

In the Main.m, we have

$$\text{val_Hz} = \text{eig}(A, B);$$

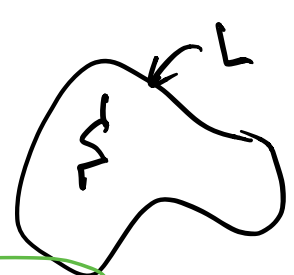
$$\text{val_Ez} = \text{eig}(A_{\text{nat}}, B_{\text{nat}});$$

that correspond to

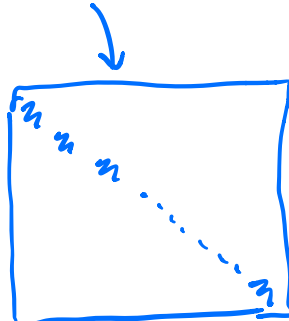
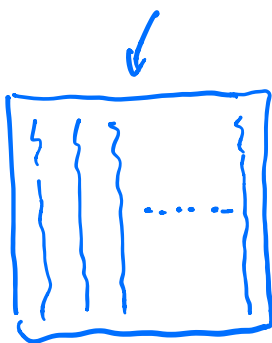
$$\begin{cases} -\nabla^2 H_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} H_2 & \text{in } \Omega \\ \hat{n} \cdot \nabla H_2 = 0 & \text{on } L \end{cases}$$

$\Rightarrow A h = \begin{pmatrix} 2 \\ 1 \end{pmatrix} B h$

$H_2(x, y) = \sum_{j=1}^n h_j \varphi_j(x, y)$
with $w_i(x, y) = \varphi_i(x, y)$



$$[\text{vtr_Hz}, \text{val_Hz}] = \text{eig}(A, B);$$

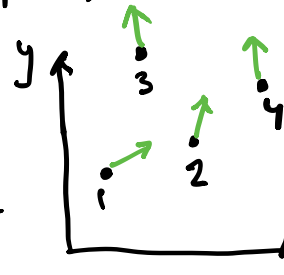


Useful MATLAB commands for visualization :

trisurf (TRI, X, Y, Z, C)

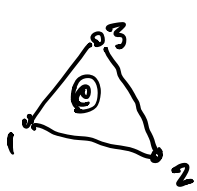
quiver (X, Y, U, V)

$$\nabla H_z = \hat{x} \frac{\partial H_z}{\partial x} + \hat{y} \frac{\partial H_z}{\partial y} + \underbrace{\hat{z} \frac{\partial H_z}{\partial z}}_{=0} \quad \text{since } H_z \text{ const. wrt } z$$



$$= \left\{ H_z(x, y) = \sum_{j=1}^N h_j \psi_j(x, y) \right\}$$

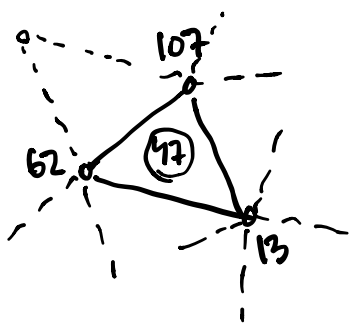
$$= \nabla \left(\sum_{j=1}^N h_j \psi_j(x, y) \right) = \sum_{j=1}^N h_j \nabla \psi_j(x, y)$$



$$\psi_j^{(e)}(x, y) = a_j^{(e)} + b_j^{(e)} x + c_j^{(e)} y \quad \text{with } j=1,2,3$$

$$\nabla \psi_j^{(e)}(x, y) = \hat{x} \frac{\partial \psi_j^{(e)}(x, y)}{\partial x} + \hat{y} \frac{\partial \psi_j^{(e)}(x, y)}{\partial y}$$

$$= \hat{x} b_j^{(e)} + \hat{y} c_j^{(e)}$$



$$H_2(x, y) \Big|_{e=47} = \sum_{j=1}^3 h_j \varphi_j(x, y) \Big|_{e=47}$$

$$= h_{62} \varphi_{62}(x, y) + h_{13} \varphi_{13}(x, y) + h_{107} \varphi_{107}(x, y) \Big|_{e=47}$$

$$= h_{62} \varphi_1^{(47)}(x, y) + h_{13} \varphi_2^{(47)}(x, y) + h_{107} \varphi_3^{(47)}(x, y)$$

$$elmo = \begin{bmatrix} 3 & \dots & \dots & 3 & 62 & 3 & \dots & \dots & 3 \\ 3 & \dots & \dots & 3 & 13 & 3 & \dots & \dots & 3 \\ 3 & \dots & \dots & 3 & 107 & 3 & \dots & \dots & 3 \end{bmatrix}$$

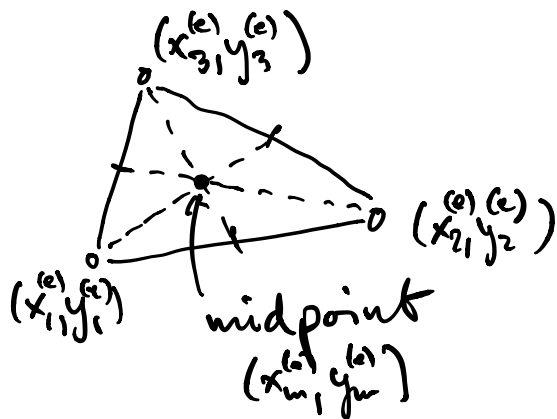
↑ column #47

$$\nabla H_2(x, y) \Big|_{e=47} =$$

$$= h_{62} \underbrace{\nabla \varphi_1^{(47)}(x, y)}_{= \hat{x} b_1^{(47)} + \hat{y} c_1^{(47)}} + h_{13} \underbrace{\nabla \varphi_2^{(47)}(x, y)}_{= \hat{x} b_2^{(47)} + \hat{y} c_2^{(47)}} + h_{107} \underbrace{\nabla \varphi_3^{(47)}(x, y)}_{= \hat{x} b_3^{(47)} + \hat{y} c_3^{(47)}}$$

$$= \hat{x} (h_{62} b_1^{(47)} + h_{13} b_2^{(47)} + h_{107} b_3^{(47)})$$

$$+ \hat{y} (h_{62} c_1^{(47)} + h_{13} c_2^{(47)} + h_{107} c_3^{(47)})$$

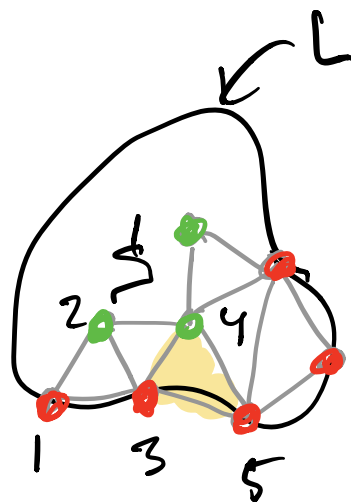


$$x_m^{(e)} = \frac{1}{3} \sum_{j=1}^3 x_j^{(e)}$$

$$y_m^{(e)} = \frac{1}{3} \sum_{j=1}^3 y_j^{(e)}$$

~ ~ ~

$$\begin{cases} -\nabla^2 E_z = k_t^2 E_z \\ E_z = 0 \end{cases} \quad \begin{array}{l} \text{in } \Omega \\ \text{on } L \end{array}$$



$$A_{\text{nat}} \Phi_{\text{nat}} = k_t^2 B_{\text{nat}} \Phi_{\text{nat}}$$

$$E_z(x, y) = \sum_{j=1}^N e_j \psi_j(x, y)$$

only
internal
node
coefficients

$$\Phi = \begin{bmatrix} 0 \\ e_{\text{nat},1} \\ 0 \\ \vdots \end{bmatrix}$$

all
coefficients