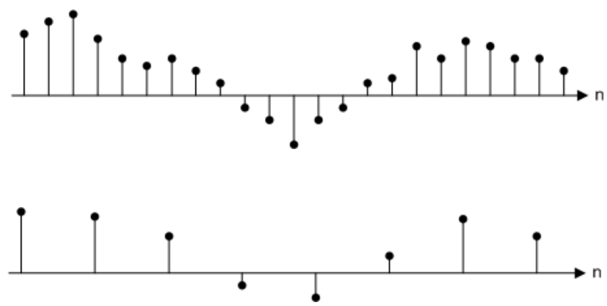
This will still result in mirror images of the spectra but the images will be somewhat attenuated which might make them easier to filter away

This is not necessarily simplier since in the first case, when we insert zeros, I-1 out of I terms in the filter will have zero valued samples and these factors can be left out of the filter implementation and we can have fewer calculations

Demonstration!

Decimation

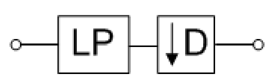
In decimation we lower the sampling frequency by a integer factor D by leaving out D-1 out of every D samples



Decimation cont.

To be allowed to do this we need to make sure that we have no signal frequencies above half the new sampling frequency f_s/D

This means that **before** the lowering of the sampling rate we need to low pass filter the signal with a filter with the cut off frequency just below $f_s/2D$

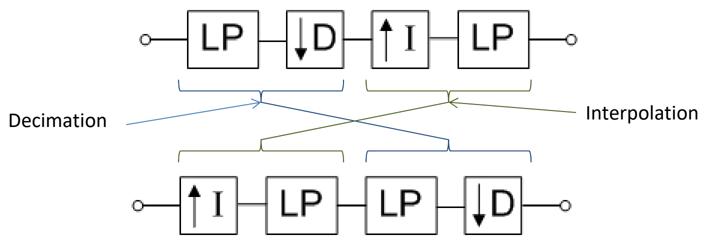


Demonstration!

Multirate

We can change the sampling rate by a rational number (I/D) by using one interpolator and one decimator.

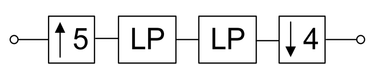
This can be done in two different ways



Does the order between the decimation and the interpolation blocks matter?

Yes it does!

Multirate



Example 1

Let´try to raise the sampling frequency and change the sampling frequency from 40 kHz to 50 kHz

That is we interpolate by 5 and decimate by 4

Let´s start by interpolating first and decimate afterwards

To start with we have signals up to 20 kHz

We interpolate the sampling frequency to 5·40 = 200 kHz

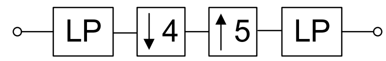
We have to lowpass filter at 20 kHz after the interpolation to avoid aliazing

We decimate the sampling frequency to 200/4 = 50 kHz

We have to lowpass filter at the new $f_s/2$, 25 kHz before the decimation

We loose nothing of the signal we started with

Multirate



Example 1 cont.

Now let´s start by decimating and interpolate afterwards

To start with we have signals up to 20 kHz

We decimate the sampling frequency to 40/4 = 10 kHz

We have to lowpass filter at 5 kHz before the decimation to avoid aliazing

We loose the frequency intervall 5 – 20 kHz

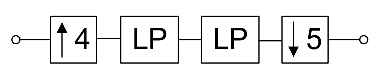
We interpolate the sampling frequency to 5·10 = 50 kHz

We have to lowpass filter at 5 kHz after the interpolation to avoid aliazing

As a whole we loose the frequency intervall 5 – 20 kHz

Not good!

Multirate



Example 2

Now let´s try to lower the sampling frequency and
change the sampling frequency from 40 kHz to 32 kHz

That is we interpolate by 4 and decimate by 5

Let´s start by interpolating and then decimate afterwards

To start with we have signals up to 20 kHz

We interpolate the sampling frequency to 4·40 = 160 kHz

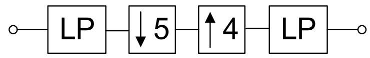
We have to lowpass filter at 20 kHz after the interpolation to avoid aliazing

We decimate the sampling frequency to 160/5 = 32 kHz

We have to lowpass filter at the new $f_s/2$, 16 kHz before the decimation

As expected we loose the frequency intervall 16 – 20 kHz
since the sampling frequency has been lowered

Multirate



Example 2 cont.

Now let´s start by decimating and interpolating afterwards

To start with we have signals up to 20 kHz

We decimate the sampling frequency to 40/5 = 8 kHz

We have to lowpass filter at 4 kHz before the decimation to avoid aliazing

We loose the frequency intervall 4 – 20 kHz

We interpolate the sampling frequency to 4·8 = 32 kHz

We have to lowpass filter at 4 kHz after the interpolation to avoid aliazing

As a whole we loose the frequency intervall 4 – 20 kHz

Not good!

Multirate

When we decimate first we need to filter out everything above the frequency $f_s/2D$.

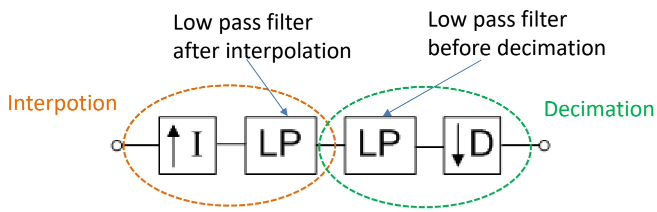
This means that these frequences are gone and can never come back even when we interpolate the sampling rate.

If we interpolate first on the other hand none of our signal frequencies disappear in the interpolation and some frequencies may disappear in the decimation, but only if the new sampling rate is lower than the one we started with

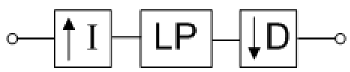
Conclusion: Interpolate first and decimate after that

Multirate

Let´s look at the block diagram again



It is obvious that we can leave out one of the low pass filters and only keep the one with the lowest cut off frequency



Demonstration!

Multirate

Example

We want to transfer some files from CD, where the standard sampling rate is 44.1 kHz, to digital audio tape (DAT), where the standard sampling rate is 48 kHz. What sampling rates are needed and what are the filter specifications?

We have to change the sampling rate by a factor 48/44.1. This is not a rational number so we have to try and find common factors

$$\frac{48}{44.1}=\frac{480}{441}=\frac{160}{147}$$

It seems like we would have to raise the intermediate sampling rate to

$$160\cdot 44.1 \propto 7.056 \text{ MHz}$$

and we would need a low pass filter with a normalized cut off frequency of $f_s/320$

We might be able to use this high sampling rate but the filter demands are totally unrealistic

What can we do?

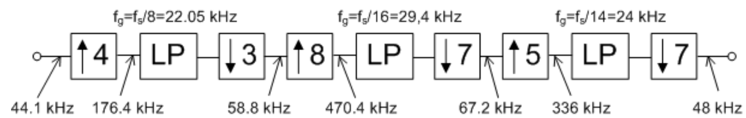
Multirate

Example cont.

Let´s look at the common factors

$$\frac{160}{147}=\frac{2\cdot2\cdot2\cdot2\cdot2\cdot5}{3\cdot7\cdot7}=\frac{4}{3}\cdot\frac{8}{7}\cdot\frac{5}{7}$$

We can do our change of sample rate in a number of steps. Once again the order is important, in order to not loose high frequency signals the lowering of the sampling rate should come **last**



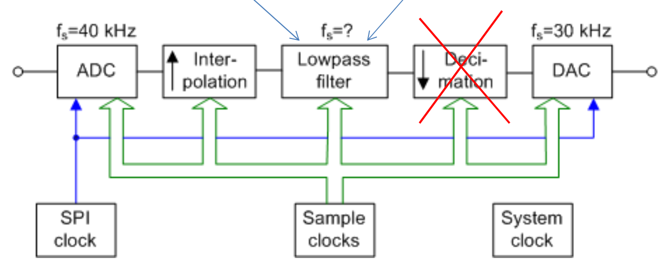
Multirate

Lab assignment 6 cont.

If we insert zeros in our interpolation we can simplify the design since many of the samples in the filtering are zeroes

Keep the shifting at the higher rate

Combine with decimation and filter at the lower rate



Multirate

Assignment

Create a multirate system where we sample the input signal at 40 kHz and after manipulation output it at a sampling rate of 30 kHz.

The signal before and after the system should be the same with the obvious restriction that the maximal signal frequency at the output will be lower than at the input